Staging Two Coolers through a Quarter-Wave Tube

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ABSTRACT

Stirling coolers have high efficiency, in part because the displacer recovers acoustic power \((pV)\) work from the gas near the cold heat exchanger, delivering it to the ambient end of the cooler. However, pulse-tube coolers have lower cost because they have no cold moving parts. In a new scheme with both high efficiency and no cold moving parts, acoustic power from the gas near the cold heat exchanger of a first cooler is recovered via a tube almost a quarter-wavelength long. The long tube’s acoustic characteristics boost the pressure amplitude, lower the volume-flow-rate amplitude, and alter the time phasing between them, while transmitting acoustic power from the first cooler’s cold temperature to a higher temperature, typically ambient temperature. The transformed oscillation is optimal to drive a second cooler, such as a pulse-tube cooler. The efficiency of this combination is higher than that of two separate pulse-tube coolers, because the second cooler is driven entirely by acoustic power that would otherwise be dissipated in the termination impedance (e.g., orifice) of the first pulse-tube cooler.

Experiments with two coupled coolers operating from room temperature to 200 K confirm the basic concept, demonstrating good performance despite a turbulent boundary layer in the long tube and even when the long tube is coiled for compactness.

The technique is attractive for powers around 1 kW or higher, where only a small fraction of the recovered acoustic power is dissipated on the walls of the long tube. The technique seems most useful for first-stage cold temperatures above 80 K, for which the acoustic power available at the first cold heat exchanger is worth recovering. Production of liquefied natural gas at 111 Kelvin is one candidate.

INTRODUCTION

In a Stirling or orifice pulse-tube cooler, the cycle thermodynamics requires that the acoustic power flowing out of the cold heat exchanger be at least as large as the cooling power. A displacer or other piston recovers this power in a Stirling cooler, so it is not wasted. However, this power is dissipated—converted to ambient-temperature heat—in an orifice pulse-tube cooler, sacrificing efficiency for the simplicity of eliminating cold moving parts.

Attempts to recover that acoustic power by feeding it back to the ambient end of the cooler via lumped or distributed acoustic elements\(^1\) have been unsatisfactory because that topology
allows Gedeon streaming,\textsuperscript{2} which must be suppressed by a moving diaphragm or a dissipative fluid diode.

Thus, we were motivated to try to use the acoustic power exiting the cold heat exchanger of one cooler to power a second cooler, thereby enjoying both high efficiency and no cold moving parts. This approach must face the impedance-mismatch challenge illustrated in Figs. 1(a) and (b). The nonzero gas volume in the regenerator and heat exchangers requires a nonzero phase difference $\phi_A - \phi_C$ between the incoming (ambient) and outgoing (cold) volume-flow-rate phasors, where $\phi$ is the phase by which volume flow rate $U_1$ leads pressure $p_1$. Meanwhile, the need to minimize viscous losses causes the two $U_1$ phasors to straddle the pressure phasors. Thus, $U_{1C}$ should lag $p_1$ at the exit of the first cooler, but should lead $p_1$ at the entrance to the second cooler. Designing a connection between the two coolers that creates the necessary phase shift is the key challenge. Thermal isolation of the first cold heat exchanger from ambient with a pulse tube, as illustrated in Fig. 1(c), makes the challenge even greater.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{(a) The heart of a Stirling or pulse-tube cooler comprises a regenerator sandwiched between an ambient heat exchanger (HX) and a cold heat exchanger, with a source of oscillating power, such as a piston, on the ambient side. For efficient operation, the oscillating volume flow rate $U_1$ leads the oscillating pressure $p_1$ at the ambient end and lags $p_1$ at the cold end. (Throughout this paper, phasor diagrams reflect the $e^{i\omega t}$ sign convention, and positive $\phi$ indicates that $U_1$ leads $p_1$). (b) The acoustic power exiting the cold heat exchanger of the cooler in (a) could be used as a source of oscillating power in a second cooler, if the wave could be transformed so that $U_1$ leads $p_1$. Only the relative phase $\phi_A$ is important; the overall phases can have any values. (c) Adding a pulse tube delays the phase of $U_1$ exiting the first cooler even more.}
\end{figure}
The wave nature of the propagation of oscillations in a tube approximately a quarter wavelength long offers one approach to transforming the phasors at the cold end of one cooler into something suitable for driving a second cooler. The phasor diagrams in Fig. 2 illustrate some basic physics of propagation in such a tube, under the simplifying assumptions that viscous drag and thermal-hysteresis losses at the tube wall can be neglected and that the density and sound speed are independent of location $x$ along the tube.

