Staging Two Coolers through a Quarter-Wave Tube

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ABSTRACT

Stirling coolers have high efficiency, in part because the displacer recovers acoustic power (pV work) from the gas near the cold heat exchanger, delivering it to the ambient end of the cooler. However, pulse-tube coolers have lower cost because they have no cold moving parts. In a new scheme with both high efficiency and no cold moving parts, acoustic power from the gas near the cold heat exchanger of a first cooler is recovered via a tube almost a quarter-wavelength long. The long tube's acoustic characteristics boost the pressure amplitude, lower the volume-flow-rate amplitude, and alter the time phasing between them, while transmitting acoustic power from the first cooler's cold temperature to a higher temperature, typically ambient temperature. The transformed oscillation is optimal to drive a second cooler, such as a pulse-tube cooler. The efficiency of this combination is higher than that of two separate pulse-tube coolers, because the second cooler is driven entirely by acoustic power that would otherwise be dissipated in the termination impedance (e.g., orifice) of the first pulse-tube cooler.

Experiments with two coupled coolers operating from room temperature to 200 K confirm the basic concept, demonstrating good performance despite a turbulent boundary layer in the long tube and even when the long tube is coiled for compactness.

The technique is attractive for powers around 1 kW or higher, where only a small fraction of the recovered acoustic power is dissipated on the walls of the long tube. The technique seems most useful for first-stage cold temperatures above 80 K, for which the acoustic power available at the first cold heat exchanger is worth recovering. Production of liquefied natural gas at 111 Kelvin is one candidate.

INTRODUCTION

In a Stirling or orifice pulse-tube cooler, the cycle thermodynamics requires that the acoustic power flowing out of the cold heat exchanger be at least as large as the cooling power. A displacer or other piston recovers this power in a Stirling cooler, so it is not wasted. However, this power is dissipated—converted to ambient-temperature heat—in an orifice pulse-tube cooler, sacrificing efficiency for the simplicity of eliminating cold moving parts.

Attempts to recover that acoustic power by feeding it back to the ambient end of the cooler via lumped or distributed acoustic elements[1] have been unsatisfactory because that topology

