Quantum interaction of a few particle system mediated by photons

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D-Wave Debrief
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Proposed work and goals

- Familiarize with D-Wave computing capabilities;
- Analyze a few particle system mediated by photons;
- Milestones:
  - a free space evolution of a single quantum particle; same for a photon state.
  - a photon-electron interaction in the context of x-ray free electron laser startup;
  - the capability limit – how many electrons and photons could be included in the description?

Quantum state evolution on D-Wave

- Familiarization with D-Wave capabilities has lead me to the conclusion that *it is not suitable to study evolution;*
- It is however appropriate for finding steady states of a system;
- 1st milestone could not be reached at the current level of understanding.

Photon-electron interaction

- X-ray FEL startup is described by Hamiltonian:

$$H = \frac{1}{2\bar{\rho}}\hat{p}^2 + \bar{\rho}\left(\hat{a}e^{i\theta} + \hat{a}^\dagger e^{-i\theta}\right)$$

- $\hat{p}$ and $e^{i\theta}$ are electron operators; $\hat{a}^\dagger$ - photon creation operator
- Given quantum FEL parameter, $\bar{\rho} = 10$,what is a state of electrons and photons?

Photon-electron interaction: Approach 1

- Assuming classical field such that $\hat{a}^\dagger \rightarrow \alpha^*$ and electron states with a momentum $k = \{-1,0,1\}$, our Hamiltonian is:

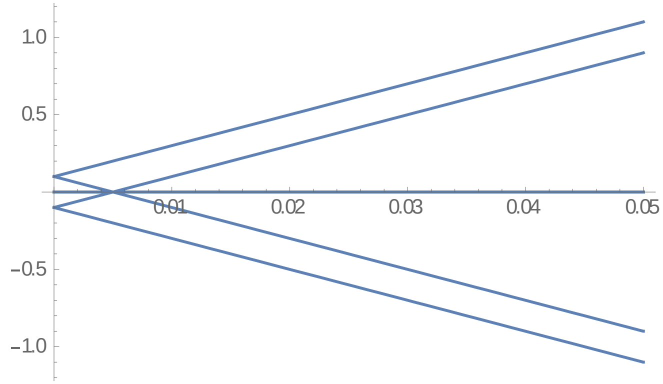
$$H = \begin{pmatrix} \frac{1}{2\bar{\rho}} & \bar{\rho}\alpha & 0 \\ \bar{\rho}\alpha^* & 0 & \bar{\rho}\alpha \\ 0 & \bar{\rho}\alpha^* & \frac{1}{2\bar{\rho}} \end{pmatrix}$$

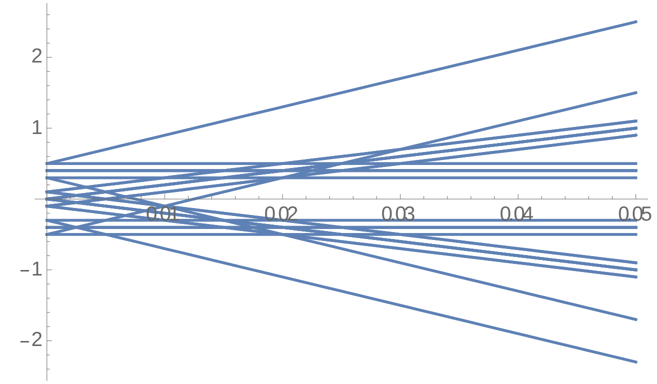
Photon-electron interaction: Approach 1

- If a basis state could be represented by a spin operator then an Ising Hamiltonian will be:

$$H = \frac{1}{2\bar{\rho}}(s_1 + s_3) + \bar{\rho}\alpha(s_1 s_2 + s_2 s_3)$$

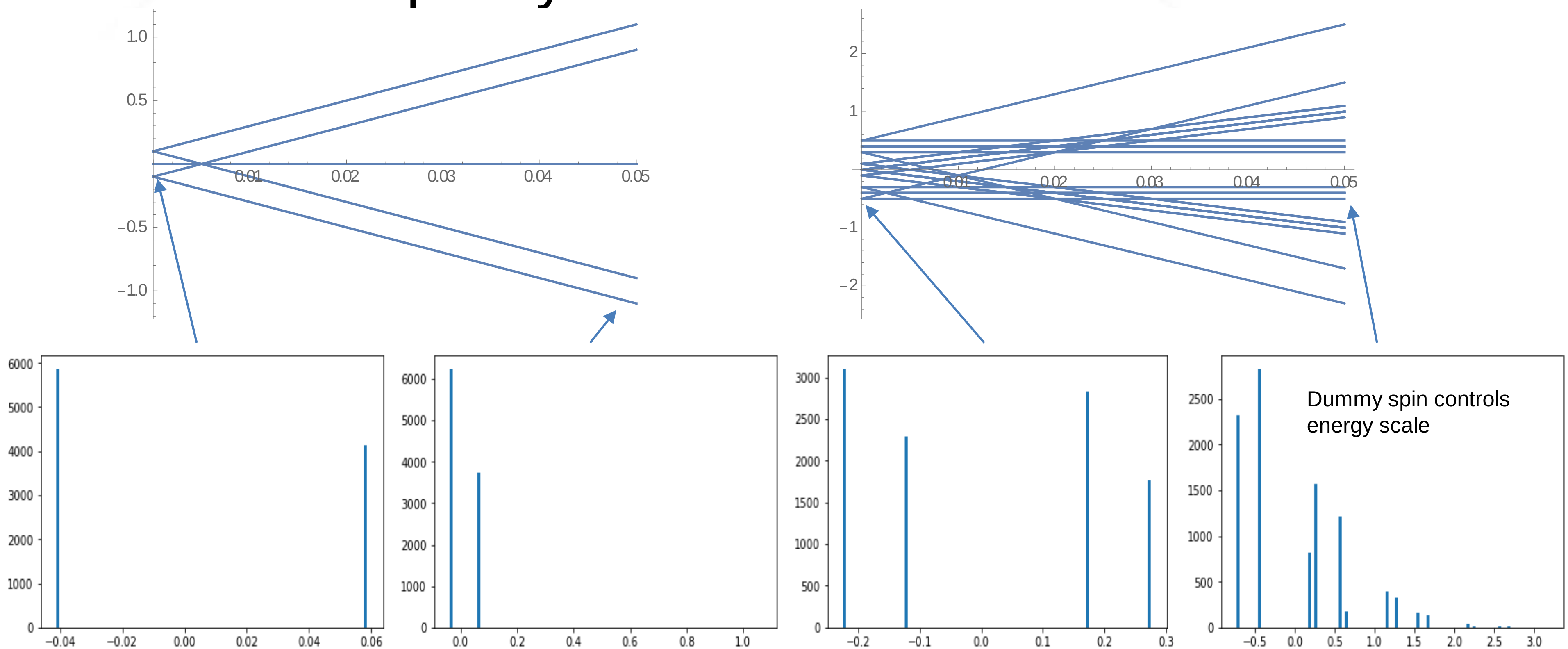
- The resulting Hamiltonian spectrum for 3 and 5:





Photon-electron interaction: Approach 1

- 3 and 5 spin systems:



Photon-electron interaction: Approach 2

- Let us represent $\hat{p}$ and $e^{i\theta}$ operators in a momentum basis:

$$\hat{p} = \sum_k k|k\rangle\langle k| \qquad \text{and} \qquad e^{i\theta} = \sum_k |k+1\rangle\langle k|$$

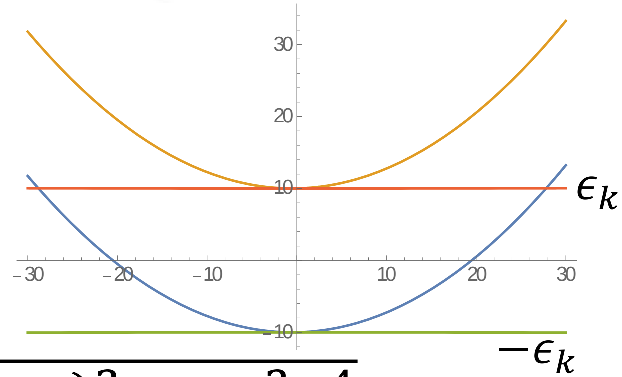
- The electron interaction with a classical field becomes:

$$H = \frac{1}{2\bar{\rho}}\sum_k \left(\frac{(k+1)^2}{2}|k+1\rangle\langle k+1| + \frac{k^2}{2}|k\rangle\langle k|\right) + \bar{\rho}\alpha\sum_k \left(|k+1\rangle\langle k| + |k\rangle\langle k+1|\right)$$

Photon-electron interaction: Approach 2

- We will thus relate a spin operator to each k-subspace:

$$H = \sum_k (E_k + \epsilon_k s_k)$$

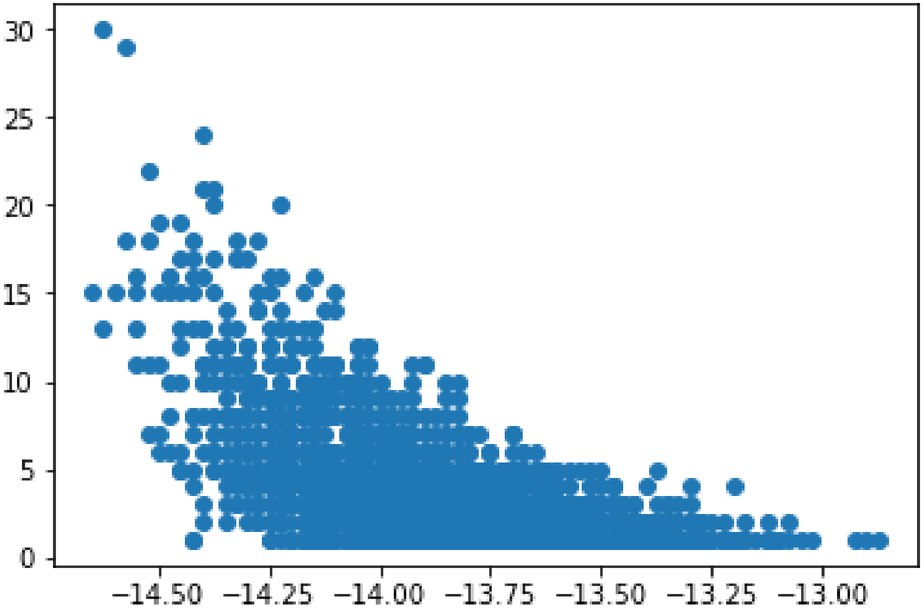


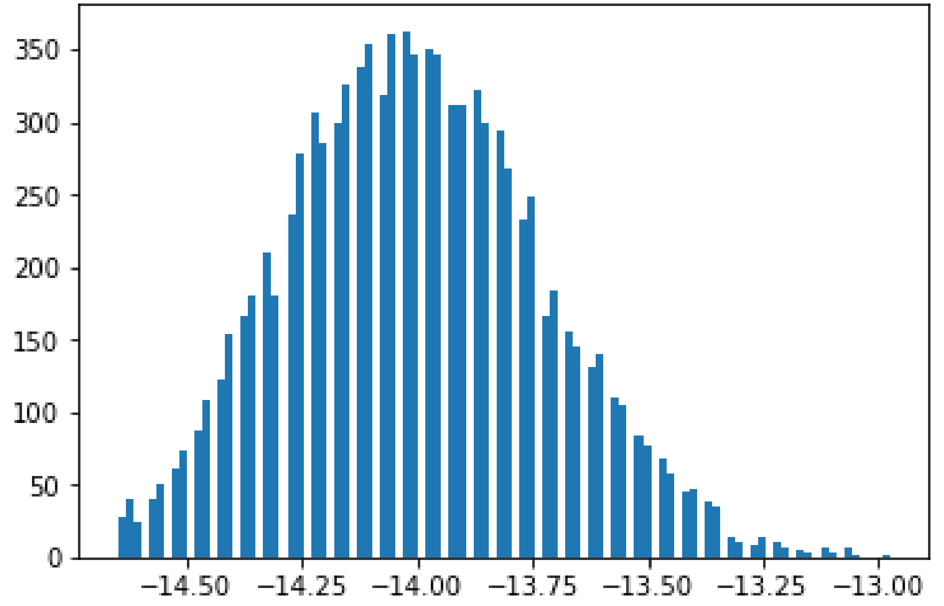
where $E_k = \frac{(k+1)^2+k^2}{8\overline{\rho}}$ and $\epsilon_k = \frac{\sqrt{(2k+1)^2+64\alpha^2\overline{\rho}^4}}{8\overline{\rho}}$.

- An expected soliton is $s_k = -1$ for all spins.

Photon-electron interaction: Approach 2

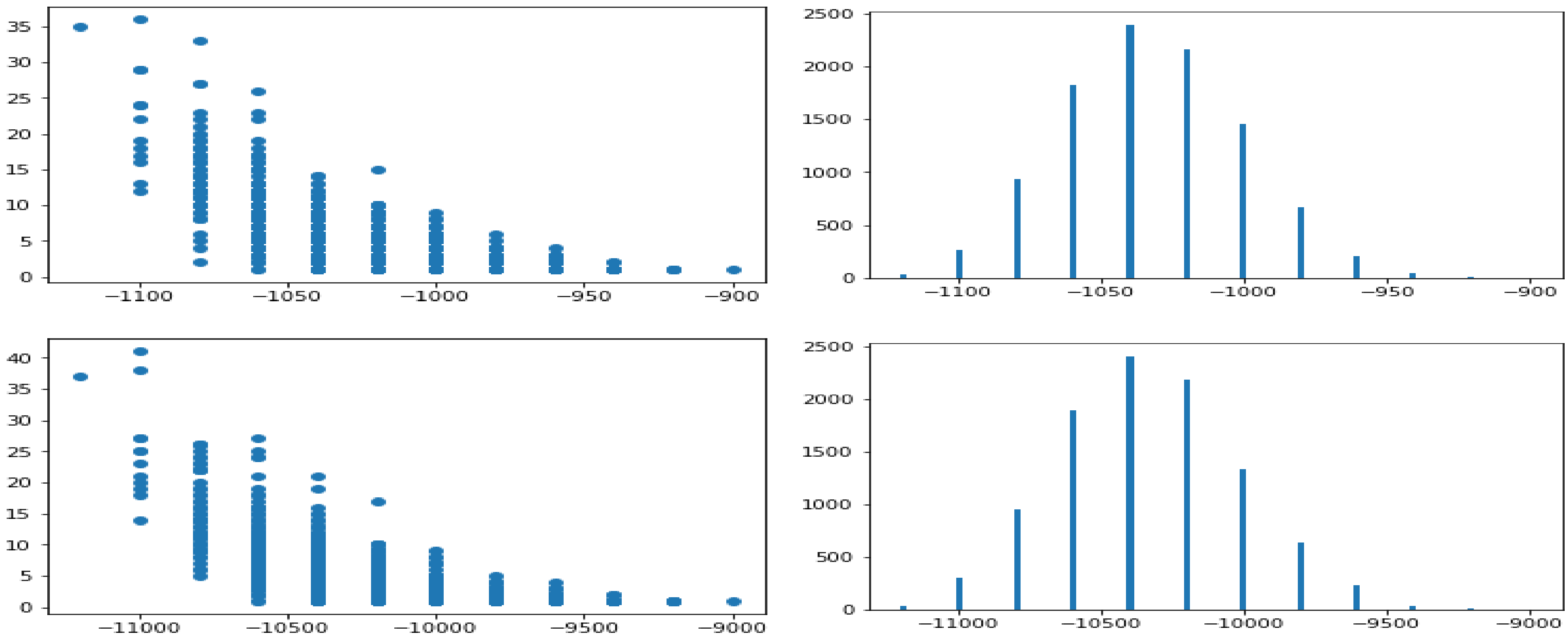
- Introducing a dummy spin, we can sample Boltzmann distribution beyond a ground state.
- In the absence of field and $k = \{-6, .., 5\}$:





