Hadron collisions in SCET

Grigory Ovanesyan

UC Berkeley/LBL

July 6, 2010

In collaboration with B.Lange and C.Bauer

earlier collaboration with C.Lee and Z.Ligeti

Santa Fe 2010 Summer Workshop

"LHC: From Here to Where?"

On Exclusive Drell-Yan in SCET

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"LHC: From Here to Where?"

Outline

- Introduction
- Drell-Yan factorization: a Review
- Breakdown of SCET in exclusive Drell-Yan
- Conclusions

Introduction

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- Most of the interesting BSM signals on Tevatron and LHC suffer from large QCD backgrounds
- Factorization is the main tool to understanding QCD backgrounds

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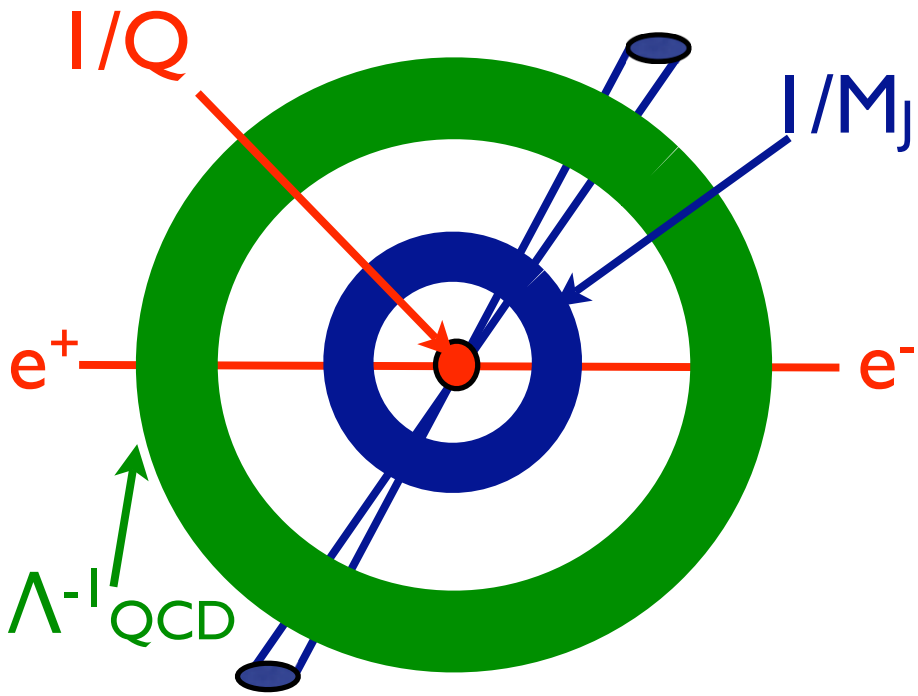
$$\sigma \sim \sigma^{part} \otimes f_1 \otimes f_2$$

Collins, Soper, Sterman, 82-85

Effective theories

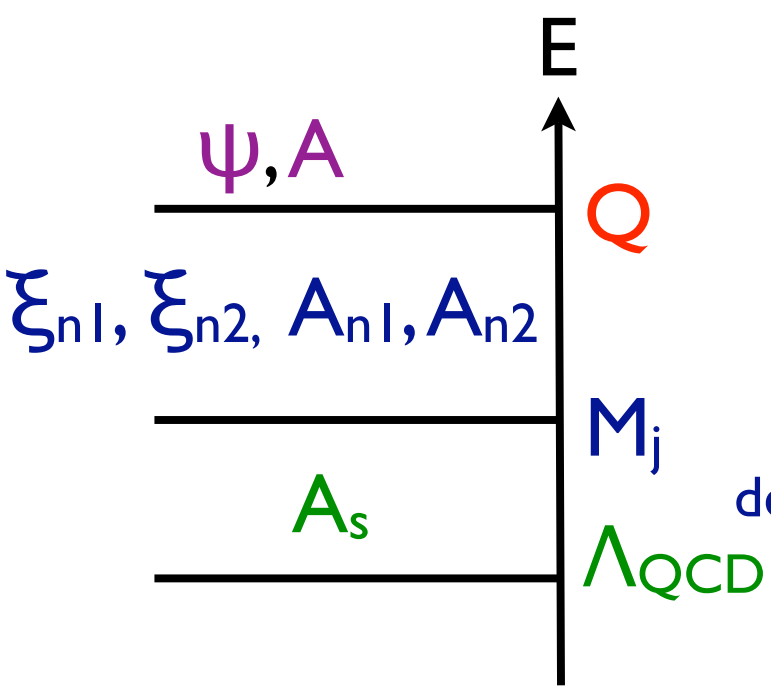
- In recent years Effective Field Theories (EFT) have been developed to derive factorization
- Method of EFT relies on the hierarchy of physical scales, a small parameter
- Natural language to derive factorization theorems
- Allows clean resummation of large logarithms (Chris Lee's talk)
- A systematic field theoretic tool that is straightforwardly expanded to higher orders and higher twist

Soft Collinear Effective Theory



Bauer, Fleming, Luke, Pirjol, Stewart, (00-01)

- Clear separation of scales between hard emission, collinear splittings and soft radiation
- In SCET the small parameter λ describes how close to the jet axis the collinear emissions occur
- Power counting of SCET requires couplings between collinear quarks, gluons, and soft gluons



In Bauer, Cata, GO, (08), the SCET Lagrangian was derived for the first time on the functional integral level

Introduction

Factorization theorems in SCET

lepton collisions:

Bauer, Manohar, Wise, 02
Bauer, Lee, Manohar, Wise, 03
Lee, Sterman, 07
Becher, Schwartz, 08 / Stewart et al (10)
Bauer, Fleming, Lee, Sterman, 08
Fleming, Hoang, Mantry, Stewart, 07 (I,II)
Hornig, Lee, GO 09
Ellis, Hornig, Lee, Vermilion, Walsh, 09,10

DIS

Manohar 03
Becher, Neubert, Pecjak 06

hadron collisions:

Becher Neubert, 07
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this talk: Exclusive Drell-Yan in SCET

A new mode in the SCET

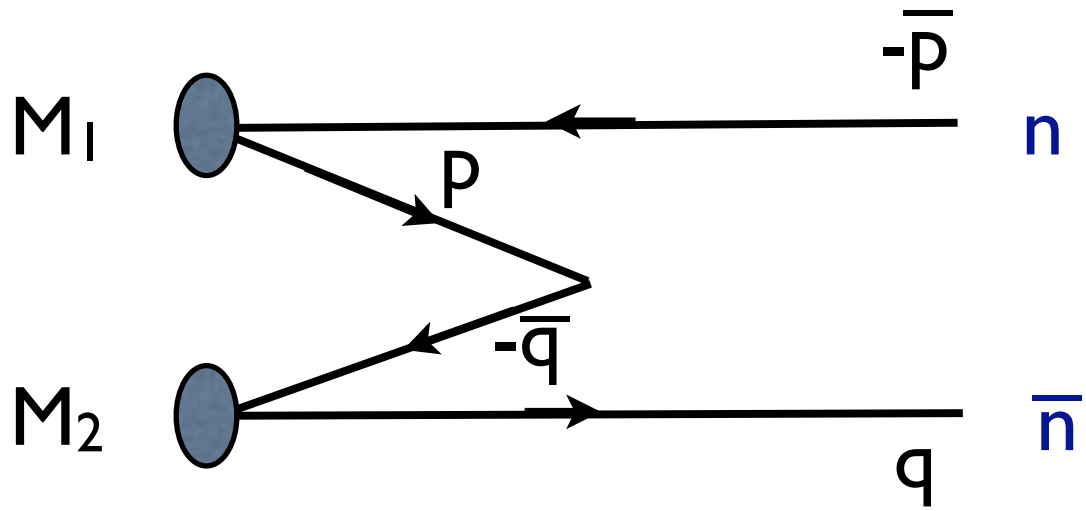
Introduction

A new mode in the SCET?

- In going from lepton to hadron collisions there are many important obstacles
- We are trying to resolve one of them: are we sure we have all the the important degrees of freedom in the theory?
- In this talk we will show that we need a new mode for one observable: exclusive Drell-Yan cross-section
- We will perform an explicit one loop matching calculation between QCD and SCET

Drell-Yan factorization: Review

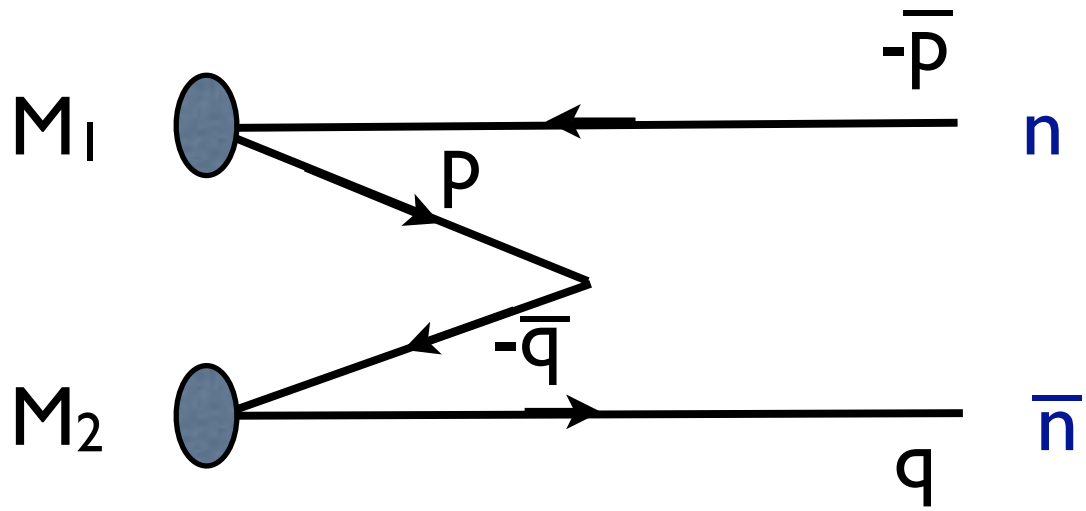
Tree level Drell-Yan



$\bar{p}, p \propto (1, \lambda^2, \lambda)$ $\quad P_{M1} = \bar{p} + p$

$\bar{q}, q \propto (\lambda^2, 1, \lambda)$ $\quad P_{M2} = \bar{q} + q$

Tree level Drell-Yan



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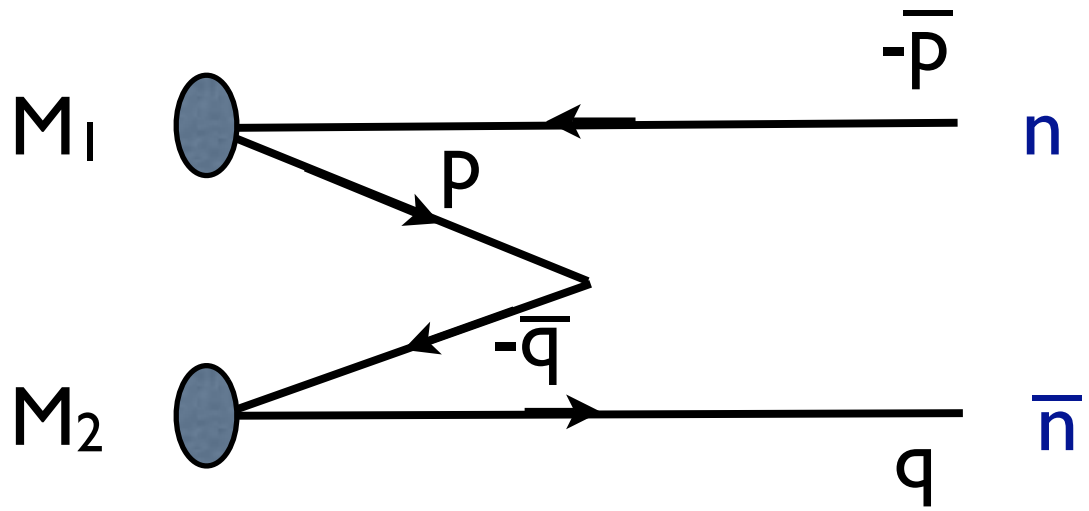
Glauber gluon: $k \propto (\lambda^2, \lambda^2, \lambda)$