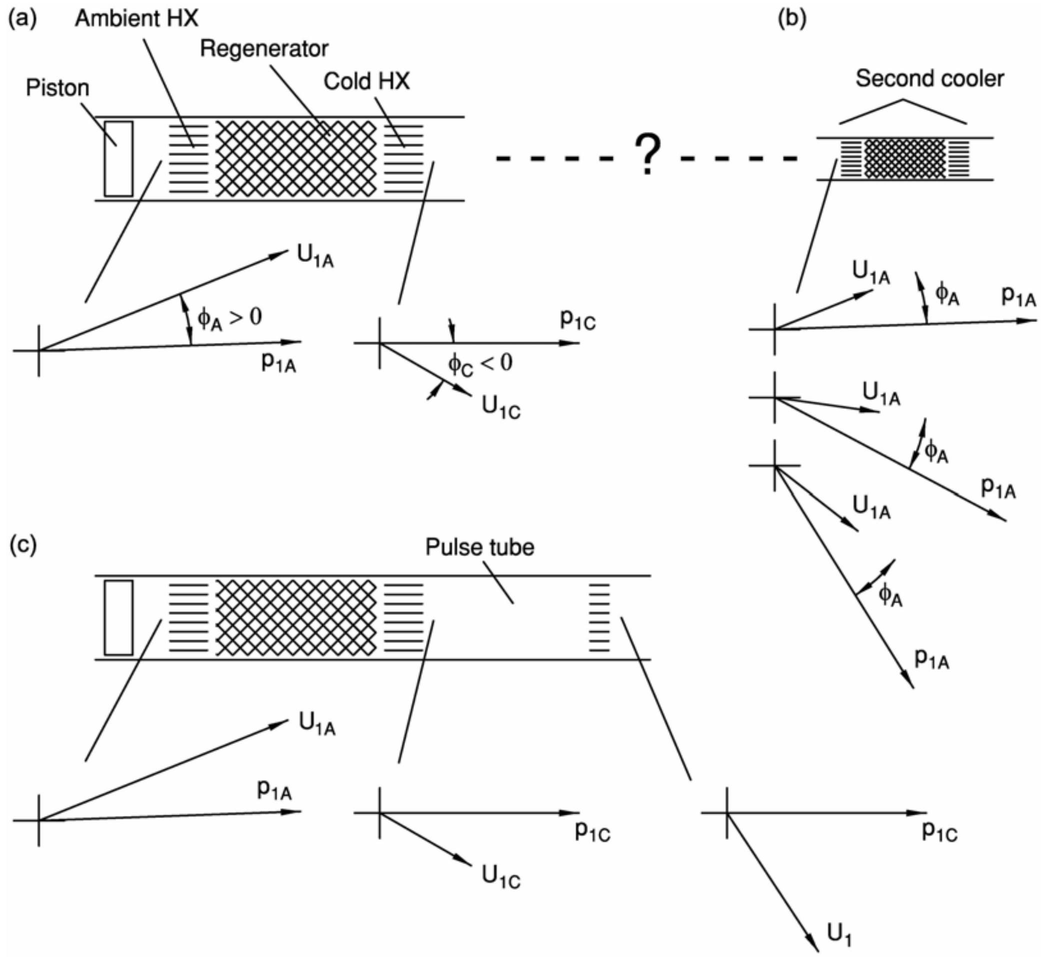

Figure 1. (a) The heart of a Stirling or pulse-tube cooler comprises a regenerator sandwiched between an ambient heat exchanger (HX) and a cold heat exchanger, with a source of oscillating power, such as a piston, on the ambient side. For efficient operation, the oscillating volume flow rate U_1 leads the oscillating pressure p_1 at the ambient end and lags p_1 at the cold end. (Throughout this paper, phasor diagrams reflect the $e^{i\omega t}$ sign convention, and positive ϕ indicates that U_1 leads p_1). (b) The acoustic power exiting the cold heat exchanger of the cooler in (a) could be used as a source of oscillating power in a second cooler, if the wave could be transformed so that U_1 leads p_1. Only the relative phase ϕ_A is important; the overall phases can have any values. (c) Adding a pulse tube delays the phase of U_1 exiting the first cooler even more.

allows Gedeon streaming,[2] which must be suppressed by a moving diaphragm or a dissipative fluid diode.

Thus, we were motivated to try to use the acoustic power exiting the cold heat exchanger of one cooler to power a second cooler, thereby enjoying both high efficiency and no cold moving parts. This approach must face the impedance-mismatch challenge illustrated in Figs. 1(a) and (b). The nonzero gas volume in the regenerator and heat exchangers requires a nonzero phase difference $\phi_A - \phi_C$ between the incoming (ambient) and outgoing (cold) volume-flow-rate phasors, where ϕ is the phase by which volume flow rate U_1 leads pressure p_1. Meanwhile, the need to minimize viscous losses causes the two U_1 phasors to straddle the pressure phasors. Thus, U_{1C} should lag p_1 at the exit of the first cooler, but should lead p_1 at the entrance to the second cooler. Designing a connection between the two coolers that creates the necessary phase shift is the key challenge. Thermal isolation of the first cold heat exchanger from ambient with a pulse tube, as illustrated in Fig. 1(c), makes the challenge even greater.

WAVE PROPAGATION IN QUARTER-WAVE TUBES

The wave nature of the propagation of oscillations in a tube approximately a quarter wavelength long offers one approach to transforming the phasors at the cold end of one cooler into something suitable for driving a second cooler. The phasor diagrams in Fig. 2 illustrate some basic physics of propagation in such a tube, under the simplifying assumptions that viscous drag and thermal-hysteresis losses at the tube wall can be neglected and that the density and sound speed are independent of location x along the tube.

With area A independent of x, the phasors evolve in such an ideal tube according to

$$p_1(x) = p_1(0) \cos 2\pi x/\lambda - i\rho c\, [U_1(0)/A] \sin 2\pi x/\lambda, \qquad (1)$$

$$U_1(x) = U_1(0) \cos 2\pi x/\lambda - [iA/\rho c]\, p_1(0) \sin 2\pi x/\lambda, \qquad (2)$$

where ρ is the density of the gas, c is its sound speed, and $\lambda = c/f$ is the wavelength, with f the frequency of the oscillations. The $x = \lambda/4$ end points of the cases (a)–(d) below are easy to calculate analytically from these equations by setting $\cos 2\pi x/\lambda = 0$ and $\sin 2\pi x/\lambda = 1$. The endpoints in case (e), and the elliptical phasor trajectories in all cases, are more easily calculated in a spreadsheet.

