Balanced and Imbalanced Magnetohydrodynamics Turbulence

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MHD is a simple one-fluid description of a conductive media. Most of the ordinary matter in the universe is ionized–that is, in a state of plasma, due to the presence of ionizing radiation such as ultraviolet, X-rays, and cosmic rays. Turbulent plasma is known to generate its own magnetic fields and most of the observed astrophysical plasma, in such objects as ISM, ICM, accretion disks, and molecular clouds, is magnetized to a degree such that the magnetic field is dynamically important. MHD can also be helpful in some plasma experiments that are either large-scale or feature large density, so that the mean-free path of the particle is much smaller than the scale of the problem.

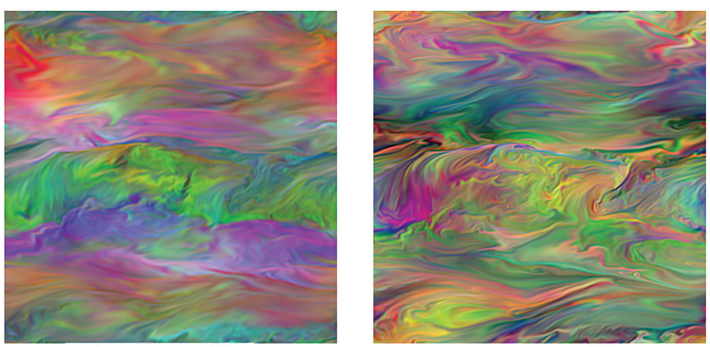
Studying magnetohydrodynamics (MHD) turbulence is interesting for two reasons. First, in MHD, just as in hydrodynamics, turbulence changes the overall properties of the flow by changing transport properties, such as effective viscosity or thermal conduction. Second, the magnetic perturbations that are the essential part of the MHD turbulence cascade affect the dynamics of charged particles, most notably cosmic rays. Indeed, in many of the astrophysical environments that we know, cosmic rays are dynamically important. Efficient acceleration of cosmic rays in shock require the back-reaction of cosmic rays to turbulence. Also, MHD turbulence is a well-observed phenomenon. Magnetic, velocity, and density perturbations covering a huge range of scales have been observed in the interstellar medium (ISM), intracluster medium (ICM), and solar wind.

A big difference between hydrodynamics and MHD is that ideal MHD equations have three ideal invariants: energy, cross-helicity, and magnetic helicity. While magnetic helicity is often important on larger scales, its influence on the inertial range of turbulence can be largely ignored. Cross-helicity is different. The conservation of energy and cross-helicity could be reformulated as the conservation of two Elsasser energies $(\omega^{\pm})^2$, where Elsasser variables are defined as:

$\mathbf{w}^+ = \mathbf{v} + \mathbf{B} / \sqrt{4\pi\rho}$ and

$\mathbf{w}^- = \mathbf{v} - \mathbf{B} / \sqrt{4\pi\rho}$

Fig. 1. A slice through a 3D imbalanced MHD simulation showing the structure of Elsasser fields. Right: dominant field, left: sub-dominant field. Magnetic field is directed horizontally.



$\mathbf{w}^+ = \mathbf{v} + \frac{\mathbf{B}}{\sqrt{4\pi\rho}}$ $\qquad$ $\mathbf{w}^- = \mathbf{v} - \frac{\mathbf{B}}{\sqrt{4\pi\rho}}$

Perturbations of $\mathbf{w}^+$ propagate against local mean magnetic field, whereas perturbations of $\mathbf{w}^-$ propagate along the field. We often say that turbulence is balanced when the flow of $\mathbf{w}^+$ statistically balances the flow of $\mathbf{w}^-$, which corresponds to the limit of zero cross-helicity. In nature, however, MHD turbulence is very often imbalanced, resulting from the presence of a strong localized source of perturbations, for example, the central engine of active galactic nuclei. In our own solar system, solar wind turbulence is measured to be imbalanced, because most perturbations are emitted from the Sun.

Imbalanced MHD turbulence has been largely unexplored until recently. Similar to the standard phenomenology of hydrodynamic turbulence, MHD has the Goldreich-Sridhar model [1]. This model, however, only treats the balanced case and is conceptually incomplete; because turbulence is a stochastic phenomenon, an average zero cross-helicity does not preclude fluctuations of this quantity in the turbulent volume. In this situation, studying imbalanced turbulence with direct numerical simulations provides valuable hints on imbalanced dynamics.

We used the highly-parallelizable, very precise MHD pseudospectral code that was described in great detail in our earlier publications [4-5]. We performed the highest resolution MHD simulations to date—in the imbalanced case, resolution was up to 1536^3, and in the balanced case, as large as $3072^2 \times 1024$. The balanced runs were evolved, typically for 10 Alfvenic times, and the imbalanced runs were evolved for 10–40. The energy injection rates were kept constant. Figure 1 features the slice from a 3D simulation of imbalanced MHD turbulence, which

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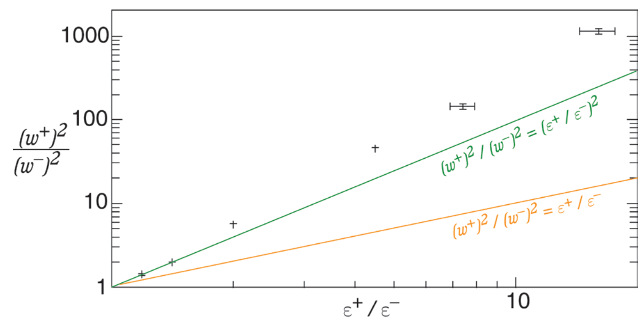

Fig. 2. The energy imbalance versus dissipation rate imbalance. Error bars are mostly due to fluctuations in time.

demonstrated that the structure of Elsasser fields is different due to imbalance.