Tree level Drell-Yan



$\sim \lambda^{-4}$

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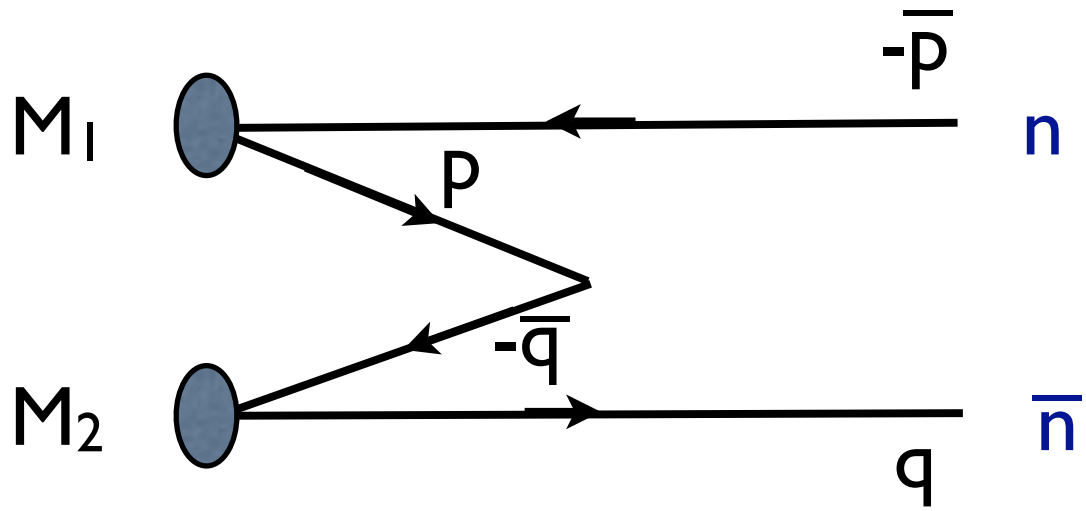

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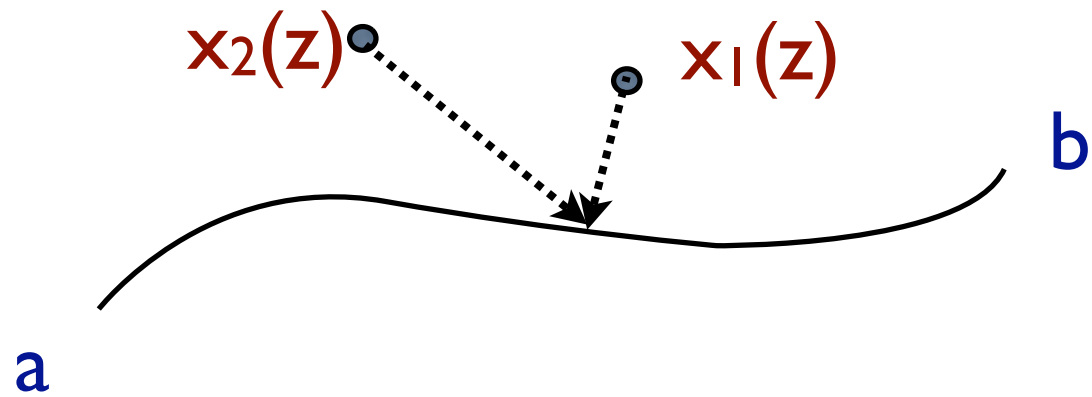
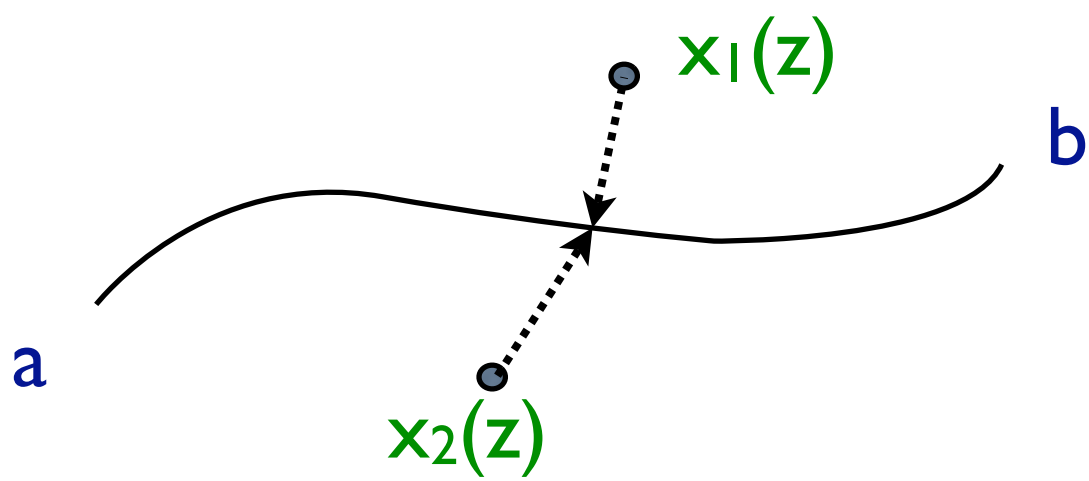
Off-shellness as an infrared regulator

Pinch analysis of loop integrals

$$I(z) = \int_C \mathrm{d}x\, f(x, z)$$

$$I(z \to z_0) = ?$$





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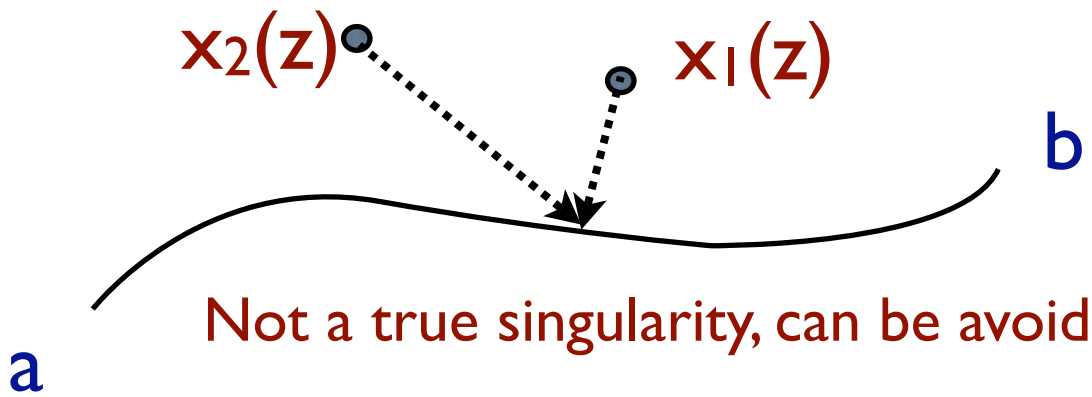
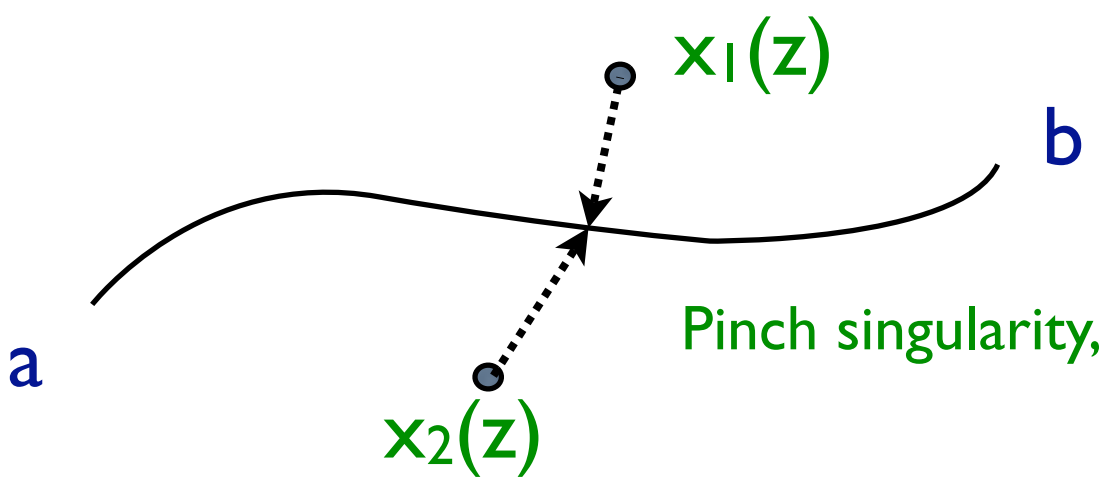


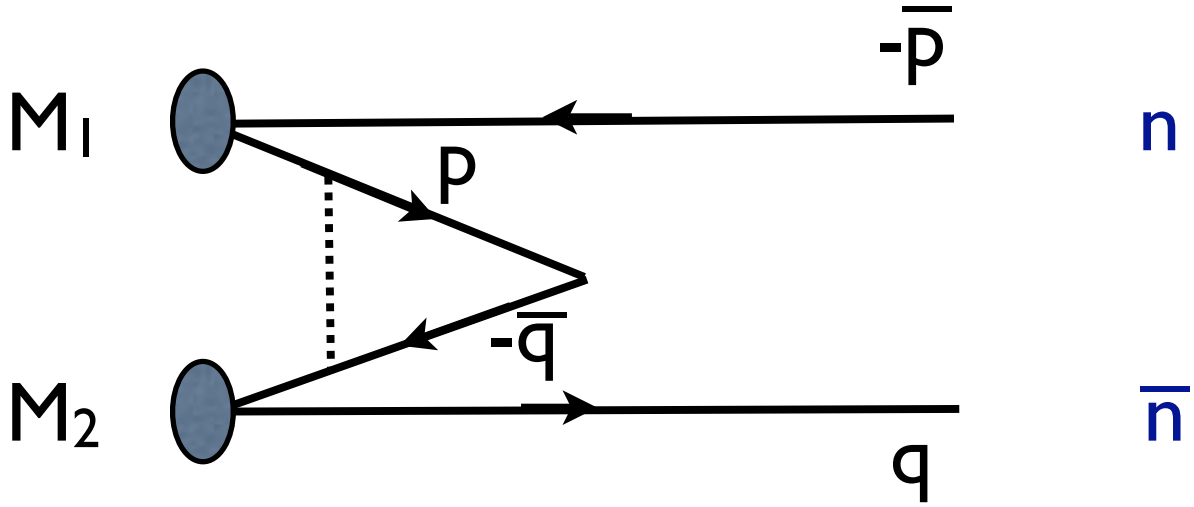
Not a true singularity, can be avoided by deforming the contour



Pinch singularity, leads to a true pole

Active-Active Interactions

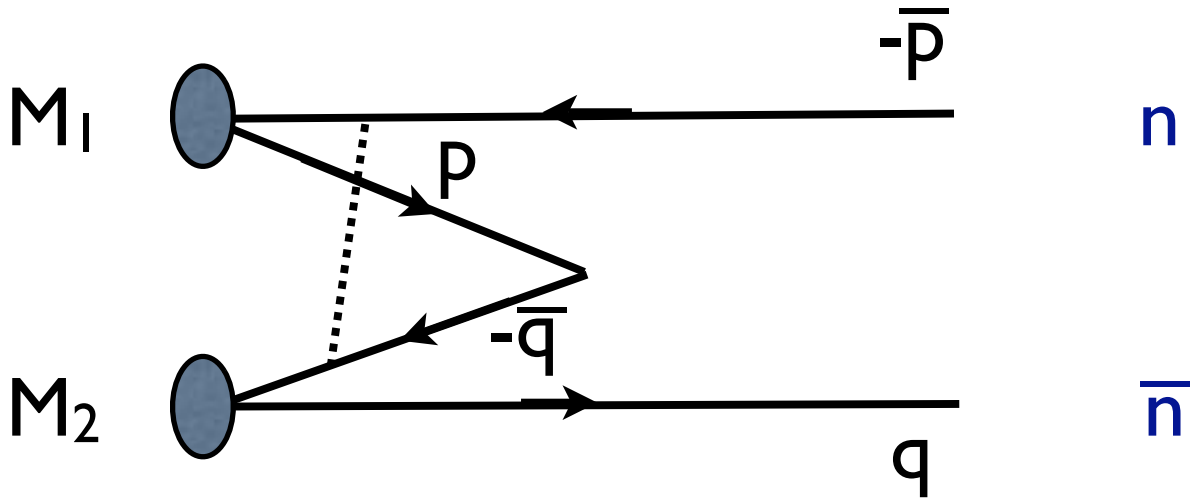
Collins, Soper, Sterman, `82-`85



- The pinched singularities appear only in the collinear n, collinear $\bar{n}$ and soft regions
- Glauber region is not pinched