As shown in Fig. 2(a), phasors that are initially in phase evolve according to $e^{-2\pi i\, x/\lambda}$ along a tube with $A = \rho c|U_1|/|p_1|$. This particular area and initial condition preserve $|p_1|$, $|U_1|$, and the 0^{o} phase between them. It is useful to think of this situation as representing a perfect match between the inertial impedance per unit length, which governs how dp_1/dx responds to U_1, and the

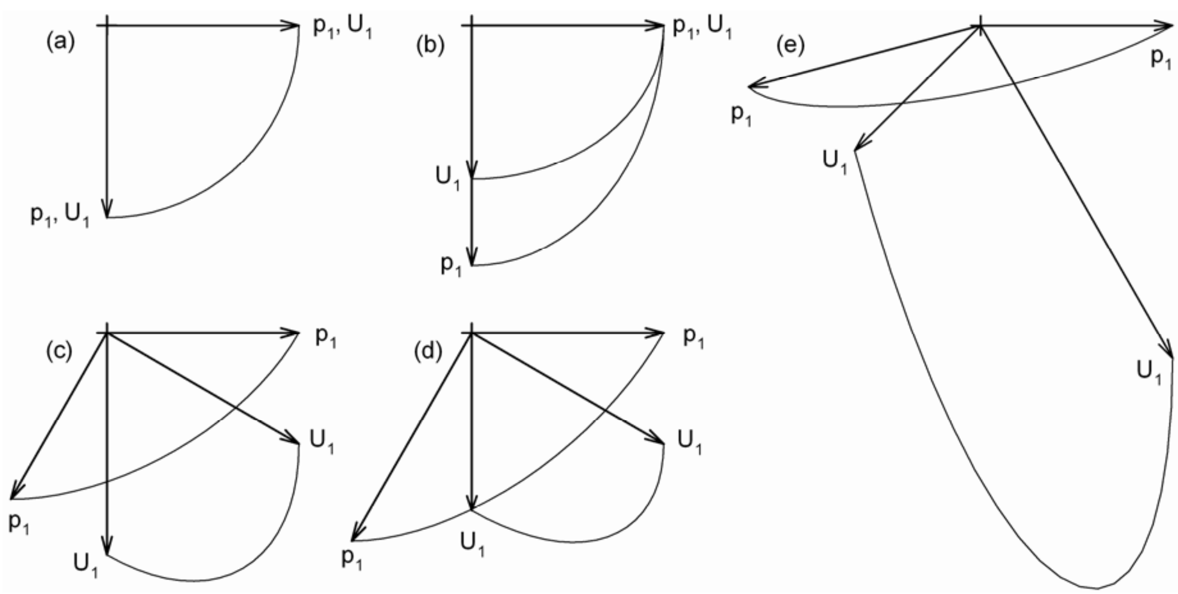

Figure 2. Phasors illustrating the basic physics of wave propagation in ideal tubes a quarter wavelength long [(a)–(d)] or slightly longer [(e)]. All five cases have the same magnitude and phase (0^{o}) for the initial p_1, and all five carry the same acoustic power ($|p_1||U_1|\cos\phi)/2$. All five are drawn to the same scale. Phasor evolution with x is clockwise from the initial end of the tube to its final end in all five cases. (a) If p_1 and U_1 are initially in phase, choosing the tube length to be $\lambda/4$ and the area to be $\rho c|U_1|/|p_1|$ causes both phasors to propagate at constant amplitude as their phases evolve together through 90^{o}. (b) With the same initial conditions as in (a), a smaller-area tube increases $|p_1|$ (here by a factor of 1.25) and decreases $|U_1|$, keeping their product constant. This can be useful to compensate for viscous pressure drop in a first-stage cooler. (c) When $A = \rho c|U_1|/|p_1|$, a negative ϕ at the initial end is transformed into an equal positive ϕ at the final end (here both are 30^{o}). (d) Combining the reduced tube area of (b) with the initial phase lag of (c) transforms both magnitudes and phases appropriately. (e) Attempting to change a large initial phase lag (here $\phi = -60^{\mathrm{o}}$) into a smaller final phase lead (here $\phi = +30^{\mathrm{o}}$) while boosting $|p_1|$ in a uniform-area tube requires a longer tube with much higher intermediate $|U_1|$.

