Towards Parallelized Convolutional Dictionary Learning
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Motivation
Sparse coding and dictionary learning are two powerful techniques used to develop efficient representations of images. These representations can be used in a wide variety of applications, including image restoration problems such as denoising, and the identification of features in biological or medical images. These techniques seek to learn a dictionary, a set of patches or convolutional filters, as well as a set of coefficients that constitute the sparse representation of a signal. The original signal can then be reconstructed by convolving the dictionary and sparse representation (operation denoted by $\ast$).

Parallelization
ADMM consensus allows us to pose the dictionary update problem as $k$ subproblems that can be solved independently from each other up until the consensus step, which averages the individual dictionaries created for the individual images into one overall dictionary, and therefore can be computed in parallel. We used the mpi4py package, which allows Python to use MPI functionality, to distribute the data over several nodes, with each rank holding the data from one signal.

Background
These methods are widely used in signal and image processing applications, since they provide a compact, low dimensional representation. The sparse coding problem can be stated as:

$$\arg\min_{x} \frac{1}{2} \sum_{i} d_{m} \ast x_{m} - s_{i}^{2} + \lambda \sum_{m} \|x_{m}\|_{1}$$

Where $d_{m}$ is a set of M dictionary filters, $a$ is the signal, $|x_{m}|$ is a set of coefficients maps, and the $\ell_{1}$ norm represents a sparsity inducing penalty function. The sparse coding problem can be solved with the Alternating Direction Method of Multipliers (ADMM). As each of the $k$ signals (images) will have its own sparse representation, we can solve all of the sparse coding problems in parallel, and then use these representations to calculate an updated dictionary.

We can state the dictionary update problem as

$$\arg\min_{d_{m}} \frac{1}{2} \sum_{k} \|d_{m} \ast x_{m} - s_{k}\|_{2}^{2} \quad \text{such that} \quad \|d_{m}\|_{2} = 1 \forall m$$

The dictionary filters are constrained so that their Euclidian norm must be equal to one to avoid the scaling ambiguity between $(d_{m} \ast x_{m})$ and $x_{m}$. Together, these two problems can be combined to form the dictionary learning problem, which is a biconvex optimization problem:

$$\arg\min_{(d_{m} \ast x_{m})} \frac{1}{2} \sum_{k} \|d_{m} \ast x_{m} - s_{k}\|_{2}^{2} + \lambda \sum_{m} \|x_{m}\|_{1} \quad \text{such that} \quad \|d_{m}\|_{2} = 1 \forall m$$

where we can alternate between solving for $D$ and $x$ until we reach a good enough solution or reach a target number of iterations.

Results

To our knowledge, this is the first experiment performed to test the efficacy of learned convolutional dictionaries based on the number of images from which they were generated as well as the number of filters they contain.

Conclusions

By using parallelization techniques to decompose the problem into independent tasks, we developed the code to enable the largest convolutional dictionary problem ever solved to our knowledge, containing 100 large (1024x1024) images. This code will be added to the open source SPORCO package to be freely used. We observed that as the number of images and filters used increased, so did the performance of the dictionary when performing sparse coding.

References


SPORCO-http://bwohlberg.github.io/sporco/

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