Spectator-Active Interactions

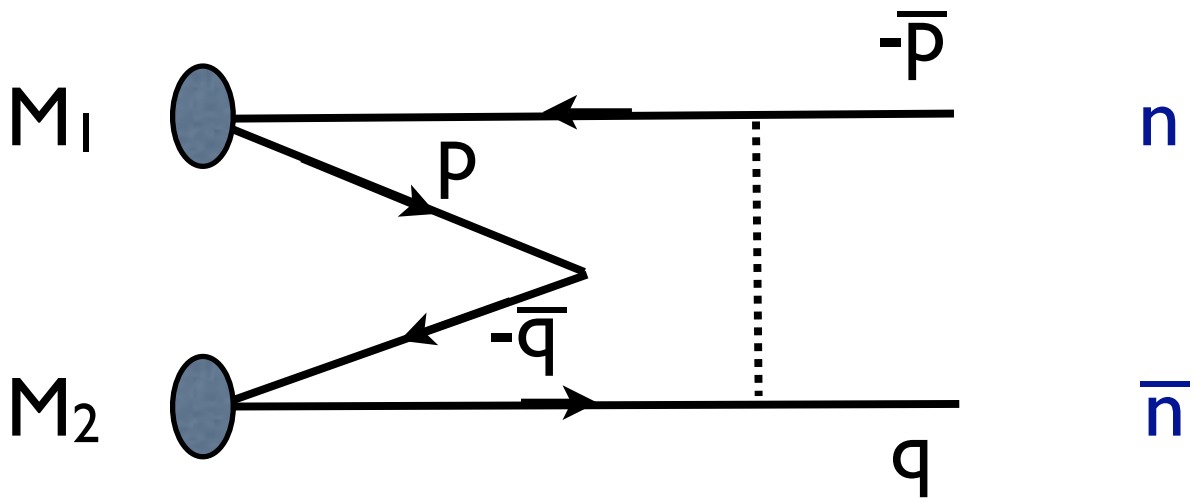
Collins, Soper, Sterman, `82-`85



- The leading pinched singularities appear only in the collinear n and soft regions
- Glauber region is not pinched

Spectator-Spectator Interactions

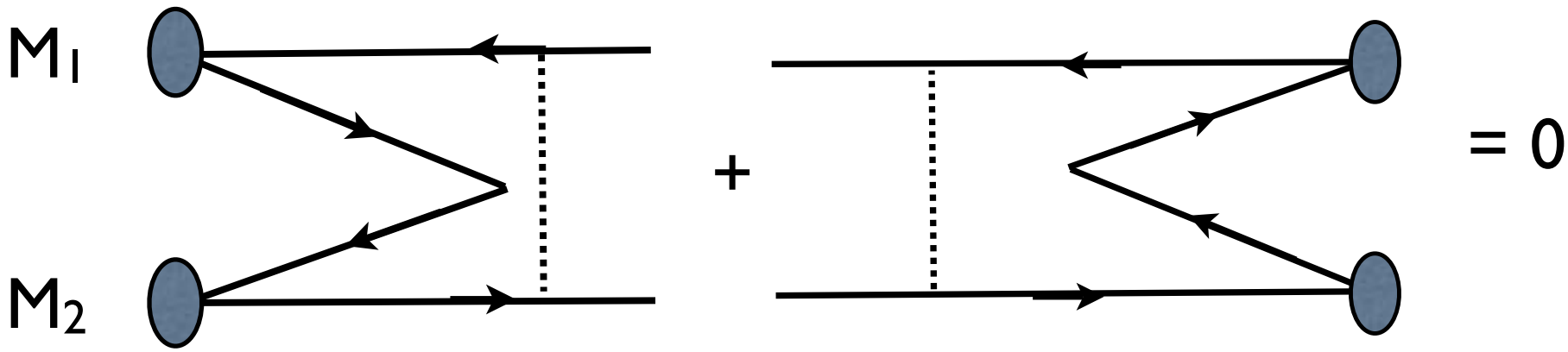
Collins, Soper, Sterman, `82-`85



- The pinched singularities appear in the Soft and Glauber regions
- This mode break the simple factorization of Drell-Yan exclusive cross-section

Cancellation of Glaubers

Collins, Soper, Sterman, `82-`85



- Glauber contribution is shown to be purely imaginary
- Thus it cancels in the inclusive cross-section
- These one-loop results are generalized to all orders

Our goal: should we include Glauber modes into Effective Theory?

Drell-Yan factorization

Collins, Soper, Sterman, `82-`85

inclusive: $\sigma_{DY}(Q^2) \sim \sigma^{part} \otimes f_1(x_1) \otimes f_2(x_2)$

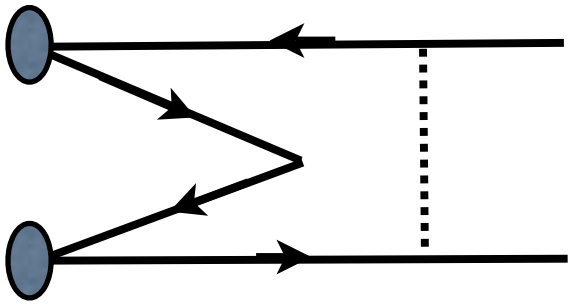
(Glauber gluons cancel)

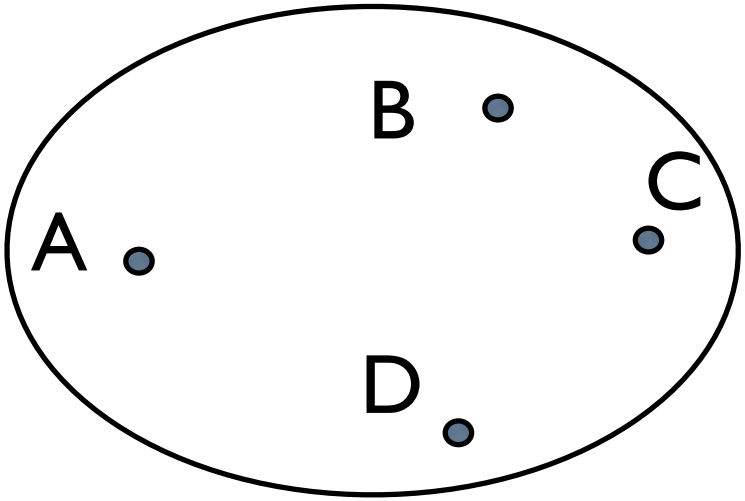
exclusive: $\sigma_{DY}(Q^2, Q_T) \sim \sigma^{part} \otimes f_1(x_1, p_{1\perp}) \otimes f_2(x_2, p_{2\perp})$

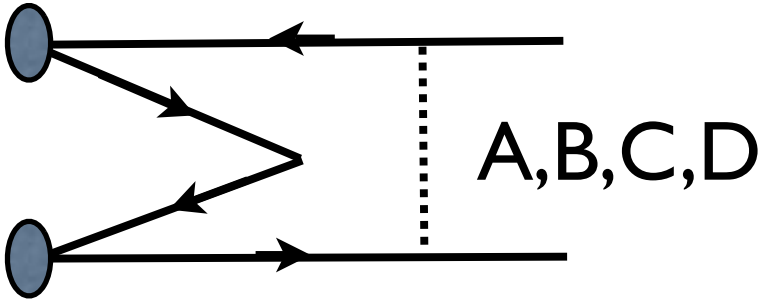
(Glauber gluons contribute)

Pinches and Effective theory modes

Full Theory pinches:







Pinches and Effective theory modes

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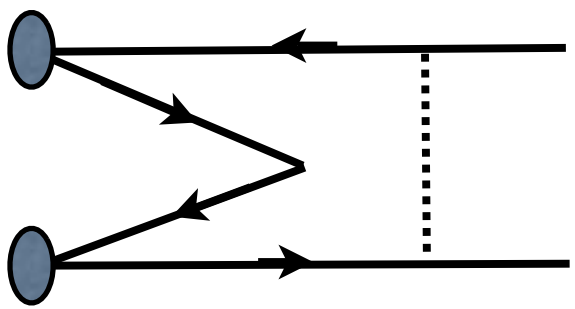
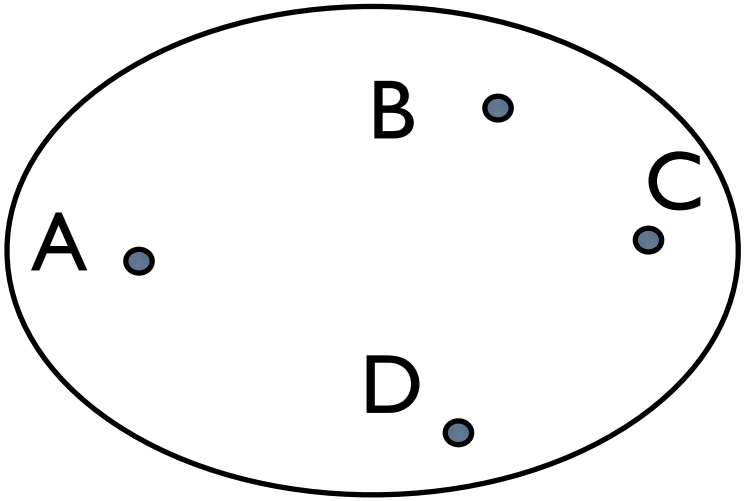
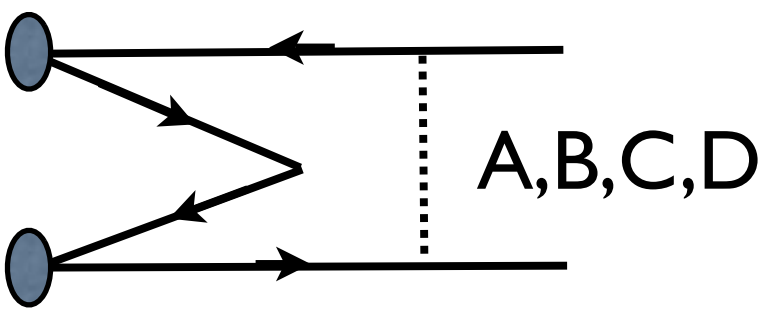
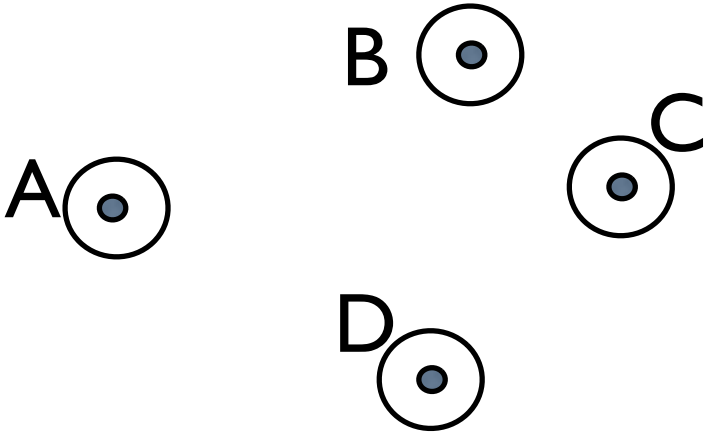




Need Effective Theory modes A, B, C, D:





Drell-Yan factorization in SCET

Bauer, Fleming, Pirjol, Rothstein, Stewart, 02
Becher, Neubert, Xu, 07
Stewart, Tackmann, Waalewijn, 09, 10
Mantry, Petriello, 09

Drell-Yan factorization in SCET

inclusive

Bauer, Fleming, Pirjol, Rothstein, Stewart, 02
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Drell-Yan factorization in SCET

inclusive threshold

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Drell-Yan factorization in SCET

inclusive	Bauer, Fleming, Pirjol, Rothstein, Stewart, 02
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(soft, collinear) exclusive — Mantry, Petriello, 09

$p_s^2 \sim p_c^2$

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$p_s^2 \sim p_c^2$

we will address the $SCET_I$ for exclusive Drell-Yan

(ultrasoft, collinear $p_{us}^2 << p_c^2$)

Breakdown of SCET in exclusive Drell-Yan

Why do we expect SCET to break down?