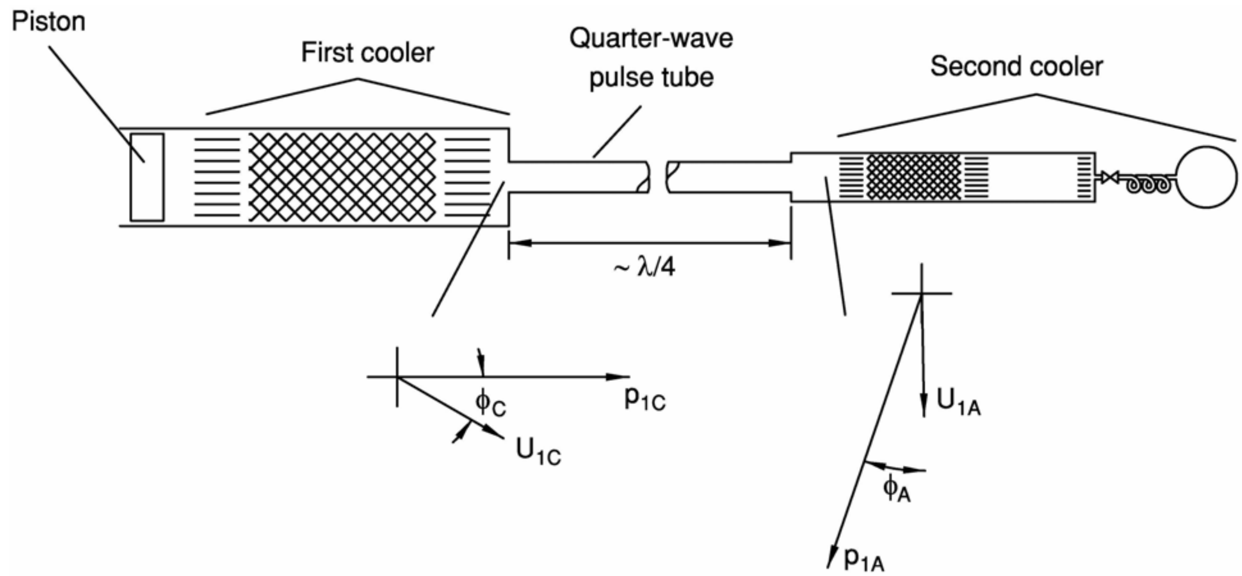

Figure 3. Combining the oscillation-transformation function of the quarter-wavelength tube with the thermal-isolation function of the pulse tube allows coupling two coolers together, the second cooler here being a traditional orifice pulse-tube refrigerator (PTR).

compliant impedance per unit length, which governs how dU_1/dx responds to p_1. Figure 2(b) shows that a tube with slightly smaller area boosts $|p_1|$ and reduces $|U_1|$ as the wave propagates along a quarter wavelength. Compared with the area of (a), the smaller area of (b) boosts the velocity for a given U_1, thus accentuating the inertial impedance of the wave and leading to a larger dp_1/dx, and simultaneously reduces the compliance per unit length, so a given p_1 yields a smaller dU_1/dx. Figure 2(c) returns to the ideal area $A = \rho c|U_1|/|p_1|$, showing how an initial phase lag of U_1 is transformed into an equal phase lead by a quarter-wave tube, while preserving $|p_1|$ and $|U_1|$. Figure 2(d) illustrates how a quarter-wave tube with a smaller area can simultaneously boost $|p_1|$ [as in Fig. 2(b)] and flip the sign of ϕ [as in Fig. 2(c)].

