Compressible Turbulence: The Cascade and Nonlinear Scale Interactions

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We combined rigorous mathematics and very high resolution simulations to analyze nonlinear scale interactions of physical processes in compressible turbulent flows. We utilized and refined a novel coarse-graining approach, rooted in a commonly used technique in the subject of partial differential equations (PDE), to allow for the analysis of scale-coupling in highly compressible flows, and subsequently investigated the transfer of kinetic energy between different scales. We also carried out state-of-the-art high-resolution simulations of compressible turbulence on LANL's HPC clusters. By applying our mathematical tools to numerical data, we obtained the first empirical evidence that mean kinetic energy cascades conservatively beyond a transitional "conversion" scale range, despite not being an invariant of the compressible flow dynamics.

Turbulence is a phenomenon that pervades most liquid, gas, and plasma flows in engineering and nature. It often involves a huge range of spatio-temporal scales that cannot be simulated directly with today's computational resources–a limitation which is unlikely to cease in the foreseeable future. It is therefore imperative that we achieve a solid fundamental understanding of such flows in order to formulate realistic models. While the traditional Richardson-Kolmogorov-Onsager picture is a successful theory of incompressible turbulence, most flows in nature and engineering are characterized by significant compressibility effects. These range from high-speed flight, to nuclear fusion power reactors, combustion, star formation in molecular clouds, and supernovae.

Compared to incompressible flows, our theoretical understanding of compressible turbulence is still underdeveloped. It is known that such flows involve important physical processes such as shock formation, sound waves, distinct dissipation mechanisms, and additional nonlinearities that can significantly influence the dynamics and statistics of compressible turbulence.

Using tools from the field of partial differential equations (PDE), we were able to make tangible progress in understanding some of these basic physical processes [1,2]. A primary goal of PDE analysis is the formulation of models that can faithfully capture the effect of small (fast) scales on large-scale (slow) dynamics without explicitly resolving all degrees of freedom in the system. A novel and very general approach, inspired by methods from PDE analysis, was introduced by [3] to study nonlinear scale interactions in flow fields. It was further refined and extended to magnetohydrodynamic [4] and geophysical [5] flows. Guided by physical and mathematical considerations, we recently showed [1] how a Favre or density-weighted decomposition can be employed to extend the coarse-graining approach to highly compressible turbulence. A Favre filtered field is weighted by density as:

$$\tilde{a}_l(x) = \int dr\, G_l(r)\rho(x+r)a(x+r) \,/\, \int dr\, G_l(r)\rho(x+r)$$

where $G_l(\mathbf{r})$ is a normalized convolution kernel.

By employing this mathematical technique to analyze numerical data, we were able to gain new insights into the physics of compressible turbulence [2]. In inviscid compressible flows, only the sum of kinetic and internal energy is a global invariant. The idea of a cascade, a central notion in incompressible turbulence, is therefore without physical basis since kinetic energy is not conserved separately. Compressible flows allow for an exchange between kinetic and internal energy through two mechanisms: viscous dissipation and pressure dilatation. While the former process is localized to the smallest scales as in incompressible turbulence, the latter is a hallmark of compressibility and can a priori allow for an exchange at any scale through compression and rarefaction. By applying the coarse-graining technique to high-resolution 3D simulations, we obtained the first direct evidence that mean kinetic energy cascades conservatively beyond a transitional scale range, despite not being an invariant of the compressible flow dynamics [2]. The key quantity we measured is pressure dilatation co-spectrum (Fig. 1). The plot shows that it decays at a rate faster than k^{1} as a function of wavenumber. This is sufficient to imply that mean pressure dilatation acts primarily at large scales and vanishes at small scales beyond the transitional "conversion" scale-range. At these

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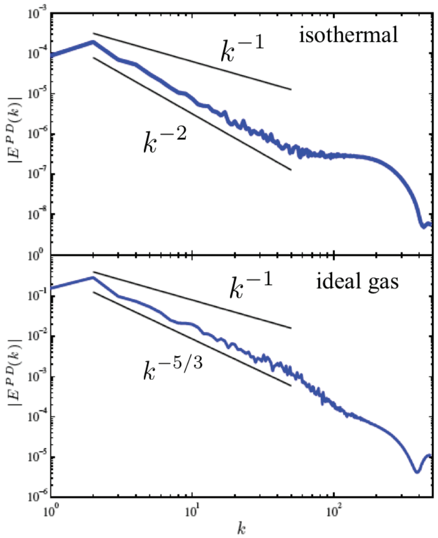

Fig. 1. Plots of pressure dilatation co-spectrum, showing a decay faster than k^{-1} as a function of wavenumber in isothermal (top) and ideal gas (bottom) turbulent flows. This is sufficient to imply that mean pressure dilatation acts primarily at large scales and vanishes at small enough scales, over which mean kinetic and internal energy budgets statistically decouple and mean kinetic energy undergoes a scale-local conservative cascade.

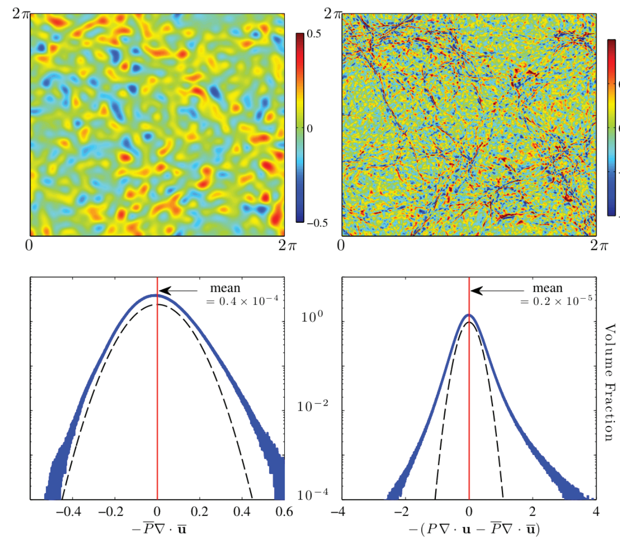

Fig. 2. Pointwise visualization of pressure dilatation in a 1024^2 cross-section of our 3D simulation at large (top-left) and small (top-right) scales. Lower two panels show the respective distribution of pressure dilatation values in the simulation domain. While pressure dilatation has large values at small scales in the vicinity of shocks, such small-scale contributions vanish when averaging over the flow domain due to cancellations between compression and rarefaction regions.

smaller scales, mean kinetic and internal energy budgets statistically decouple, giving rise to an inertial range over which mean kinetic energy undergoes a scale-local conservative cascade.

Our work is based on a rigorous mathematical analysis of the dynamics supported by high-resolution numerical simulations. The results and the physical picture we are advancing might seem counterintuitive at first. After all, a hallmark of compressible turbulence is the formation of shocks and the generation of sound waves. Such phenomena involve compression and rarefaction at all scales and are not restricted to large scales. However, our results concern global pressure dilatation, $-\langle P\nabla \bullet u \rangle$, and not the pointwise quantity. We showed (Fig. 2) that while pressure dilatation has large values at small scales in the vicinity of shocks, such small-scale contributions vanish when averaging over the flow domain due to cancellations between compression and rarefaction regions. Our findings should have significant implications on devising reduced models of compressible turbulence as well as providing physical insight into this rich problem.

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