Bodwin, Brodsky, Lepage, `81
Collins, Soper, Sterman, `82,`85
Bodwin, `85

- Factorization of the Drell-Yan process
- Loop diagrams contain a Glauber region which gives a leading order IR divergent contribution (on top of Soft and Collinear regions)
- "Glaubers" break the traditional factorization of the exclusive Drell-Yan cross-section
- In the inclusive cross-section this contribution cancels: $G+G^{*}=0$ and factorization is restored

Do we need Glauber modes in the Effective Theory?

Why is the presence of Glauber modes important?

- Glauber interactions happen between initial state spectator partons and they break the simple factorization in the exclusive cross-section
- Factorization is the key ingredient to make predictions for high energy QCD cross-sections
- Factorization of any process in hadron-hadron collisions needs analysis of Glauber modes
- Conceptual issue: do we have all the necessary low energy modes included into SCET?
- "Glaubers" play an important role for jet propagating in dense QCD media
  - Idilbi, Majumder `08,
  - D`Eramo, Liu, Rajagopal, 10

A Matching calculation for Drell-Yan

Idea of the calculation

- We want to set up a matching calculation which involves "Drell-Yan"-like one loop diagrams
- Should be a matching between QCD and EFT_1, and EFT_2, where

EFT_1=SCET (collinear, ultrasoft)

EFT_2=SCET+Glaubers

- By comparing the two matching calculations we should be able to find out which effective theory consistently describes the Drell-Yan amplitude

Operator O_2

Operator O_2 arises from matching the QCD current onto a 2-jet Effective Theory:

$$\mathcal{J} = \bar{q}\,\Gamma q$$

$$\mathcal{J} = C_2 O_2$$

where O_2 is defined as:

$$O_2 = \chi_n\,\Gamma\chi_{\bar{n}}$$

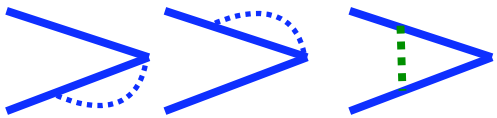
C_2 is Wilson coefficient which is well known to higher orders

Operator O_2

The Idea

Simplest final states to calculate C_2 are $\langle 0|$ and $|q\bar{q}\rangle$:

$$\langle q\bar{q}|J|0\rangle = C_2 \langle q\bar{q}|O_2|0\rangle$$



For our purpose we will chose instead states: $\langle \gamma\gamma|$ and $|q\bar{q}\rangle$

$$\langle \gamma\gamma|J|q\bar{q}\rangle = C_2 \langle \gamma\gamma|O_2|q\bar{q}\rangle$$

EFT_1 $\quad C_2 = ?$

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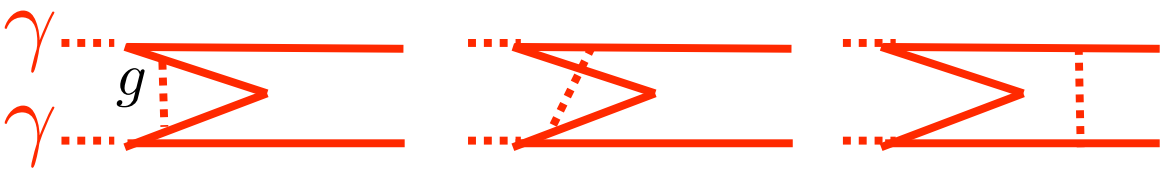

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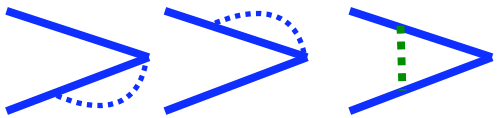

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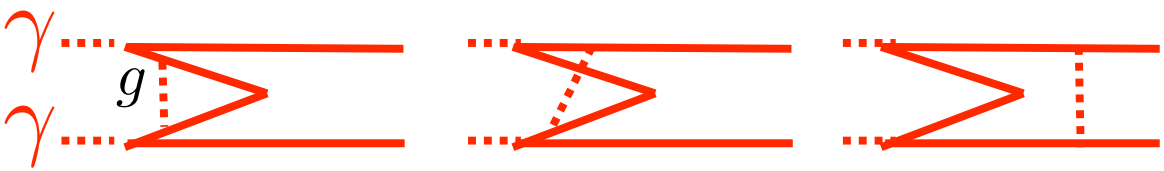
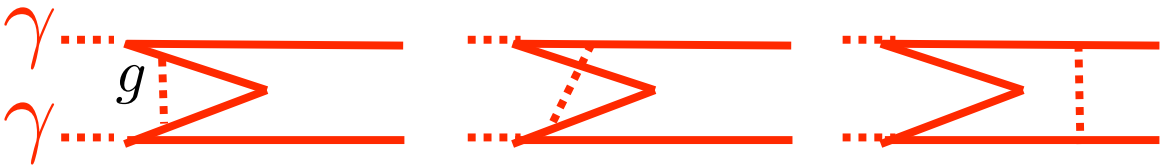


We know the answer for C_2

Outline of the matching



Full Theory

$$\langle\gamma^*\gamma^*|\,O_2\,|\bar{q}q\rangle_{\rm FT} = \frac{1}{p^2\bar{q}^2}I_3 + \frac{1}{\bar{q}^2}I_4^{(n\bar{n})} + \frac{1}{p^2}I_4^{(\bar{n}n)} + I_5$$

EFT$_{\rm I}$

$$\langle\gamma^*\gamma^*|\,O_2\,|\bar{q}q\rangle_{\rm EFT_1} = \frac{1}{p^2\bar{q}^2}(I_3^c + I_3^{\bar{c}} + I_3^s) + \frac{1}{\bar{q}^2}(I_4^{(n\bar{n})c} + I_4^{(n\bar{n})s}) + \frac{1}{p^2}(I_4^{(\bar{n}n)\bar{c}} + I_4^{(\bar{n}n)s}) + I_5^s$$

EFT$_{\rm II}$

$$\begin{aligned}\langle\gamma^*\gamma^*|\,O_2\,|\bar{q}q\rangle_{\rm EFT_2} \quad = \quad & \frac{1}{p^2\bar{q}^2}(I_3^{c'} + I_3^{\bar{c}'} + I_3^g + I_3^s) + \frac{1}{\bar{q}^2}(I_4^{(n\bar{n})c'} + I_4^{(n\bar{n})g} + I_4^{(n\bar{n})s}) \\ & + \frac{1}{p^2}(I_4^{(\bar{n}n)\bar{c}'} + I_4^{(\bar{n}n)g} + I_4^{(\bar{n}n)s}) + I_5^g + I_5^s\,.\end{aligned}$$

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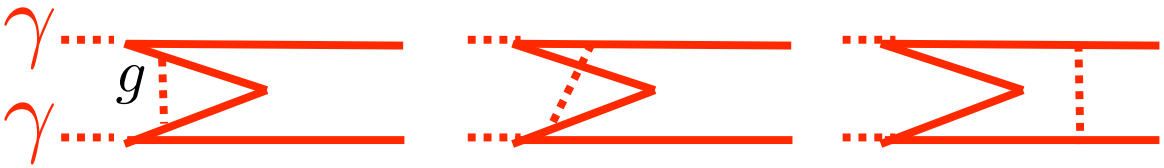

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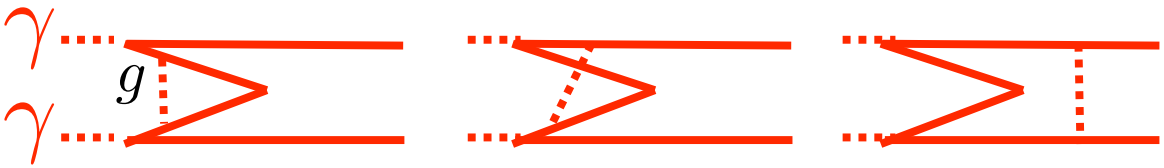


Full Theory

$$\langle\gamma^*\gamma^*|\,O_2\,|\bar{q}q\rangle_{\rm FT} = \frac{1}{p^2\bar{q}^2}I_3 + \frac{1}{\bar{q}^2}I_4^{(n\bar{n})} + \frac{1}{p^2}I_4^{(\bar{n}n)} + I_5$$

EFT$_{\rm I}$

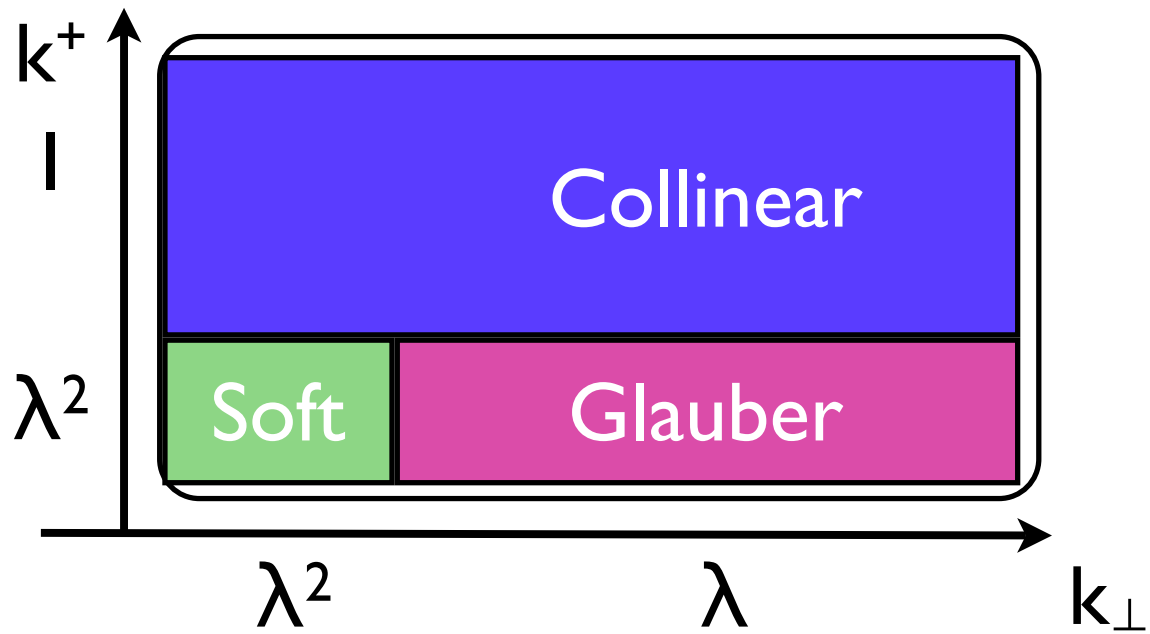
$$\langle\gamma^*\gamma^*|\,O_2\,|\bar{q}q\rangle_{\rm EFT_1} = \frac{1}{p^2\bar{q}^2}(I_3^c + I_3^{\bar{c}} + I_3^s) + \frac{1}{\bar{q}^2}(I_4^{(n\bar{n})c} + I_4^{(n\bar{n})s}) + \frac{1}{p^2}(I_4^{(\bar{n}n)\bar{c}} + I_4^{(\bar{n}n)s}) + I_5^s$$

EFT$_{\rm II}$

$$\begin{aligned}\langle\gamma^*\gamma^*|\,O_2\,|\bar{q}q\rangle_{\rm EFT_2} \quad = \quad & \frac{1}{p^2\bar{q}^2}(I_3^{c'} + I_3^{\bar{c}'} + I_3^g + I_3^s) + \frac{1}{\bar{q}^2}(I_4^{(n\bar{n})c'} + I_4^{(n\bar{n})g} + I_4^{(n\bar{n})s}) \\ & + \frac{1}{p^2}(I_4^{(\bar{n}n)\bar{c}'} + I_4^{(\bar{n}n)g} + I_4^{(\bar{n}n)s}) + I_5^g + I_5^s\,.\end{aligned}$$

Zero-bin subtractions in EFT_2

Manohar, Stewart (`06)