Figure 2(e) illustrates how much harder it is to transform a wave with a larger initial phase lag, such as results from using the pulse tube as shown in Fig. 1(c). To arrive at the same final $|p_1|$, $|U_1|$, and ϕ as in Fig. 2(d), i.e. with $|p_1|$ boosted by a factor of 1.25 and with $\phi = +30^\circ$, requires a tube with an area 2.6 times larger than that of Fig. 1(d), and a length 1.5 times longer. In addition, $|U_1|$ reaches more extreme values near the middle of the tube. Turbulent viscous dissipation dominates thermal-hysteresis dissipation in these cases, and the dissipation in the tube of Fig. 1(e) would be 1.6 times higher than that of Fig. 1(d). Thus, both energy efficiency and hardware compactness suggest that avoiding the extra phase lag resulting from the use of the pulse tube as shown in Fig. 1(c) is desirable. This led us to consider using the long tube for both the thermal isolation and the wave transformation, as shown in Fig. 3: a quarter-wave pulse tube.

EFFECTS OF TURBULENCE AND COILING THE PULSE TUBE

The basic wave-propagation physics needed to design a long pulse tube according to the concept shown in Fig. 3 seemed almost as simple as the description of Fig. 2 above, even using numerical integration to take into account the axial temperature gradient and attendant gradients in ρ and c. However, two aspects of the concept shown in Fig. 3 seemed risky to us. First, the small area needed to transform the wave was significantly smaller than the area that would typically be used for a pulse tube, so the velocity was high—high enough to create turbulence in the tube. Turbulence usually causes increased heat transfer, which we feared might spoil the axial thermal isolation of the pulse tube. Second, the length of the tube seemed impractically large, but we feared that coiling the pulse tube might also increase the end-to-end heat transfer and spoil the axial thermal isolation.

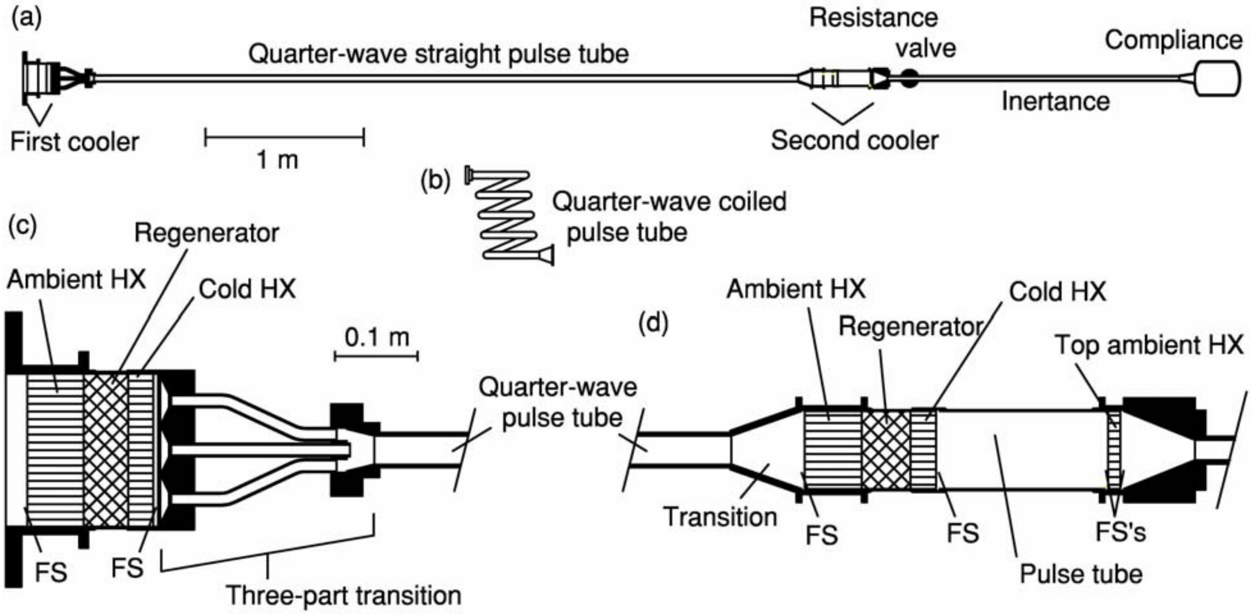

Figure 4. Scale drawing of the apparatus used to test the long, turbulent pulse tube. HX = heat exchanger; FS = flow straightener. Actual orientation was vertical. (a) Overall view with the straight tube. (b) The coiled tube, at the same scale as (a). (c) Close-up of the first cooler. (d) Close-up of the second cooler, at the same scale as (a).