One of the most robust quantities in numerical simulations of MHD turbulence is the energy cascading rate or dissipation rate. In hydrodynamic turbulence, the dissipation rate and the spectrum of velocity are connected by the well-known Kolmogorov constant: $E(k) = C_K\varepsilon^{2/3}k^{-5/3}$. The important fact that strong hydrodynamic turbulence dissipates in one dynamic time scale l/v is reflected by C_K being close to unity (~1.6). In MHD turbulence, however, there are two energy cascades (or "Elsasser cascades") and there are two dissipation rates, ε^+ and ε^-. The question of how these rates are related to the velocity-like Elsasser amplitudes $\mathbf{w}^+$ and $\mathbf{w}^-$ is one of the central questions of imbalanced MHD turbulence.

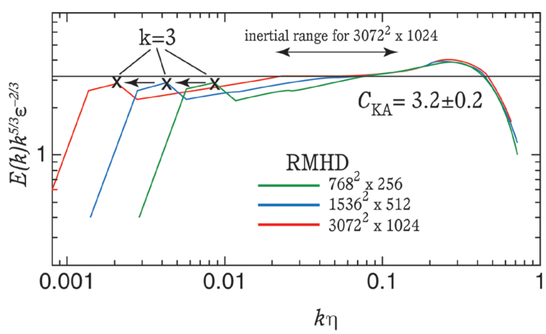

Fig. 3. The dimensionless compensated spectra in balanced simulations plotted versus dimensionless $k\eta$ where η is a Kolmogorov scale. We reproduce the asymptotic -5/3 slope of MHD turbulence. The Kolmogorov constant for this purely Alfvenic turbulence is $C_{KA} \approx 3.2$, which is much higher than for hydroturbulence. This fact was missed in earlier lower resolution simulations. Also, higher C_K means less efficient energy transfer, which is consistent with our picture of diffuse locality [5]. This is a first measurement of the Kolmogorov constant in MHD turbulence.

The Goldreich-Sridhar model predicts that in the balanced case the cascading is strong and each wave is cascaded by the shear rate of the opposite wave, that is, $\varepsilon^+ = (w^+_l)^2 w^-_l / l, \varepsilon^- = (w^-_l)^2 w^+_l / l$. It is similar to the Kolmogorov cascade with w's replacing v. If this model still works for the imbalanced case, we can obtain $(w^+)^2 / (w^-)^2 = (\varepsilon^+ /\varepsilon^-)^2$, which was proposed in [2]. Figure 2 shows the relation between the ratio of energies and ratio of fluxes from direct numerical simulations. As we see, most points lie above the prediction of [2], which is consistent with our model's [3] prediction. In [3] we argued that the classical critical balance of [1] becomes inconsistent in the imbalanced case and a different relation should be used–this was further confirmed by direct measurements of anisotropy [4], which showed different anisotropies for $\mathbf{w}^+$ and $\mathbf{w}^-$, while [2] was predicting the same anisotropy. Another model, based on "dynamic alignment" was predicting a viscous-type dissipation law $(w^+)^2 / (w^-)^2 = \varepsilon^+ /\varepsilon^-$ and is completely inconsistent with numerics.

Another challenging problem is to measure the Kolmogorov constant of MHD turbulence. Earlier simulations showed a spectral slope, which was shallower that Kolmogorov's -5/3. Some models, based on "dynamic alignment" claimed that the asymptotic slope is not -5/3, but rather -3/2. As we discovered, shallower slopes in previous simulations were artifacts of a low resolution. We also discovered that MHD turbulence is less local that hydrodynamic turbulence [5]–a higher resolution is therefore necessary. Figure 3 shows a measurement of the Kolmogorov constant in a purely Alfvenic turbulence (also called reduced MHD). From this measurement we can derive a Kolmogorov constant for full MHD, taking the amount of slow mode between 1 and 1.3 of Alfvenic mode. The final value for the Kolmogorov constant for MHD is, therefore, $C_K = 4.1 \pm 0.3$. Remarkably, it is much higher than the hydrodynamic value of 1.6.

[1] Goldreich, P., Sridhar, S., *Astrophys J* **438,** 763 (1995).
[2] Lithwick, Y., et al., *Astrophys J* **655,** 269 (2007).
[3] Beresnyak, A., Lazarian, A., *Astrophys J* **678,** 961 (2008).
[4] Beresnyak, A., Lazarian, A., *Astrophys J* **702,** 460 (2009).
[5] Beresnyak, A., Lazarian, A., *Astrophys J* **702,** 1190 (2009).

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