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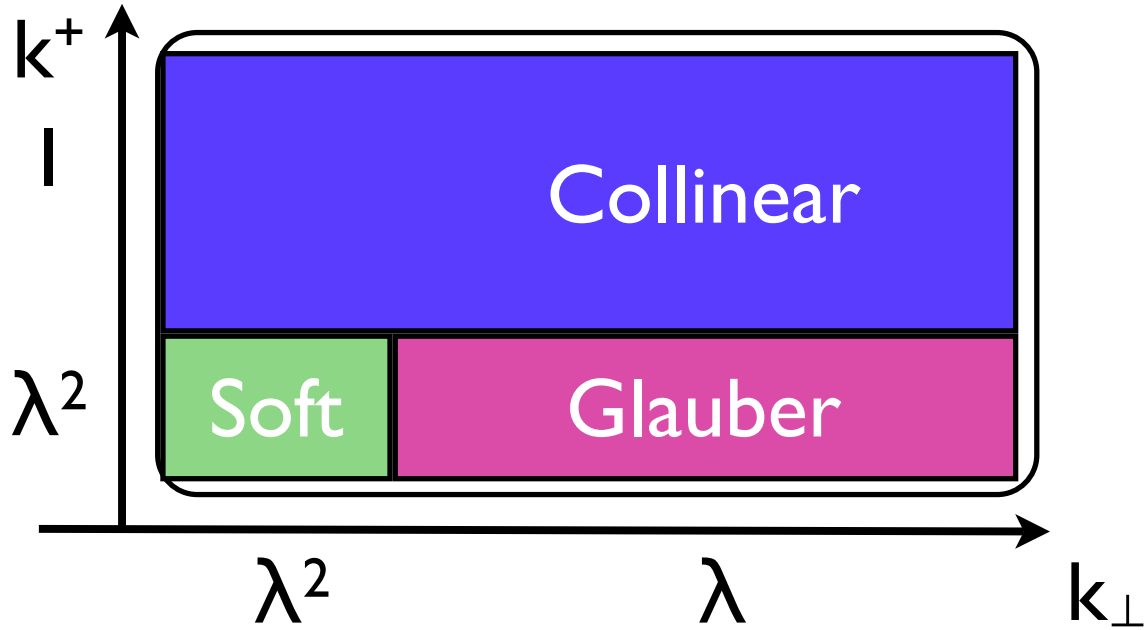
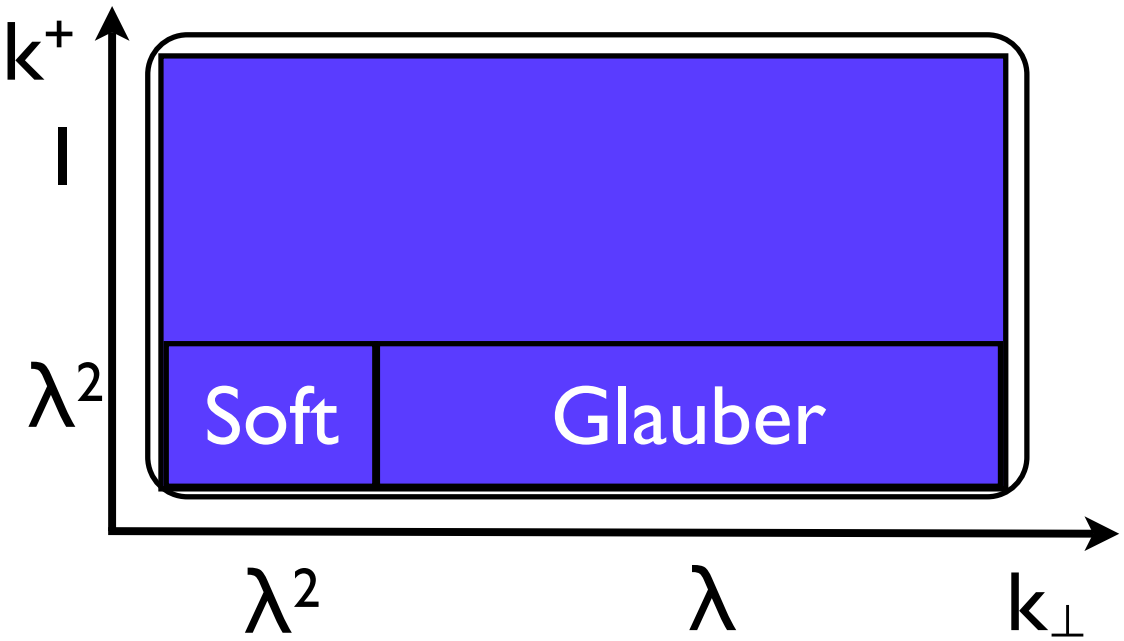
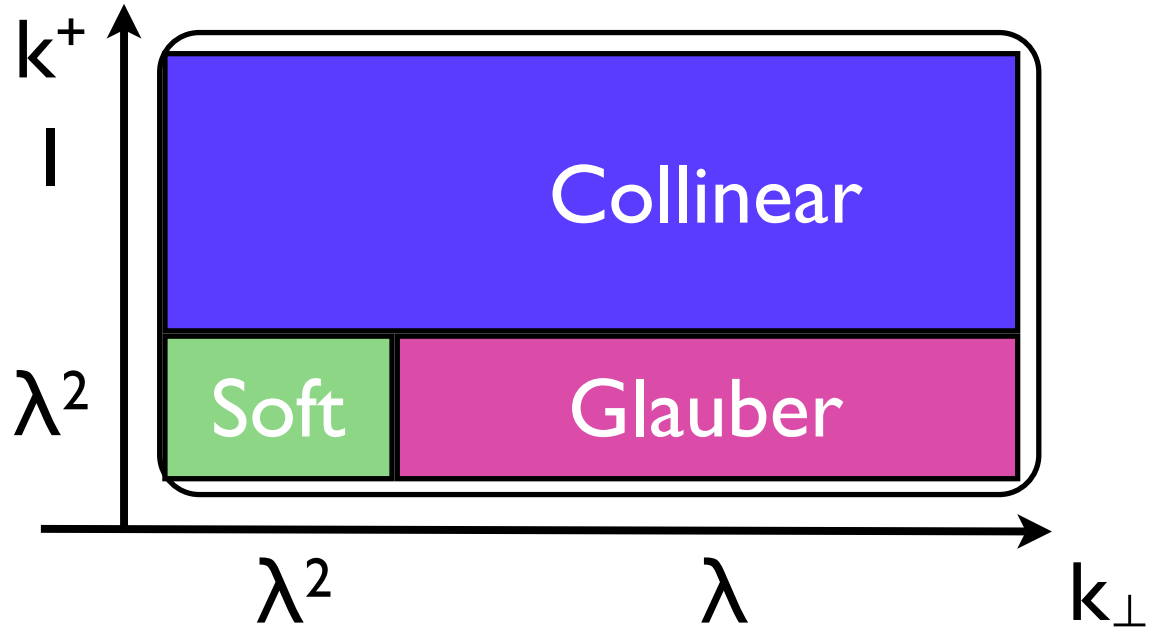


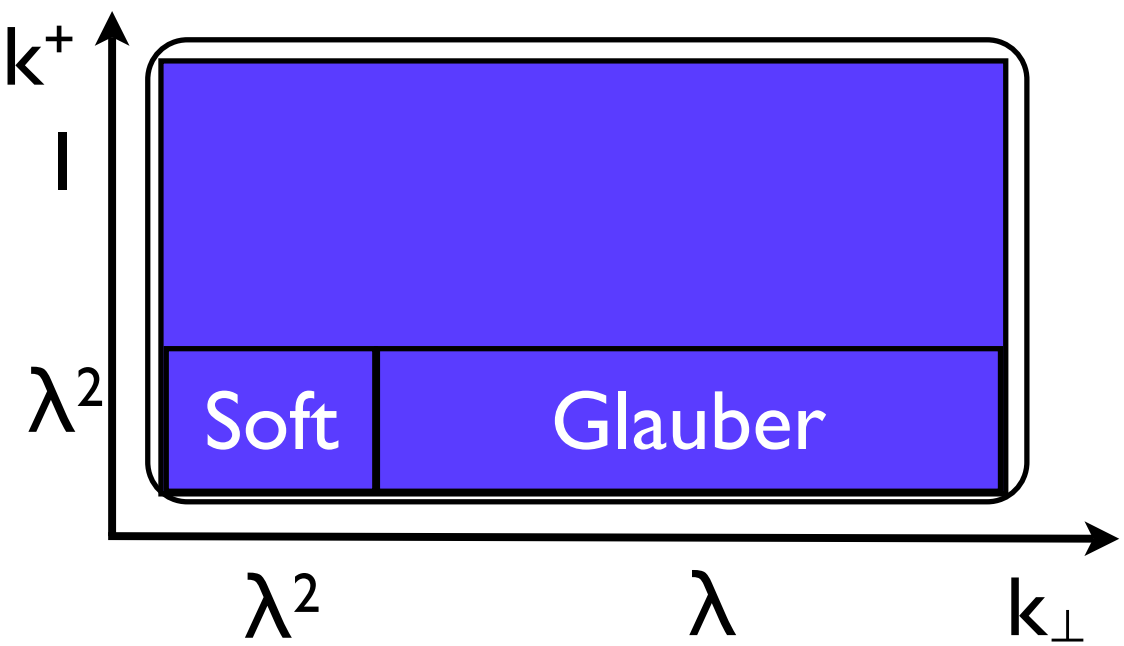


Zero-bin subtractions in EFT_2

Manohar, Stewart (`06)

$$C`_n = C - (C_g - C_{gs} + C_s)$$





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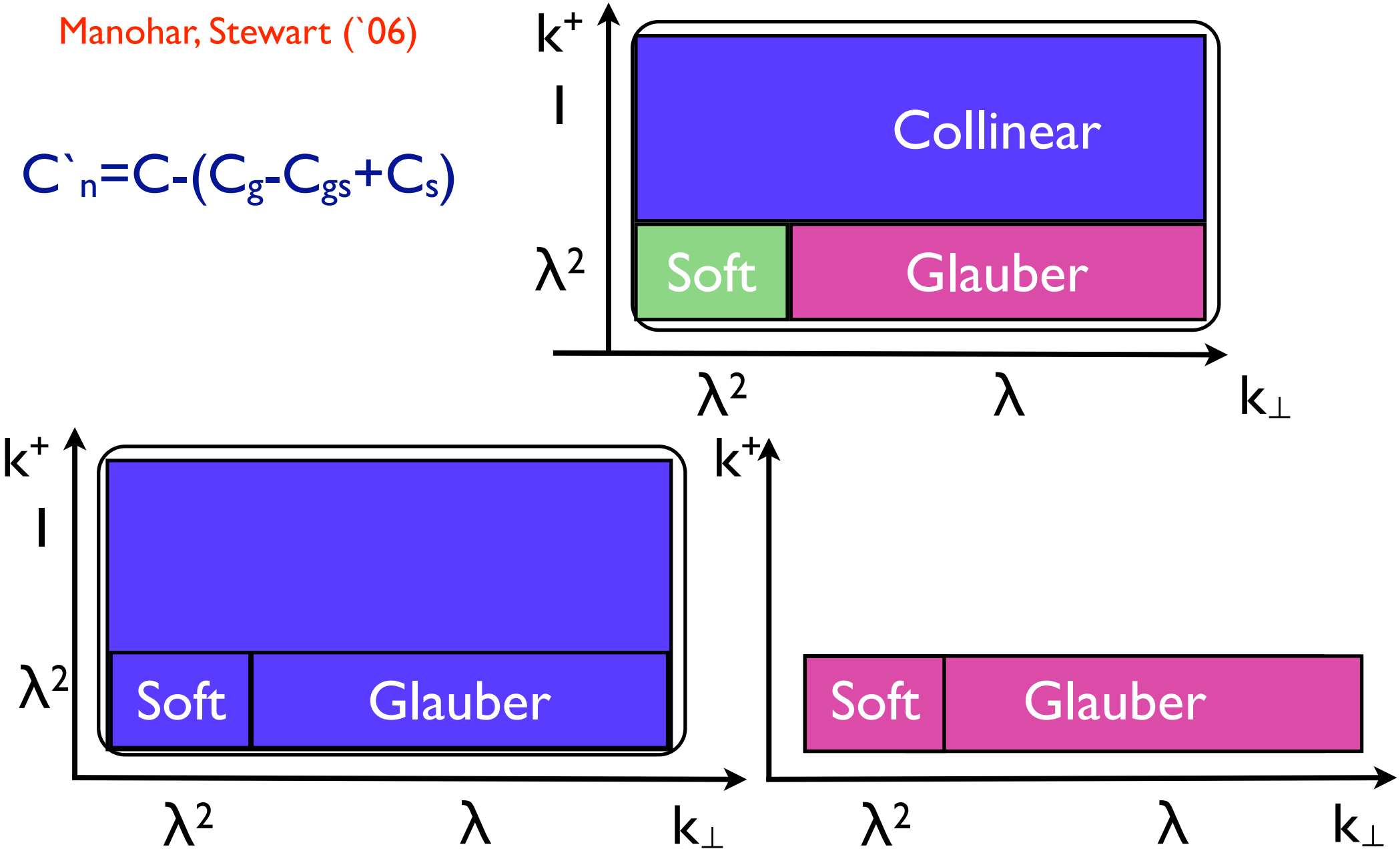
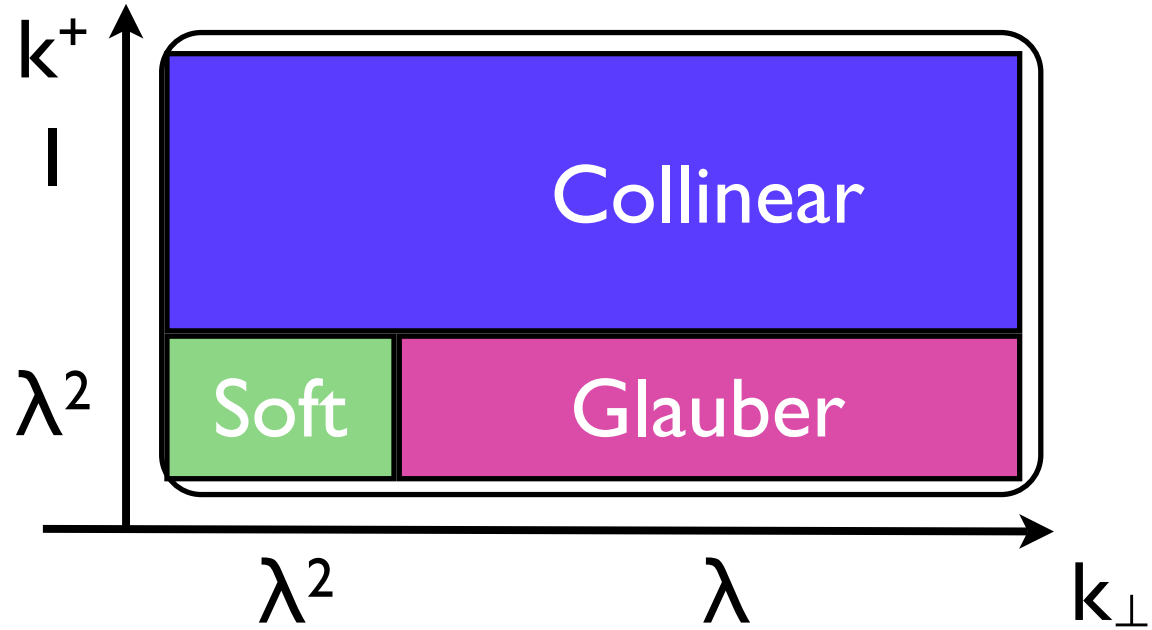