To explore these two risks, we built and tested the two-stage cooler shown in Fig. 4. Details of this hardware and the experiments with it are given in Ref. 3. In brief:

Constrained by a small budget, we used regenerators and drivers left over from earlier work, and insulated the cold parts with hardware-store fiberglass and foam instead of with a vacuum can. Limitations on the drivers' power caused us to use an approximately similitude-matched He–Ar mixture instead of pure helium, leading to reduced cooling power. We used T_C = 200 K for most measurements and 138 K for the rest.

Measured pressure phasors indicated that the wave-transformation properties of the long tube worked as expected.

Three aspects of the experiments were consistent with weak heat transport along the straight quarter-wave tube, indicating that turbulence had no serious consequences: the time dependence of the development of the temperature profile in the tube after the cooler was turned on, the shape of the steady-state temperature profile in the tube, and the cooling power of the first stage.

Switching from the long, straight tube to a 4-turn coil showed no change in performance.

POSSIBLE USE FOR LNG

Coupling two coolers through a quarter-wave pulse tube is unlikely to be useful for the smallest cryocoolers at the lowest cryogenic temperatures, such as those used in spacecraft, because the viscous and thermal-hysteresis dissipation on all the surface area of the quarter-wave pulse tube is relatively large for small coolers, and because the acoustic power available at the cold heat exchanger, always less than T_C/T_A times the driver power,[4] is hardly worth recovering at the lowest T_C. Thus, we considered using this concept for an application having relatively high power and high T_C: the liquefaction of natural gas. With a boiling point of 111 K at atmospheric pressure, liquefied natural gas (LNG) is at the warm end of the spectrum of cryogenic fluids.

To explore the use of a quarter-wave pulse tube to couple two coolers for LNG production, we used simple DeltaEC models[5] similar to the model used in Ref. 3, and properties of methane from Ref. 6 as a surrogate for natural gas, considering the three cases summarized in Fig. 5 and Table 1. In all three cases, we assumed that we had 10 kW of 60-Hz acoustic power available to drive the first cooler's ambient heat exchanger with 310 kPa amplitude oscillations in 3.1 MPa