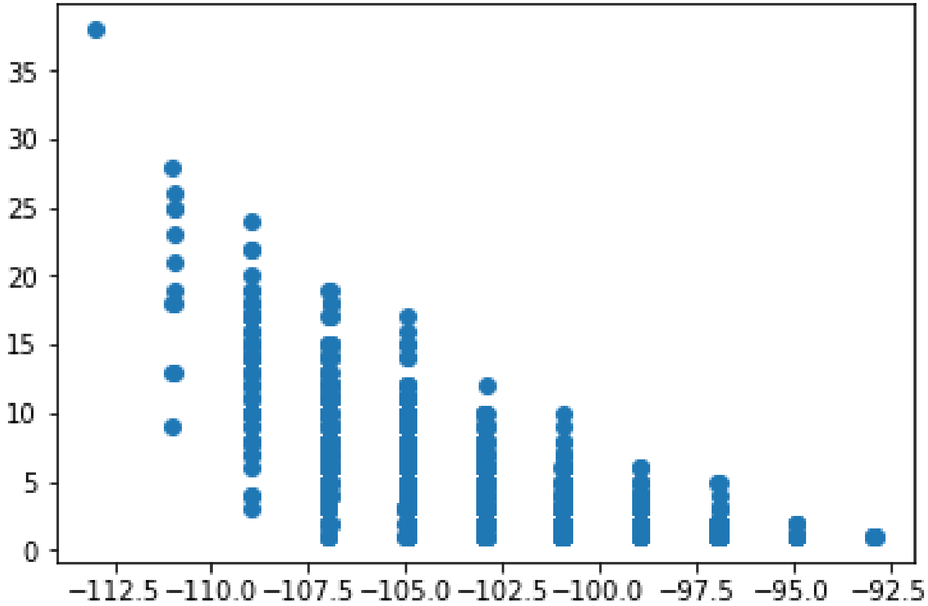
Photon-electron interaction: Approach 2

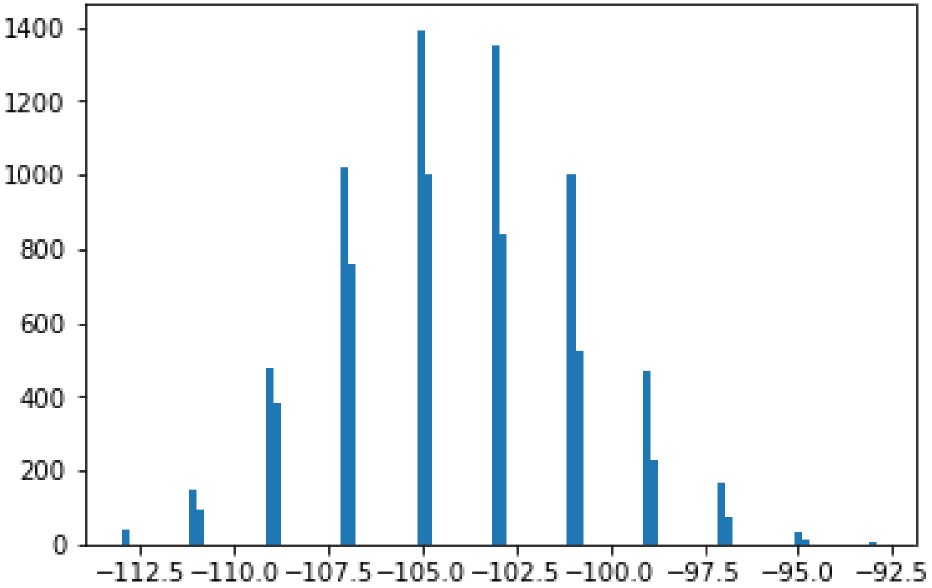
- Coherent field $\alpha = 1\ or\ 10$ saturates solutions:



Photon-electron interaction: Approach 2

- Coherent field less than 1, such as $\alpha = 0.1$, could be analyzed:

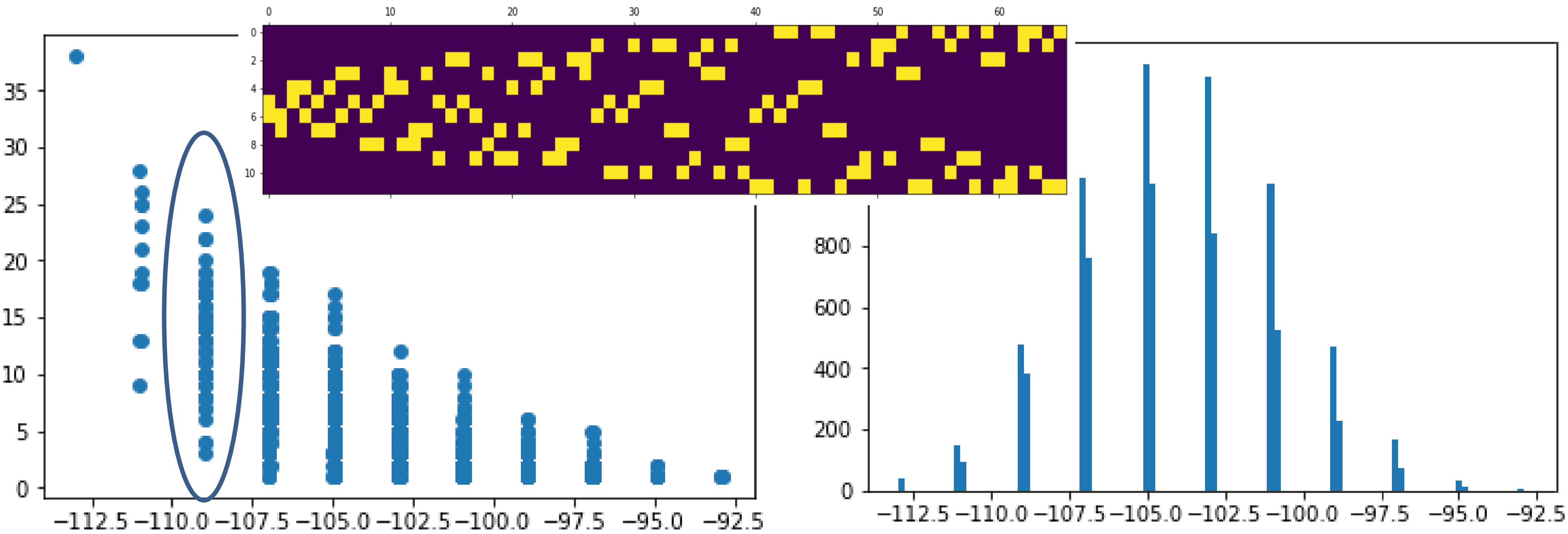




Number of lines match the number of spins used -1

Photon-electron interaction: Approach 2

- Coherent field less than 1, such as $\alpha = 0.1$, could be analyzed:



Number of lines match the number of spins used -1

Photon-electron interaction: Approach 3

- Let us represent $\hat{a}^\dagger$ and $\hat{a}$ operators in a photon basis:

$$\hat{a}^\dagger = \sum_n \sqrt{n+1}|n+1\rangle\langle n| \quad \text{and} \quad \hat{a} = \sum_n \sqrt{n+1}|n\rangle\langle n+1|$$

- The electron interaction with a classical field becomes:

$$H = \frac{1}{2\bar{\rho}} \sum_k k^2 |k\rangle\langle k| + \bar{\rho} \sum_{k,n} \sqrt{n} \left(|k+1, n-1\rangle\langle k, n| + |k, n\rangle\langle k+1, n-1| \right)$$

Photon-electron interaction: Approach 3

- We will thus relate a spin operator to each k-subspace and each n-subspace:

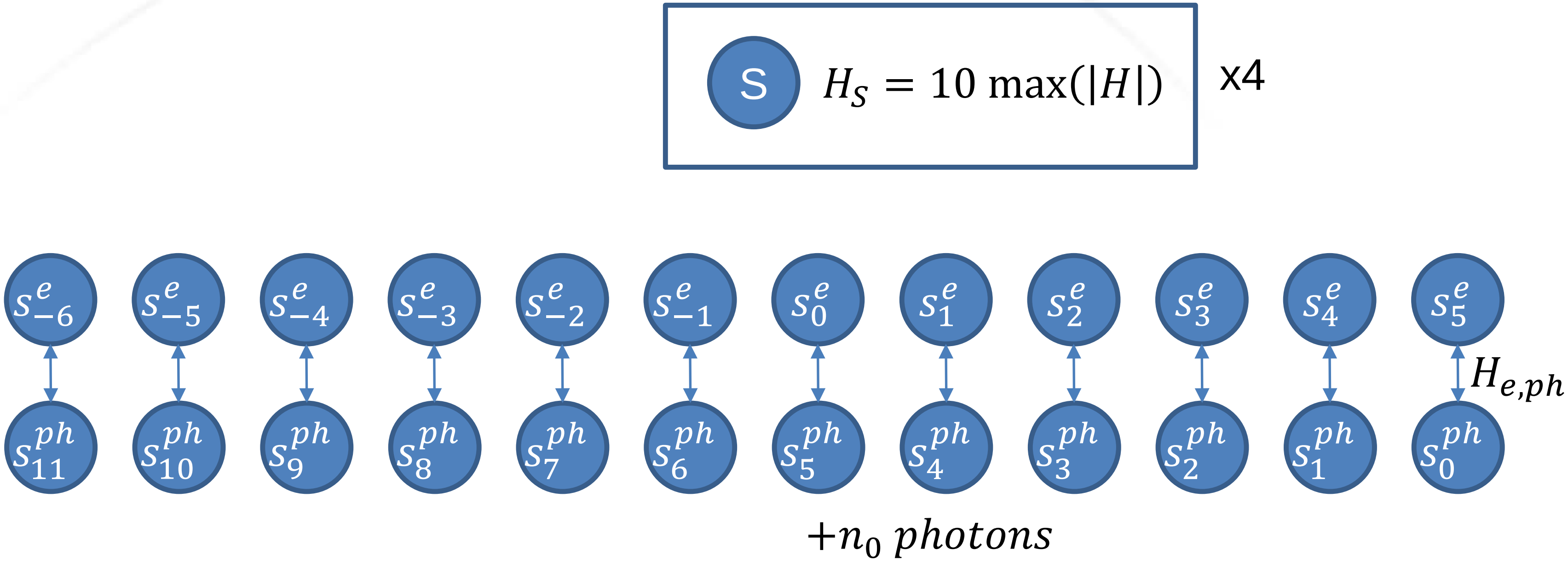
$$H = \frac{1}{2\bar{\rho}} \sum_{k} (2k+1) s_k^e + \bar{\rho} \sum_{k,n} \sqrt{n} s_k^e s_n^{ph}$$

where acceptable solution should have s_k^e and s_n^{ph} antiparallel!

- Restriction $k + n = const$ limits coupling to

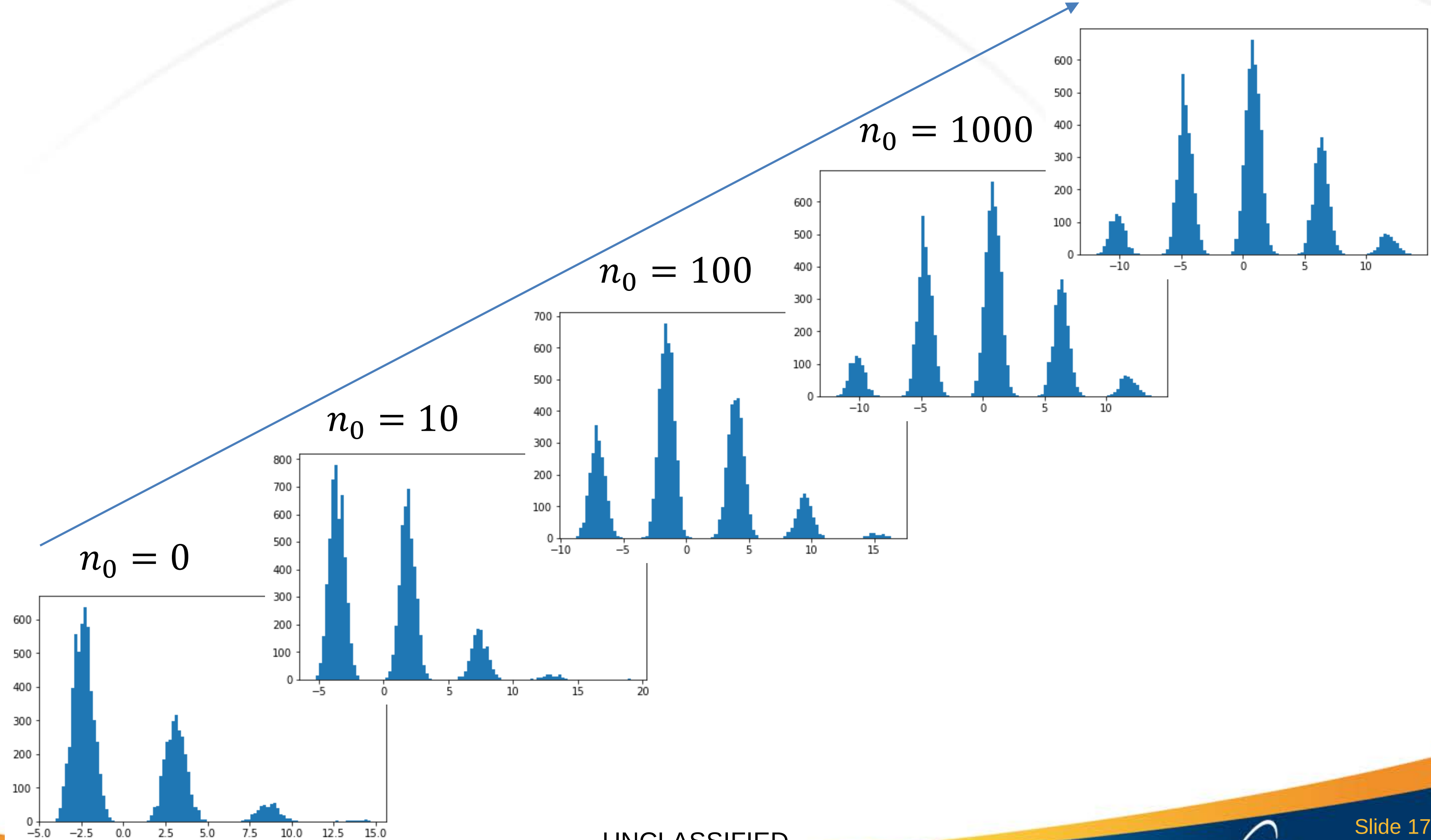
$$s_{k-1}^e \leftrightarrow s_{n+1}^{ph}, s_k^e \leftrightarrow s_n^{ph}, s_{k+1}^e \leftrightarrow s_{n-1}^{ph}$$

