Nonlinear Cascades in Rotating Stratified Boussinesq Flows

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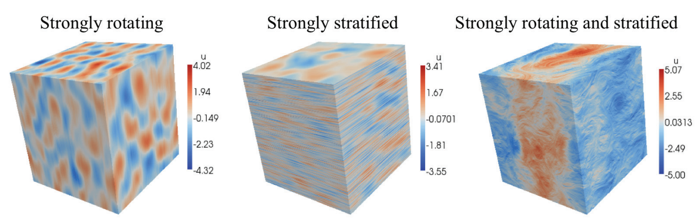
We employ state-of-the-art numerical simulations hand-in-hand with a very general mathematical approach to analyze nonlinear scale interactions in geophysical flows. We use high-resolution, high-Reynolds number direct numerical simulations of rotating stratified Boussinesq flows carried out on the 0.6-petaflops, 164-K compute cores of the IBM Blue Gene/P at ANL. We study the transfer of quadratic invariants, energy and potential enstrophy between different scales in three different regimes of stratification and rotation. Our results include the first reported measurement of potential enstrophy flux across scales. They also show joint cascades of energy and potential enstrophy from large to small scales in marked departure from 2D turbulence, which is often used as a model of geophysical flows.

Sea-water density in the oceans is ≈ 1 g/cm^3 with relatively small variations as a function of depth (of order 10^{-2} g/cm^3 or less), and even smaller fluctuations due to fluid motion (of order ≈ 4 km or less) [1]. This makes the Boussinesq equations a very accurate description of the flow dynamics in Earth's oceans. Yet the equations are prohibitively expensive to simulate–which is why most of today's climate models of the ocean rely on further simplifications. Such reduced models are a very good description for the horizontal quasi-2D fluid motions at scales much larger than the ocean depth of ≈ 4 km, but do not capture the 3D turbulent fluid motions at smaller length scales. Furthermore, their predictive power fails on long time scales, over which the effect of small scales on the global flow dynamics cannot be neglected and is, in fact, an essential factor in phenomena such as the thermohaline circulation. Such shortcomings of existing models demonstrate the need for a better physical understanding of the nonlinear coupling between the motions at different length scales, and in various regimes of rotation and stratification in turbulent Boussinesq flows. The progress we made on this front [2] relied on two key tools: state-of-the-art numerical simulations and a powerful mathematical technique to probe the data.

Fig. 1. Boussinesq flow visualizations in three distinct regimes of rotation and stratification.



Thanks to a DOE "Innovative and Novel Computational Impact on Theory and Experiment" award (PI: S. Kurien), we carried out a series of computationally intensive Boussinesq simulations (Fig. 1) on the IBM Blue Gene/P at the ANL Leadership Computing Facility. Details of the simulations are described in [3].

We also used a novel and very general mathematical approach, introduced by [4], to study fluid flows. It was further refined and extended to magnetized [5], geophysical [2], and compressible [6] flows. The method itself is simple. For any field $\mathbf{a(x)}$, a "coarse-grained" or (low-pass) filtered field, which contains modes at length-scales > l, is defined as:

$$\bar{a}_l(x) = \int dr\ G_l(r)\ a(x + r)$$

where $G_l(\boldsymbol{r})$ is a normalized convolution kernel. The coarse-graining operation may be interpreted as a local space average. Coarse-grained dynamical equations can then be written to describe the large-scale flow at every point $\boldsymbol{x}$ in the domain, at every instant in time $\boldsymbol{t}$, and at scales $> l$, for arbitrary l (Fig. 2). The approach, therefore, allows for the simultaneous resolution of dynamics both in scale and in space. Moreover, coarse-grained equations describe a range of large scales whose dynamics is coupled to the small-scale through so-called sub-scale terms. These terms depend inherently on the unresolved dynamics that has been filtered out. Traditional modeling efforts focus on devising closures for such terms that are plausible but whose regime of applicability and validity is inevitably unknown. A key feature of our formalism, which distinguishes it from such modeling efforts, is in developing a physical understanding of the sub-scale terms and estimating their contributions as a function of the resolution scale L through exact mathematical analysis and direct numerical simulations. The approach allows for studying the nature of coupling that exists between different scales and for unraveling certain scale-invariant or universal features in the dynamics. Furthermore, while traditional modeling efforts seek to formulate faithful models to describe the

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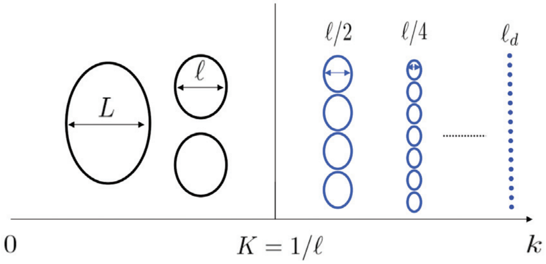

Fig. 2. A schematic depiction of the coarse-graining approach. Length scales in the system are partitioned into large, $\geq l$, and small, $< l$. Wavenumber, k, is inversely proportional to length scale; $k \sim l^{-1}$. The largest scale, that of the system's size, is L. Below scale l_d, the dynamics is linear and modes are uncoupled. Length l represents the smallest scale that is resolved after coarse-graining. Scales < l (in blue) are averaged out.

largest scales possible, typically of order L, our interest is to probe *all scales in the system*, including limits of small $l << L$. For example, in large-scale ocean simulations, the dynamical resolution is often variable in space due to adaptive mesh techniques and it is, therefore, imperative that sub-scale models used in these codes accurately reflect the latent physics as a function of resolution. Fundamental studies using the coarse-graining approach have a direct bearing on such issues and their findings may be naturally and easily translated to those modeling efforts in climate research.

We showed in [2] that, contrary to common belief, turbulent flows under strong rotation and/or stratification exhibit salient departures from 2D turbulence. In particular, an important finding of our work showed the existence of a joint cascade of energy and potential enstrophy, another dynamical invariant, from large to small scales (Fig. 3). Furthermore, these joint cascades exhibited pronounced but similar degrees of anisotropy, which are absent from the treatment of turbulent dissipation in current ocean models. The right three panels in Fig. 3 are the first measurements of potential enstrophy flux in rotating stratified Boussinesq flows. They can be regarded as the first empirical confirmation of analytical results in [7] which derived an exact law for potential enstrophy flux in physical space as a function of scale.

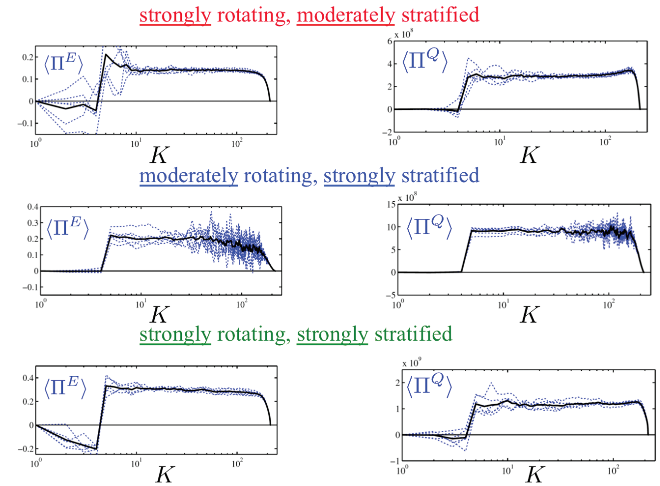

Fig. 3. Plots showing joint cascades of energy and potential enstrophy from large to small scales in all three regimes of rotation and stratification.

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