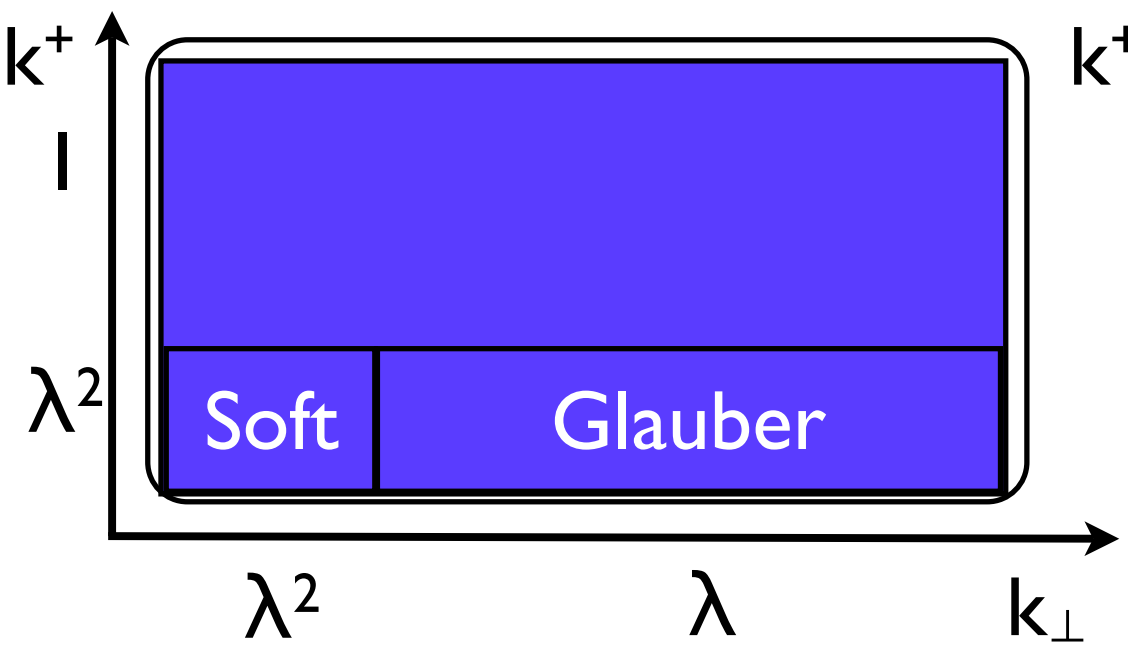
Zero-bin subtractions in EFT_2

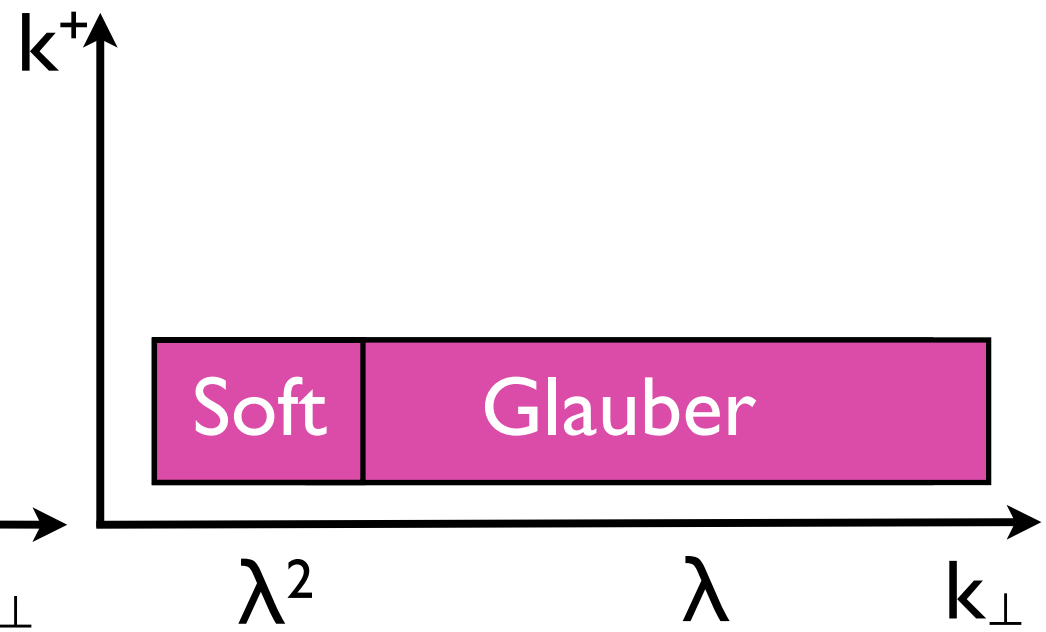
Manohar, Stewart (`06)

$C`_n = C - (C_g - C_{gs} + C_s)$

$G_n = G - G_s$

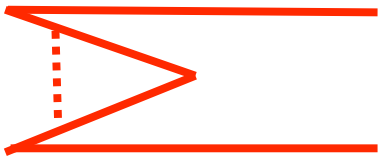






Active-Active topology

QCD=I_3= $\int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 (l+p)^2 (l-\bar{q})^2}$



n-collinear ($1,\lambda^2,\lambda$):

$$I_3^c = \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 (l+p)^2 [-\bar{q}^- l^+]} \propto \lambda^{-4}$$

$\bar{n}$-collinear ($\lambda^2, 1, \lambda$):

$$I_3^{\bar{c}} = \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 [p^+ l^-] (l-\bar{q})^2} \propto \lambda^{-4}$$

EFT$_1$

Soft ($\lambda^2,\lambda^2, \lambda^2$):

$$I_3^s = \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 [p^+ l^- + p^2][-\bar{q}^- l^+ + \bar{q}^2]} \propto \lambda^{-4}$$

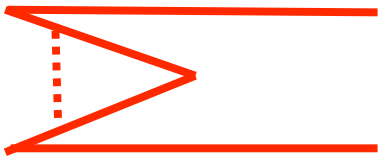
EFT$_2$

Glauber ($\lambda^2,\lambda^2,\lambda$):

$$I_3^g = \int \frac{d^D l}{(2\pi)^D} \frac{1}{[-l_\perp^2][p^+(l^- + p^-) - (l_\perp + p_\perp)^2][-\bar{q}^-(l^+ - \bar{q}^+) - (l_\perp - \bar{q}_\perp)^2]} \propto \lambda^{-4}$$

Active-Active topology

Contribution to the matching



In the First effective theory all modes including overlaps equal:

$$EFT_1 = I_3^{c} + I_3^{\bar{c}} + I_3^{s}$$

In the Second effective theory all modes including overlaps equal:

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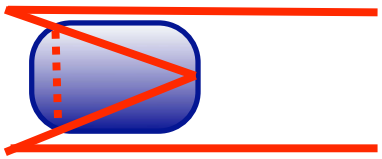
Active-Active topology

Contribution to the matching



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Active-Active topology

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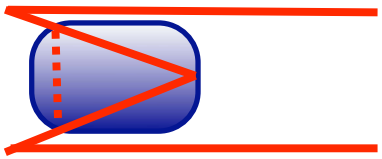

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In the Second effective theory all modes including overlaps equal:

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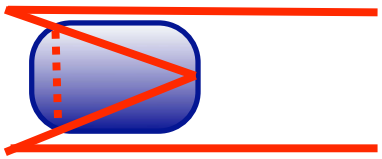
Active-Active topology

Contribution to the matching



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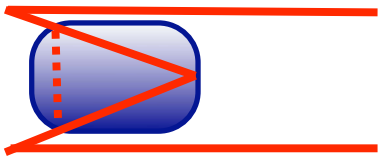
All zero bin integrals and Glauber integral scaleless!

Active-Active topology

Contribution to the matching



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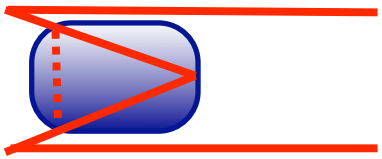

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In the Second effective theory all modes including overlaps equal:

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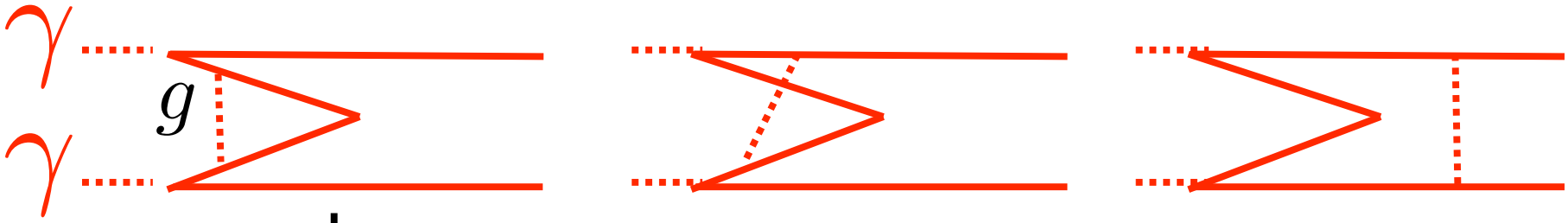
All zero bin integrals and Glauber integral scaleless!