Photon-electron interaction: Approach 3

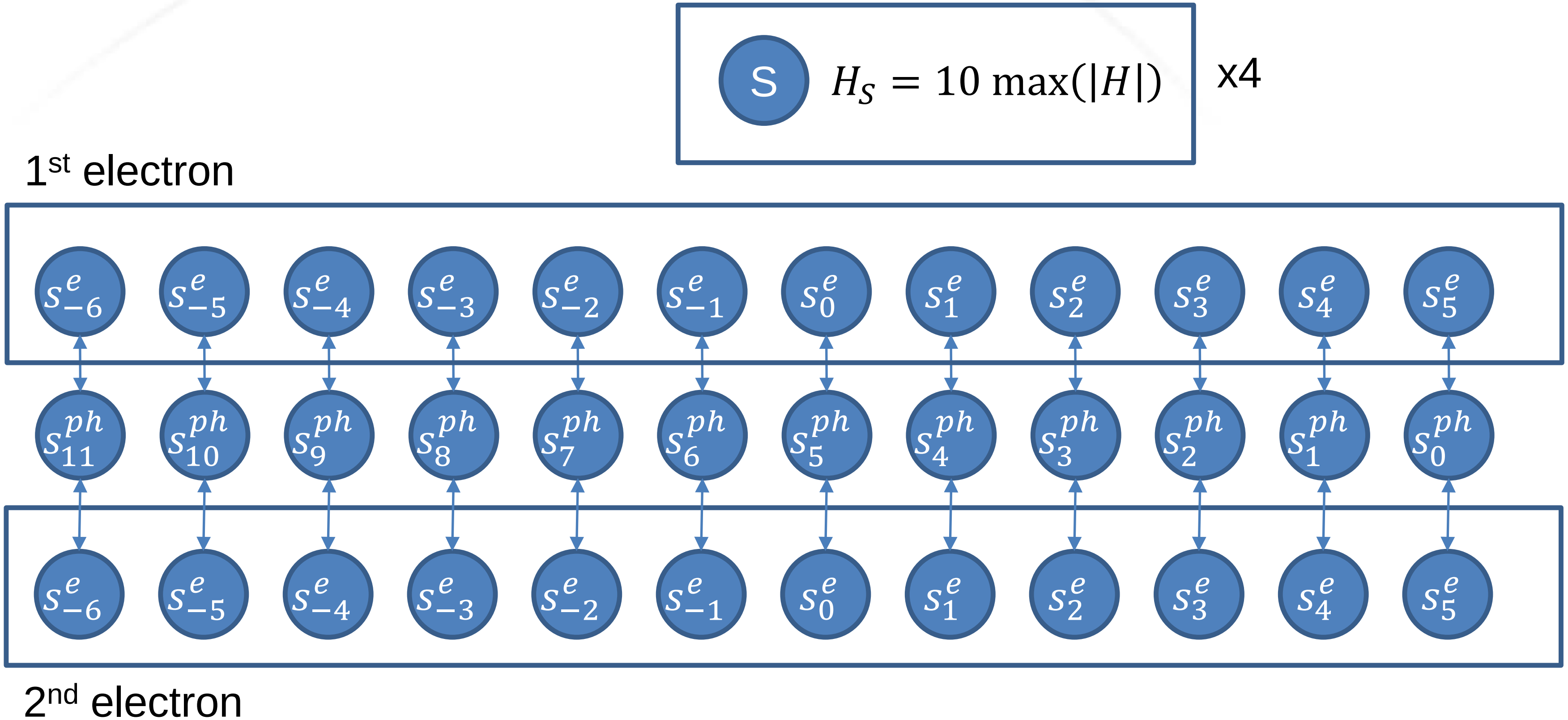


Antiparallel order is controlled by $C_{e,ph} = 6\max(|H|)$

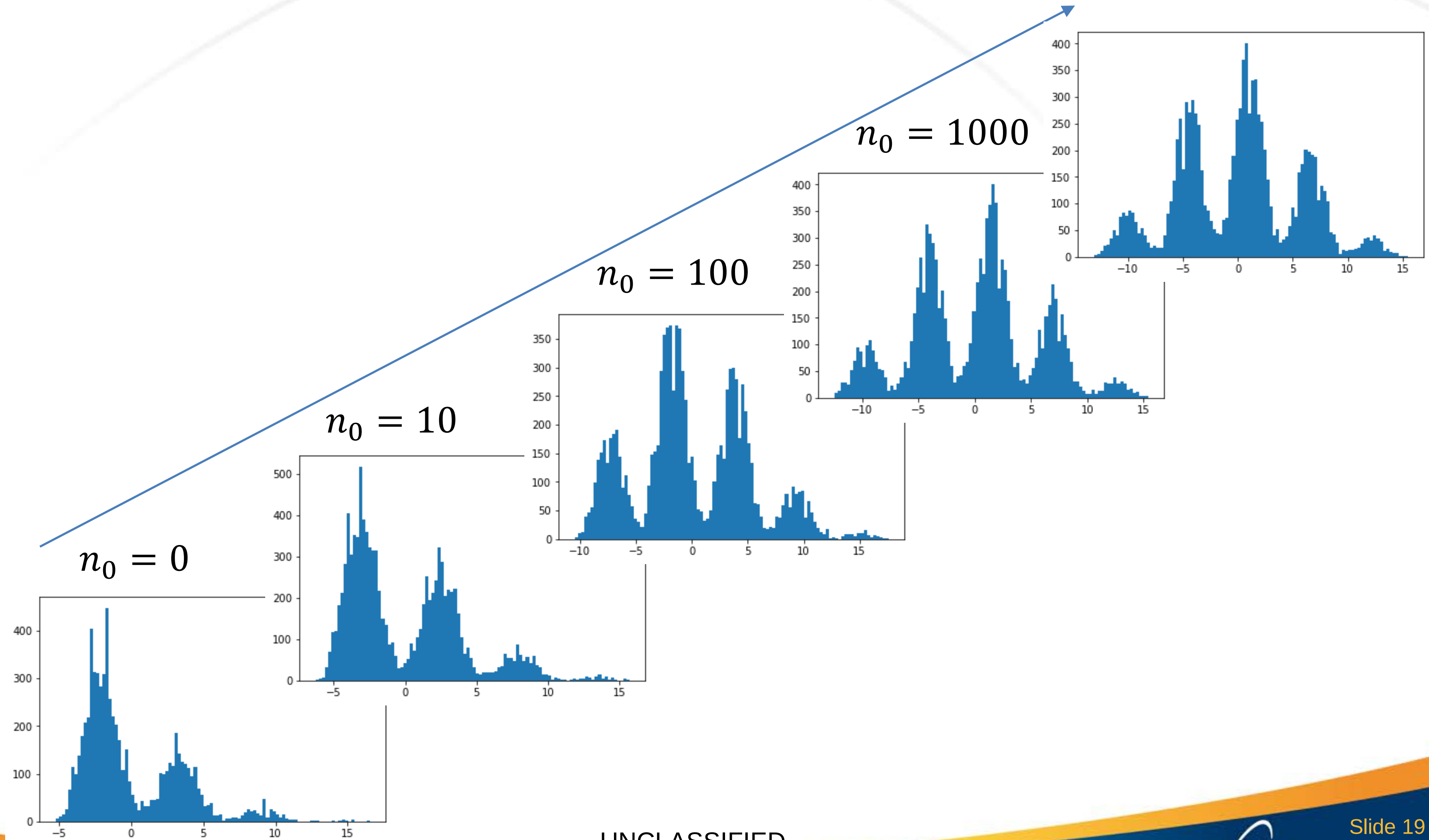
Photon-electron interaction: Approach 3



Photon-mediated interaction



Photon-mediated interaction



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Conclusions

- We could not study a state evolution on D-Wave;
- Approach 1: Reports energy spectrum and states that are hard to relate to original task;
- Approach 2: Reports electron momentum distributions for coherent amplitude <0.1;
- Approach 3: Reports electron momentum distributions and corresponding photon distributions.

Conclusions

- Photon-mediated interaction could be analyzed:
  - Adding new electrons requires $n + \sum_i k_i = const$;
  - Successful study requires cloning momentum spins or photon spins in order to allow for $n + k \neq const$;
  - Interpretation of momentum and photon distributions requires additional time to understand.