With area $A$ independent of $x$, the phasors evolve in such an ideal tube according to

$$p_1(x) = p_1(0) \cos \frac{2\pi x}{\lambda} - i\rho c \frac{U_1(0)}{A} \sin \frac{2\pi x}{\lambda}, \quad (1)$$

$$U_1(x) = U_1(0) \cos \frac{2\pi x}{\lambda} - \left[i\frac{A}{\rho c} p_1(0) \sin \frac{2\pi x}{\lambda}\right], \quad (2)$$

where $\rho$ is the density of the gas, $c$ is its sound speed, and $\lambda = c/f$ is the wavelength, with $f$ the frequency of the oscillations. The $x = \lambda/4$ end points of the cases (a)–(d) below are easy to calculate analytically from these equations by setting $\cos \frac{2\pi x}{\lambda} = 0$ and $\sin \frac{2\pi x}{\lambda} = 1$. The endpoints in case (e), and the elliptical phasor trajectories in all cases, are more easily calculated in a spreadsheet.

As shown in Fig. 2(a), phasors that are initially in phase evolve according to $e^{-2\pi i x/\lambda}$ along a tube with $A = \rho c |U_1|/|p_1|$. This particular area and initial condition preserve $|p_1|$, $|U_1|$, and the $0^\circ$ phase between them. It is useful to think of this situation as representing a perfect match between the inertial impedance per unit length, which governs how $dp_1/dx$ responds to $U_1$, and the

![Figure 2](image)