So, for active-active graph we find:

$$EFT_1 \equiv EFT_2$$

Status of the calculation



gives C_2

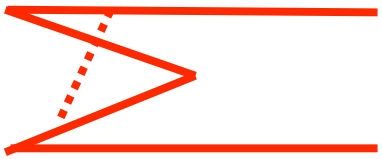
$\langle\gamma\gamma|J|q\bar{q}\rangle = C_2\langle\gamma\gamma|O_2|q\bar{q}\rangle$

EFT_1 $C_2 = C_2 + ? + ?$

EFT_2 $C_2 = C_2 + ? + ?$

Spectator-Active topology

QCD=I_4= $\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[l^2][(l-\bar{p})^2][(l+p)^2][(l-\bar{q})^2]}$



n-collinear ($1,\lambda^2,\lambda$):

$$I_4^c = \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[l^2][(l-\bar{p})^2][(l+p)^2][-l^+\bar{q}^-]} \propto \lambda^{-4}$$

$\bar{n}$-collinear ($\lambda^2,1,\lambda$):

$$I_4^{\bar{c}} = \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[l^2][-\bar{p}^+l^-][p^+l^- + p^2][(l-\bar{q})^2]} \propto \lambda^{-2}$$

Soft ($\lambda^2,\lambda^2,\lambda^2$):

$$I_4^s = \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[l^2][-\bar{p}^+l^- + \bar{p}^2][p^+l^- + p^2][-\bar{q}^-l^+ + \bar{q}^2]} \propto \lambda^{-4}$$

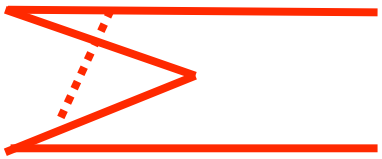
Glauber ($\lambda^2,\lambda^2,\lambda$):

$$I_4^g = \int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[-l_\perp^2][-\bar{p}^+(l^- - \bar{p}^-) - (l_\perp - \bar{p}_\perp)^2][p^+(l^- + p^-) - (l_\perp + p_\perp)^2][-\bar{q}^-(l^+ - \bar{q}^+) - (l_\perp - \bar{q}_\perp)^2]} \propto \lambda^{-4}$$

EFT_1 EFT_2

Spectator-Active topology

Contribution to the matching



In the First effective theory all modes including overlaps equal:

$$EFT_1 = I_4^c + I_4^s$$

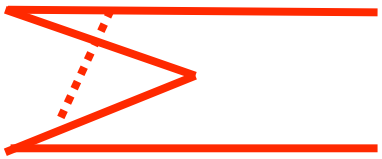
In the Second effective theory all modes including overlaps equal:

$$EFT_2 = I_4^{c`} + I_4^g + I_4^s$$

The only non-zero zero bin is: $(I_3^{c`})_{0g} = I_4^g$

Spectator-Active topology

Contribution to the matching



In the First effective theory all modes including overlaps equal:

$$EFT_1 = I_4^c + I_4^s$$

In the Second effective theory all modes including overlaps equal:

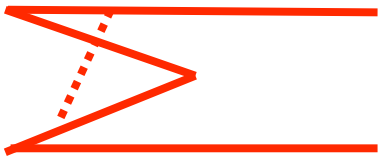
$$EFT_2 = I_4^{c`} + I_4^g + I_4^s$$

The only non-zero zero bin is: $(I_3^{c`})_{0g} = I_4^g$

So, for spectator-active graph we find: $EFT_1 \equiv EFT_2$

Spectator-Active topology

Contribution to the matching



In the First effective theory all modes including overlaps equal:

$$EFT_I = I_4^c + I_4^s$$

In the Second effective theory all modes including overlaps equal:

$$EFT_2 = I_4^{c`} + I_4^g + I_4^s$$

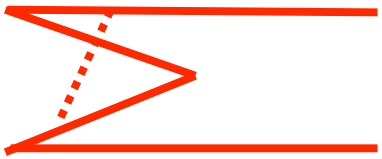
The only non-zero zero bin is: $(I_3^{c`})_{0g} = I_4^g$

So, for spectator-active graph we find: $EFT_I \equiv EFT_2$

$$\Delta C_2 = (I_4 - EFT_{I,II})/Tree = ?$$

Spectator-Active topology

Contribution to the matching



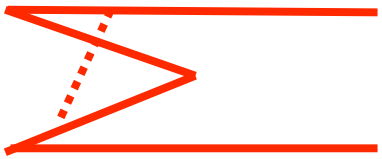
$$I_4 = \frac{i}{16\pi^2}\frac{1}{\bar{q}^-}\frac{1}{\bar{p}^2 p^+ + p^2\bar{p}^+}\left(\frac{\pi^2}{3} - 2\,\mathrm{Li}_2\left(-\frac{p^2\bar{p}^+}{\bar{p}^2 p^+}\right) + \left(\ln\left(\frac{\bar{p}^2 p^+}{p^2\bar{p}^+}\right) - i\,\pi\right)\ln\left(\frac{\bar{q}^-\left(p^+\bar{p}^2 + \bar{p}^+ p^2\right)^2}{\bar{q}^2(p+\bar{p})^2 p^+\bar{p}^2}\right)\right) + \mathcal{O}\left(\epsilon, \lambda^0\right).$$

$$I_4^{(n\bar{n})s} = \frac{i}{16\pi^2}\frac{1}{\bar{q}^-}\frac{1}{\bar{p}^2 p^+ + p^2\bar{p}^+}\left(-\frac{\ln\frac{p^2\bar{p}^+}{\bar{p}^2 p^+} + i\pi}{\epsilon} + \frac{1}{2}\ln\frac{\bar{p}^2 p^+}{p^2\bar{p}^+}\ln\frac{\mu^4 p^+\bar{p}^+(\bar{q}^-)^2}{p^2\bar{p}^2(\bar{q}^2)^2} - i\pi\ln\frac{\mu^2 p^2(\bar{p}^+)^2\bar{q}^-}{\bar{q}^2(\bar{p}^2)^2 p^+} + \frac{3}{2}\pi^2\right)$$

$$I_4^{(n\bar{n})c} = \frac{i}{16\pi^2}\frac{1}{\bar{q}^-}\frac{1}{\bar{p}^2 p^+ + p^2\bar{p}^+}\left(\frac{\ln\frac{p^2\bar{p}^+}{\bar{p}^2 p^+} + i\pi}{\epsilon} - \frac{7\pi^2}{6} - 2\,\mathrm{Li}_2\left(-\frac{p^2\bar{p}^+}{\bar{p}^2 p^+}\right) + i\pi\ln\frac{\mu^2 p^2(p+\bar{p})^2(\bar{p}^+)^2}{(\bar{p}^2 p^+ + p^2\bar{p}^+)^2\bar{p}^2} + \right.$$
$$\left.\ln\frac{\bar{p}^2 p^+}{p^2\bar{p}^+}\left(\ln\frac{(\bar{p}^2 p^+ + p^2\bar{p}^+)^2}{(p+\bar{p})^2 p^+\bar{p}^+\bar{p}^2} - \frac{1}{2}\ln\frac{\mu^4 p^+}{p^2\bar{p}^2\bar{p}^+}\right)\right)$$

Spectator-Active topology

Contribution to the matching



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$$I_4^{(n\bar{n})s} = \frac{i}{16\pi^2}\frac{1}{\bar{q}^-}\frac{1}{\bar{p}^2 p^+ + p^2\bar{p}^+}\left(-\frac{\ln\frac{p^2\bar{p}^+}{\bar{p}^2 p^+} + i\pi}{\epsilon} + \frac{1}{2}\ln\frac{\bar{p}^2 p^+}{p^2\bar{p}^+}\ln\frac{\mu^4 p^+\bar{p}^+(\bar{q}^-)^2}{p^2\bar{p}^2(\bar{q}^2)^2} - i\pi\ln\frac{\mu^2 p^2(\bar{p}^+)^2\bar{q}^-}{\bar{q}^2(\bar{p}^2)^2 p^+} + \frac{3}{2}\pi^2\right)$$

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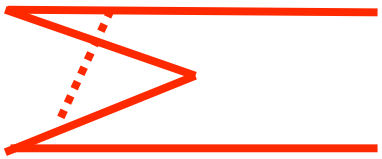
$$\left.\ln\frac{\bar{p}^2 p^+}{p^2\bar{p}^+}\left(\ln\frac{(\bar{p}^2 p^+ + p^2\bar{p}^+)^2}{(p+\bar{p})^2 p^+\bar{p}^+\bar{p}^2} - \frac{1}{2}\ln\frac{\mu^4 p^+}{p^2\bar{p}^2\bar{p}^+}\right)\right)$$

$\Delta C_2 = (I_4 - \mathrm{EFT}_I)/\mathrm{Tree} = 0$

Spectator-Active topology

Contribution to the matching



$$I_4 = \frac{i}{16\pi^2}\frac{1}{\bar{q}^-}\frac{1}{\bar{p}^2 p^+ + p^2\bar{p}^+}\left(\frac{\pi^2}{3} - 2\,\mathrm{Li}_2\left(-\frac{p^2\bar{p}^+}{\bar{p}^2 p^+}\right) + \left(\ln\left(\frac{\bar{p}^2 p^+}{p^2\bar{p}^+}\right) - i\,\pi\right)\ln\left(\frac{\bar{q}^-\left(p^+\bar{p}^2 + \bar{p}^+ p^2\right)^2}{\bar{q}^2(p+\bar{p})^2 p^+\bar{p}^2}\right)\right) + \mathcal{O}\left(\epsilon, \lambda^0\right).$$

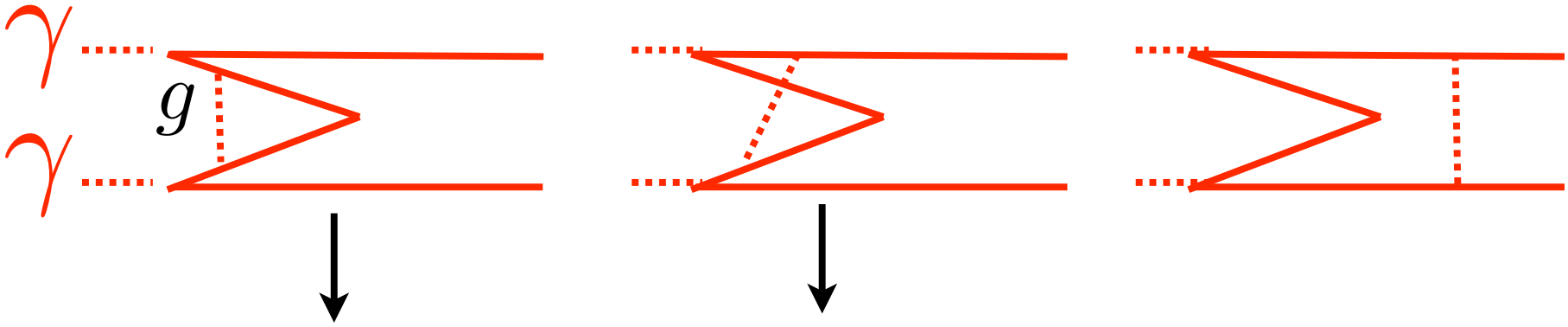
$$I_4^{(n\bar{n})s} = \frac{i}{16\pi^2}\frac{1}{\bar{q}^-}\frac{1}{\bar{p}^2 p^+ + p^2\bar{p}^+}\left(-\frac{\ln\frac{p^2\bar{p}^+}{\bar{p}^2 p^+} + i\pi}{\epsilon} + \frac{1}{2}\ln\frac{\bar{p}^2 p^+}{p^2\bar{p}^+}\ln\frac{\mu^4 p^+\bar{p}^+(\bar{q}^-)^2}{p^2\bar{p}^2(\bar{q}^2)^2} - i\pi\ln\frac{\mu^2 p^2(\bar{p}^+)^2\bar{q}^-}{\bar{q}^2(\bar{p}^2)^2 p^+} + \frac{3}{2}\pi^2\right)$$

$$I_4^{(n\bar{n})c} = \frac{i}{16\pi^2}\frac{1}{\bar{q}^-}\frac{1}{\bar{p}^2 p^+ + p^2\bar{p}^+}\left(\frac{\ln\frac{p^2\bar{p}^+}{\bar{p}^2 p^+} + i\pi}{\epsilon} - \frac{7\pi^2}{6} - 2\,\mathrm{Li}_2\left(-\frac{p^2\bar{p}^+}{\bar{p}^2 p^+}\right) + i\pi\ln\frac{\mu^2 p^2(p+\bar{p})^2(\bar{p}^+)^2}{(\bar{p}^2 p^+ + p^2\bar{p}^+)^2\bar{p}^2} + \ln\frac{\bar{p}^2 p^+}{p^2\bar{p}^+}\left(\ln\frac{(\bar{p}^2 p^+ + p^2\bar{p}^+)^2}{(p+\bar{p})^2 p^+\bar{p}^+\bar{p}^2} - \frac{1}{2}\ln\frac{\mu^4 p^+}{p^2\bar{p}^2\bar{p}^+}\right)\right)$$

