Non-Hermitian Description of a Superconducting Phase Qubit

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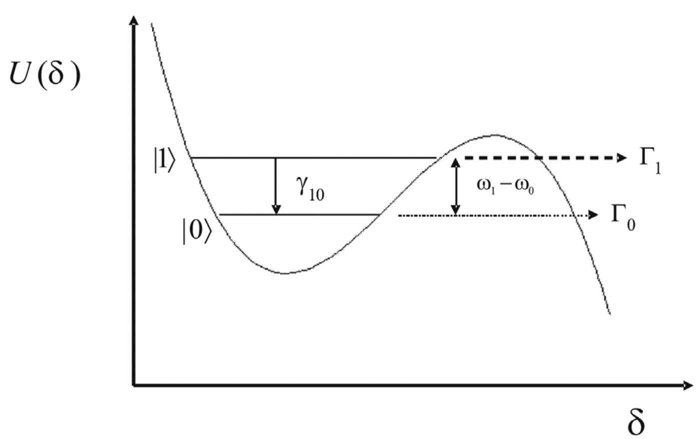
An approach based on a non-Hermitian Hamiltonian to describe the process of measurement by tunneling of superconducting phase qubit states is presented. We obtain simple analytical expressions which describe the dynamics of measurement, and compare our results with those experimentally available. In particular, we show that even for a single qubit, the analytical expressions simplify the analysis of the dynamics in comparison with the density matrix approach.

A superconducting phase qubit has significant potential applications in quantum information processing due to its convenient functional operations, its capability to reduce the effects of noise and decoherence, and its future utilization in scalable quantum computers. One of the crucial problems in quantum computation is a measurement of qubit states. The tunneling effect is used in many experiments with superconducting qubits for measuring the qubit states. Usually, to describe the dynamics of the qubit state measurement by using a tunneling from the upper level, one utilizes a density matrix approach. In spite of many advantages, this approach has some evident disadvantages: (1) the number of density matrix elements grows exponentially with the number of qubits, (2) some relaxation and decoherence coefficients are not included self-consistently, (3) the coherent (collective) effects of interaction with the continuous part of the spectrum is not always properly described, and (4) interference between different tunneling channels is not always simple to take into account. In many respects, our approach, based on the non-Hermitian Hamiltonians, addresses all these issues.

Fig. 1. A superconducting phase qubit radiated by a microwave field. $\Gamma_1(\Gamma_0)$ denotes the tunneling rate from level $|1\rangle(|0\rangle)$, and γ_{10} denotes the rate of energy relaxation from $|1\rangle$ to $|0\rangle$. The model is applicable to a Josephson junction radiated by a monochromatic microwave.



Non-Hermitian Hamiltonians naturally appear when the energy spectrum of a quantum system can be formally represented by both quasi-discrete (intrinsic) and continuous parts, and one performs a projection of the total wave function onto the quasi-discrete part of the spectrum. In this case, the corresponding intrinsic energy levels acquire finite widths, which are associated with transitions from the intrinsic states to the continuum. Then, the dynamics of the intrinsic states, including tunneling effects (coherent or incoherent) from these metastable intrinsic states to the continuum, can be described by the Schrödinger equation with an effective non-Hermitian Hamiltonian.

We propose an approach [1] based on non-Hermitian Hamiltonians to describe the dynamics of measurement of the superconducting phase qubit states by tunneling into their continuum (Fig.1).

We obtained simple analytical expressions for the probability of tunneling into the continuum. We showed that even for a single-phase qubit the analytical expressions simplify the analysis of the dynamics in comparison with the density matrix approach. We also demonstrated that the effect of the interference of tunneling channels can be easily described by using our approach based on the non-Hermitian Hamiltonian. We compared our theoretical results (Figs. 2 and 3) with experiments on superconducting phase qubits [2,3] and demonstrated that our theoretical predictions are in good agreement with the available experimental data.

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In conclusion, we have presented a non-Hermitian description of the dynamics of measurement by tunneling of a superconducting phase qubit. We have shown that our theoretical predictions are in a good agreement with the experimental data obtained in various experiments on superconducting phase qubits. We believe that our approach based on the non-Hermitian Hamiltonians can, in many cases, and especially for a multi-qubit register, simplify a theoretical description of the dynamics of a phase qubit measurement by tunneling to the continuum.

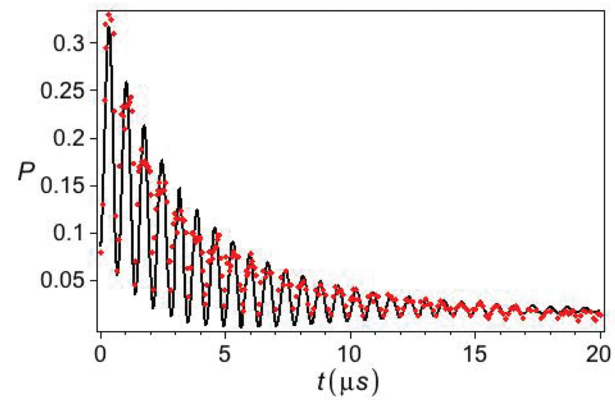

Fig.3. The probability, P(t), of tunneling from the upper level to the continuum as a function of time. The diamonds (red) are the experimental data [3] and the black solid line is our theoretical prediction [1].

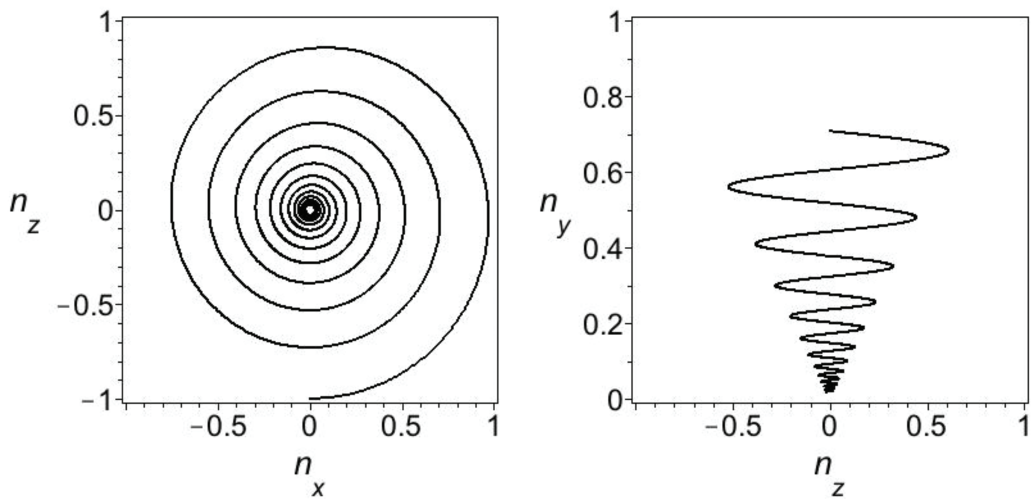

Fig. 2. Theoretical predictions of the evolution of the Bloch vector obtained from the Schrödinger equation with the non-Hermitian Hamiltonian [1] are in a good agreement with the experimental results [2].

[1] Nesterov, A.I., et al., *IJQI* **6**, 895 (2010).
[2] Steffen, M., et al., *Phys Rev Lett* **97**, 050502 (2006).
[3] Yu, Y., et al., *Science* **296**, 889 (2002).

Funding Acknowledgments
Intelligence Advanced Research Projects Activity (IARPA)