*Figure 2.* Phasors illustrating the basic physics of wave propagation in ideal tubes a quarter wavelength long [(a)–(d)] or slightly longer [(e)]. All five cases have the same magnitude and phase ($0^\circ$) for the initial $p_1$, and all five carry the same acoustic power $(|p_1||U_1|\cos \phi)/2$. All five are drawn to the same scale. Phasor evolution with $x$ is clockwise from the initial end of the tube to its final end in all five cases. (a) If $p_1$ and $U_1$ are initially in phase, choosing the tube length to be $\lambda/4$ and the area to be $\rho c |U_1|/|p_1|$ causes both phasors to propagate at constant amplitude as their phases evolve together through $90^\circ$. (b) With the same initial conditions as in (a), a smaller-area tube increases $|p_1|$ (here by a factor of 1.25) and decreases $|U_1|$, keeping their product constant. This can be useful to compensate for viscous pressure drop in a first-stage cooler. (c) When $A = \rho c |U_1|/|p_1|$, a negative $\phi$ at the initial end is transformed into an equal positive $\phi$ at the final end (here both are $30^\circ$). (d) Combining the reduced tube area of (b) with the initial phase lag of (c) transforms both magnitudes and phases appropriately. (e) Attempting to change a large initial phase lag (here $\phi = -60^\circ$) into a smaller final phase lag (here $\phi = +30^\circ$) while boosting $|p_1|$ in a uniform-area tube requires a longer tube with much higher intermediate $|U_1|$. 
compliant impedance per unit length, which governs how $dU_1/dx$ responds to $p_1$. Figure 2(b) shows that a tube with slightly smaller area boosts $|p_1|$ and reduces $|U_1|$ as the wave propagates along a quarter wavelength. Compared with the area of (a), the smaller area of (b) boosts the velocity for a given $U_1$, thus accentuating the inertial impedance of the wave and leading to a larger $dp_1/dx$, and simultaneously reduces the compliance per unit length, so a given $p_1$ yields a smaller $dU_1/dx$. Figure 2(c) returns to the ideal area $A = \rho c |U_1|/|p_1|$, showing how an initial phase lag of $U_1$ is transformed into an equal phase lead by a quarter-wave tube, while preserving $|p_1|$ and $|U_1|$. Figure 2(d) illustrates how a quarter-wave tube with a smaller area can simultaneously boost $|p_1|$ [as in Fig. 2(b)] and flip the sign of $\phi$ [as in Fig. 2(c)]. Figure 2(e) illustrates how much harder it is to transform a wave with a larger initial phase lag, such as results from using the pulse tube as shown in Fig. 1(c). To arrive at the same final $|p_1|$, $|U_1|$, and $\phi$ as in Fig. 2(d), i.e. with $|p_1|$ boosted by a factor of 1.25 and with $\phi = +30^\circ$, requires a tube with an area 2.6 times larger than that of Fig. 1(d), and a length 1.5 times longer. In addition, $|U_1|$ reaches more extreme values near the middle of the tube. Turbulent viscous dissipation dominates thermal-hysteresis dissipation in these cases, and the dissipation in the tube of Fig. 1(e) would be 1.6 times higher than that of Fig. 1(d). Thus, both energy efficiency and hardware compactness suggest that avoiding the extra phase lag resulting from the use of the pulse tube as shown in Fig. 1(c) is desirable. This led us to consider using the long tube for both the thermal isolation and the wave transformation, as shown in Fig. 3: a quarter-wave pulse tube.

**EFFECTS OF TURBULENCE AND COILING THE PULSE TUBE**

The basic wave-propagation physics needed to design a long pulse tube according to the concept shown in Fig. 3 seemed almost as simple as the description of Fig. 2 above, even using numerical integration to take into account the axial temperature gradient and attendant gradients in $\rho$ and $c$. However, two aspects of the concept shown in Fig. 3 seemed risky to us. First, the small area needed to transform the wave was significantly smaller than the area that would typically be used for a pulse tube, so the velocity was high—high enough to create turbulence in the tube. Turbulence usually causes increased heat transfer, which we feared might spoil the axial thermal isolation of the pulse tube. Second, the length of the tube seemed impractically large, but we feared that coiling the pulse tube might also increase the end-to-end heat transfer and spoil the axial thermal isolation.
To explore these two risks, we built and tested the two-stage cooler shown in Fig. 4. Details of this hardware and the experiments with it are given in Ref. 3. In brief:

Constrained by a small budget, we used regenerators and drivers left over from earlier work, and insulated the cold parts with hardware-store fiberglass and foam instead of with a vacuum can. Limitations on the drivers’ power caused us to use an approximately similitude-matched He–Ar mixture instead of pure helium, leading to reduced cooling power. We used $T_C = 200$ K for most measurements and 138 K for the rest.

Measured pressure phasors indicated that the wave-transformation properties of the long tube worked as expected.

Three aspects of the experiments were consistent with weak heat transport along the straight quarter-wave tube, indicating that turbulence had no serious consequences: the time dependence of the development of the temperature profile in the tube after the cooler was turned on, the shape of the steady-state temperature profile in the tube, and the cooling power of the first stage. Switching from the long, straight tube to a 4-turn coil showed no change in performance.

Possible use for LNG

Coupling two coolers through a quarter-wave pulse tube is unlikely to be useful for the smallest cryocoolers at the lowest cryogenic temperatures, such as those used in spacecraft, because the viscous and thermal-hysteresis dissipation on all the surface area of the quarter-wave pulse tube is relatively large for small coolers, and because the acoustic power available at the cold heat exchanger, always less than $T_C/T_A$ times the driver power, is hardly worth recovering at the lowest $T_C$. Thus, we considered using this concept for an application having relatively high power and high $T_C$: the liquefaction of natural gas. With a boiling point of 111 K at atmospheric pressure, liquefied natural gas (LNG) is at the warm end of the spectrum of cryogenic fluids.