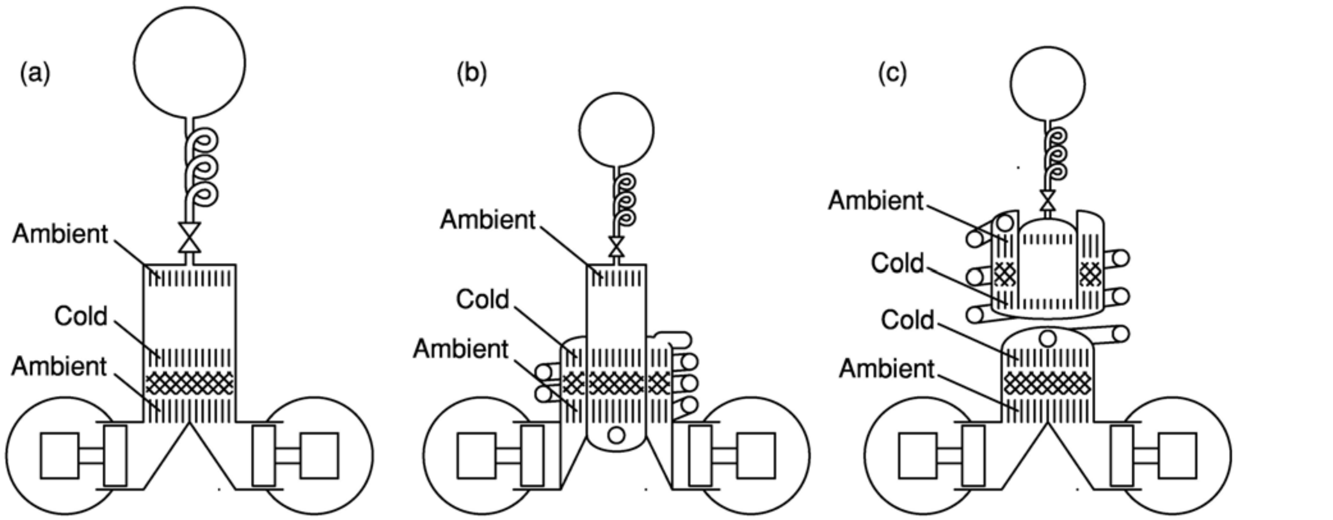

Figure 5. Three configurations for liquefaction of natural-gas. Regenerators and heat exchangers are drawn roughly to scale, with their diameter in (a) being 20 cm. (a) A traditional orifice pulse-tube cooler, with T_A = 300 K and T_C = 111 K. (b) A two-stage cooler with the topology shown in Fig. 3, with the second stage concentric with, and inside, the first stage, with the quarter-wave coupling tube wrapped around the outside, again with T_A = 300 K and with T_C = 111 K for both stages. (c) A two-stage cooler with the topology shown in Fig. 3, with the second stage above the first stage, with the second stage's pulse tube concentric with, and inside, its regenerator, with the quarter-wave coupling tube wrapped around the outside, with T_A = 300 K for both stages but with T_C = 155 K for the first stage and 111 K for the second stage.

Table 1. Assumptions and results for the natural-gas liquefiers shown in Fig. 5.

Figure number	5(a)	5(b)	5(c)
Drive power	10,000 W	10,000 W	10,000 W
T_A	300 K	300 K	300 K
1st stage T_C	111 K	111 K	155 K
1st stage cooling power	1780 W	1780 W	2880 W
2nd stage T_C	n.a.	111 K	111 K
2nd stage cooling power	n.a.	450 W	650 W
Natural-gas source p	1 bar	1 bar	13 bar
Natural-gas source T	300 K	300 K	300 K
Natural-gas delivery p	1 bar	1 bar	1 bar
Natural-gas delivery T	111 K	111 K	111 K
Natural-gas liquefaction rate	165 kg/day	210 kg/day	330 kg/day

helium gas, that ambient temperature was 300 K, and that we would deliver LNG at 1 bar and 111 K. Our DeltaEC models were not highly optimized, so there is some potential for better performance. On the other hand, our DeltaEC models did not include some minor components such as flow straighteners at radical changes in area (details that we usually add while doing detailed engineering design of pulse-tube refrigerators), so there is also some potential for worse performance. With all three models sharing the same simplifications, we expect that conclusions about relative performance are valid.

Figure 5(a) and the associatied column in Table 1 show the baseline case: a traditional orifice pulse-tube refrigerator operating between 300 K and 111 K. With a cooling power of 1780 W, it would liquefy 165 kg/day of natural gas. Some 2500 W of acoustic power would be dissipated in the impedance network.

Figure 5(b) shows one way to package two coolers coupled with a quarter-wave pulse tube. The regenerators and heat exchangers of the two coolers could be concentric, and axially aligned, to simplify headers for ambient-temperature water and for natural gas. Using the acoustic power that was dissipated in the baseline case, the second cooler would provide an extra 450 W of cooling power, raising the liquefaction rate by 27% and bringing the system's coefficient of performance up to 39% of the Carnot coefficient of performance. The extra cooling power would come at the expense of additional fabrication complexity.

Figure 5(c) shows another option, with the two coupled coolers having different cold temperatures. Long-distance natural-gas pipelines operate between about 13 and 100 bar, while local distribution pipes typically operate at 13 bar and below.[7] Thus, many LNG-production applications might have gas available at 13 bar. A two-cooler system could liquefy all the gas at 13 bar at 155 K, then isenthalpically expand the fluid (a Joule-Thomson expansion) to 1 bar and 111 K, and finally re-liquefy the small fraction of the fluid that evaporated during the isenthalpic expansion, delivering all the fluid as 111 K liquid at 1 bar. With most of the latent heat removed at the highest possible temperature, this process is inherently much more efficient than performing all of the refrigeration at 111 K. In this case, a total of 3530 W of cooling power would be provided by the 10 kW of drive power, and the liquefaction rate would be double that of case (a).

CONCLUSIONS

Despite a turbulent boundary layer and the secondary flows associated with oscillations in a curved pipe, a long, narrow pulse tube can efficiently couple two coolers while maintaining the reliability advantage of no cold moving parts. The cold temperatures of the two coolers can be the same or can differ. The long pulse tube seems useful only for large coolers and/or when the first cooler has a relatively modest cold temperatures.

ACKNOWLEDGMENT

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