To explore the use of a quarter-wave pulse tube to couple two coolers for LNG production, we used simple DeltaEC models similar to the model used in Ref. 3, and properties of methane from Ref. 6 as a surrogate for natural gas, considering the three cases summarized in Fig. 5 and Table 1. In all three cases, we assumed that we had 10 kW of 60-Hz acoustic power available to drive the first cooler’s ambient heat exchanger with 310 kPa amplitude oscillations in 3.1 MPa
helium gas, that ambient temperature was 300 K, and that we would deliver LNG at 1 bar and 111 K. Our DeltaEC models were not highly optimized, so there is some potential for better performance. On the other hand, our DeltaEC models did not include some minor components such as flow straighteners at radical changes in area (details that we usually add while doing detailed engineering design of pulse-tube refrigerators), so there is also some potential for worse performance. With all three models sharing the same simplifications, we expect that conclusions about relative performance are valid.

Figure 5(a) and the associated column in Table 1 show the baseline case: a traditional orifice pulse-tube refrigerator operating between 300 K and 111 K. With a cooling power of 1780 W, it would liquefy 165 kg/day of natural gas. Some 2500 W of acoustic power would be dissipated in the impedance network.

Figure 5(b) shows one way to package two coolers coupled with a quarter-wave pulse tube. The regenerators and heat exchangers of the two coolers could be concentric, and axially aligned, to simplify headers for ambient-temperature water and for natural gas. Using the acoustic power

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<th>5(a)</th>
<th>5(b)</th>
<th>5(c)</th>
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<tr>
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<td>10,000 W</td>
<td>10,000 W</td>
<td>10,000 W</td>
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<td>$T_A$</td>
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<td>300 K</td>
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<td>1st stage $T_C$</td>
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<td>155 K</td>
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<td>2nd stage $T_C$</td>
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<tr>
<td>2nd stage cooling power</td>
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<td>300 K</td>
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<tr>
<td>Natural-gas delivery $p$</td>
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that was dissipated in the baseline case, the second cooler would provide an extra 450 W of cooling power, raising the liquefaction rate by 27% and bringing the system’s coefficient of performance up to 39% of the Carnot coefficient of performance. The extra cooling power would come at the expense of additional fabrication complexity.

Figure 5(c) shows another option, with the two coupled coolers having different cold temperatures. Long-distance natural-gas pipelines operate between about 13 and 100 bar, while local distribution pipes typically operate at 13 bar and below. Thus, many LNG-production applications might have gas available at 13 bar. A two-cooler system could liquefy all the gas at 13 bar at 155 K, then isenthalpically expand the fluid (a Joule-Thomson expansion) to 1 bar and 111 K, and finally re-liquefy the small fraction of the fluid that evaporated during the isenthalpic expansion, delivering all the fluid as 111 K liquid at 1 bar. With most of the latent heat removed at the highest possible temperature, this process is inherently much more efficient than performing all of the refrigeration at 111 K. In this case, a total of 3530 W of cooling power would be provided by the 10 kW of drive power, and the liquefaction rate would be double that of case (a).

CONCLUSIONS

Despite a turbulent boundary layer and the secondary flows associated with oscillations in a curved pipe, a long, narrow pulse tube can efficiently couple two coolers while maintaining the reliability advantage of no cold moving parts. The cold temperatures of the two coolers can be the same or can differ. The long pulse tube seems useful only for large coolers and/or when the first cooler has a relatively modest cold temperatures.

ACKNOWLEDGMENT

This work was supported by the Technology Transfer Division at Los Alamos National Laboratory.

REFERENCES