$\Delta C_2 = (I_4 - \mathrm{EFT}_I)/\mathrm{Tree} = 0$

In both effective theories contribution to the matching coefficient is zero

Status of the calculation



gives C_2
(both theories)

gives 0
(both theories)

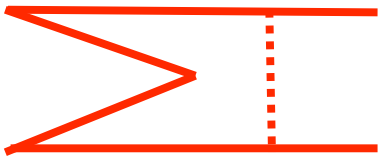
$\langle\gamma\gamma|J|q\bar{q}\rangle = C_2\langle\gamma\gamma|O_2|q\bar{q}\rangle$

EFT$_1$ $C_2 = C_2 + 0 + ?$

EFT$_2$ $C_2 = C_2 + 0 + ?$

Spectator-Spectator topology

QCD=I_5= $\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^2(l-\bar{p})^2(l+p)^2(l-\bar{q})^2(l+q)^2}$



n-collinear ($1,\lambda^2,\lambda$): $\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^2(l-\bar{p})^2(l+p)^2[-l^+\bar{q}^-][l^+q^-]}$ $\propto \lambda^{-2}$

$\bar{n}$-collinear ($\lambda^2,1,\lambda$): $\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^2[-l^-\bar{p}^+][l^-p^+](l-\bar{q})^2(l+q)^2}$ $\propto \lambda^{-2}$

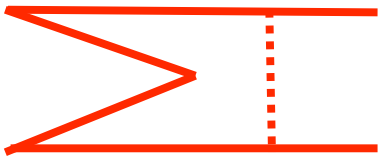
Soft ($\lambda^2,\lambda^2,\lambda^2$): $\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{l^2[-l^-\bar{p}^+ + \bar{p}^2][l^-p^+ + p^2][-l^+\bar{q}^- + \bar{q}^2][l^+q^- + q^2]}$ $\propto \lambda^{-4}$

Glauber ($\lambda^2,\lambda^2,\lambda$):

$$\int \frac{\mathrm{d}^D l}{(2\pi)^D} \frac{1}{[-l_\perp^2][-\bar{p}^+(l^- - \bar{p}^-) - (l_\perp - \bar{p}_\perp)^2][p^+(l^- + p^-) - (l_\perp + p_\perp)^2][-\bar{q}^-(l^+ - \bar{q}^+) - (l_\perp - \bar{q}_\perp)^2]} \frac{1}{[q^-(l^+ + q^+) - (l_\perp + q_\perp)^2]} \propto \lambda^{-4}$$

Spectator-Spectator topology

Contribution to the matching



In the First effective theory we have only Soft mode present:

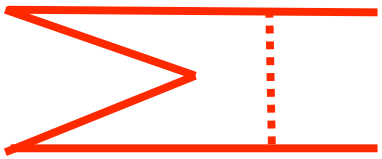
$$EFT_1 = I_5^s = (1/\varepsilon_{UV} + 1/\varepsilon_{IR} + \text{finite})$$

In the Second effective theory all modes including overlaps equal:

$$EFT_2 = I_5^g + I_5^s$$

Spectator-Spectator topology

Contribution to the matching



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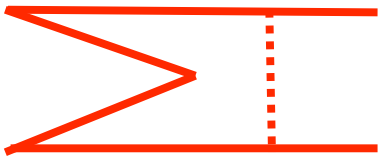
- So, for spectator-active graph EFT_1 and EFT_2 are Not equivalent

Spectator-Spectator topology

Contribution to the matching



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$$EFT_2 = I_5^g + I_5^s$$

- So, for spectator-active graph EFT_1 and EFT_2 are Not equivalent
- In the matching I_5-EFT_1 there is an extra UV divergence which will change the anomalous dimension of C_2

Spectator-Spectator topology

Matching contribution in EFT_2

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)$$

Spectator-Spectator topology

Matching contribution in EFT_2

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)$$

$$I_5^s = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

Spectator-Spectator topology

Matching contribution in EFT_2

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)$$

$$I_5^s = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$I_5^g = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon}-2\pi^2+2\pi\,i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right) +\right.$$

$$\left. + \frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

Spectator-Spectator topology

Matching contribution in EFT_2

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)$$

$$I_5^s = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$I_5^g = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon}-2\pi^2+2\pi\,i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right) +\right.$$

$$\left. + \frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $(I_5^g)_{0s}=0$

Spectator-Spectator topology

Matching contribution in EFT_2

UV divergence

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right) + \mathcal{O}\left(\epsilon,\ \frac{1}{\lambda^2}\right)$$

$$I_5^s = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$I_5^g = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon}-2\pi^2+2\pi\,i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right) +\right.$$

$$\left. + \frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $(I_5^g)_{0s}=0$

Spectator-Spectator topology

Matching contribution in EFT_2 IR divergence

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right) + \mathcal{O}\left(\epsilon,\frac{1}{\lambda^2}\right)$$

$$I_5^s = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

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$$\left. + \frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $(I_5^g)_{0s}=0$

Spectator-Spectator topology

Matching contribution in EFT_2 IR divergence

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\left(\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right)+\mathcal{O}\left(\epsilon,\frac{1}{\lambda^2}\right)$$

$$I_5^s = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

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$$\left.+\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $(I_5^g)_{0s}=0$

Spectator-Spectator topology

Matching contribution in EFT_2 IR divergence

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\left(\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right) + \mathcal{O}\left(\epsilon, \frac{1}{\lambda^2}\right)$$

$$I_5^s = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$I_5^g = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon} - 2\pi^2 + 2\pi\, i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right) +\right.$$

$$\left. + \frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2 - q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2} - \frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $(I_5^g)_{0s}=0$

Spectator-Spectator topology

Matching contribution in EFT_2

Finite term

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right) + \mathcal{O}\left(\epsilon,\frac{1}{\lambda^2}\right)$$

$$I_5^s = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$I_5^g = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon}-2\pi^2+2\pi\,i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right) +\right.$$

$$\left. + \frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right)\right]$$

zero-bin integral is scaleless: $(I_5^g)_{0s}=0$

Spectator-Spectator topology

Matching contribution in EFT_2

Finite term

Pentagon integral is reduced to sum of 5 box integrals:

$$I_5 = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left(\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-Q^4}\right)+\pi^2\right)\right] +$$

$$\frac{i}{16\pi^2}\frac{2\pi i\,Q^2}{p^+\bar{p}^+(p+\bar{p})^2-q^-\bar{q}^-(q+\bar{q})^2}\left(\frac{\ln\left(\frac{Q^4}{q^-\bar{q}^-(q+\bar{q})^2}\right)}{q^-\bar{q}^-(q+\bar{q})^2}-\frac{\ln\left(\frac{Q^4}{p^+\bar{p}^+(p+\bar{p})^2}\right)}{p^+\bar{p}^+(p+\bar{p})^2}\right) + \mathcal{O}\left(\epsilon,\ \frac{1}{\lambda^2}\right)$$

$$I_5^s = \frac{i}{16\pi^2}\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+\bar{q}^-)^2}\left[-\frac{2\,i\pi}{\epsilon}+\ln\left(\frac{\bar{p}^+p^2}{p^+\bar{p}^2}\right)\ln\left(\frac{q^-\bar{q}^2}{\bar{q}^-q^2}\right)+i\pi\ln\left(\frac{\bar{p}^2p^2\bar{q}^2q^2}{\bar{p}^+p^+\bar{q}^-q^-\mu^4}\right)+3\pi^2\right]$$

$$I_5^g = \frac{i}{16\pi^2}\left[\frac{Q^2}{p^+\bar{p}^+(p+\bar{p})^2 q^-\bar{q}^-(q+q)^2}\left(\frac{2\pi i}{\epsilon}-2\pi^2+2\pi\,i\,\ln\left(\frac{\mu^2}{Q^2}\right)\right) +\right.$$

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$$\Delta C_2 = (I_5 - I_5^s - I_5^g)/\text{Tree} = 0$$

Spectator-Spectator topology

$$EFT_1 \neq EFT_2$$

EFT_1

EFT_2

Spectator-Spectator topology

$$EFT_1 \neq EFT_2$$

EFT_1

$$\Delta C_2 = (I_5 - I_5^s)/Tree = 1/\varepsilon_{UV} + 1/\varepsilon_{IR}$$

EFT_2

Spectator-Spectator topology

$$EFT_1 \neq EFT_2$$

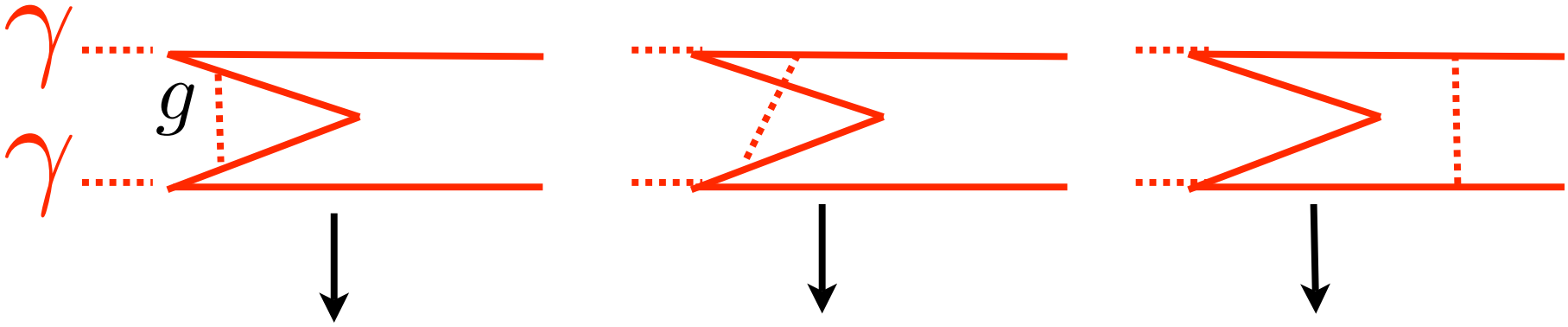
EFT_1

$$\Delta C_2 = (I_5 - I_5^s)/Tree = 1/\varepsilon_{UV} + 1/\varepsilon_{IR}$$

EFT_2

$$\Delta C_2 = (I_5 - I_5^s - I_5^g)/Tree = 0$$

Status of the calculation



gives C_2 (both theories)

gives 0 (both theories)

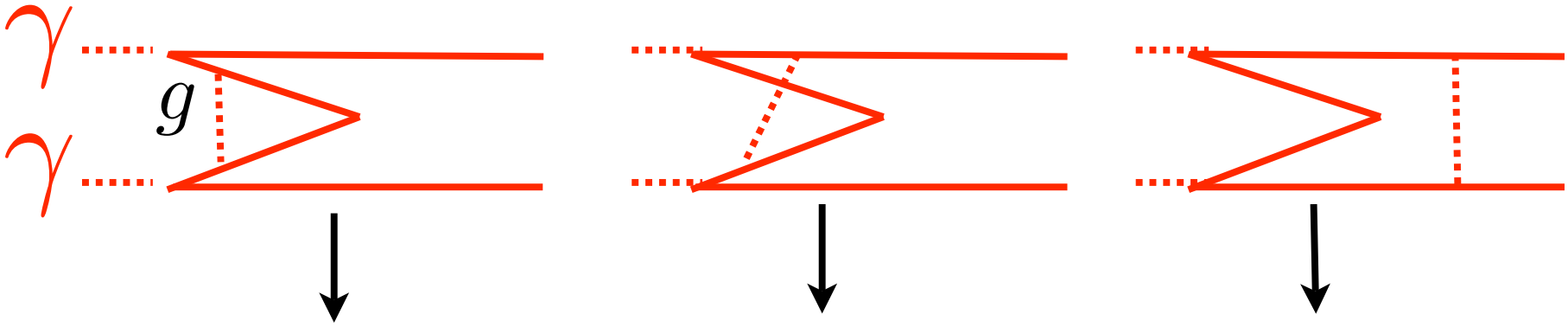
gives 0 in EFT_2 but not in EFT_1

$$\langle\gamma\gamma|J|q\bar{q}\rangle = C_2\langle\gamma\gamma|O_2|q\bar{q}\rangle$$

EFT_1 $C_2 = C_2 + 0 + 1/\varepsilon_{UV} + 1/\varepsilon_{IR}$

EFT_2 $C_2 = C_2 + 0 + 0$

Status of the calculation



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EFT_2 $C_2 = C_2 + 0 + 0$

EFT_2 is an Effective Theory with Glauber modes, and it is the Right one!

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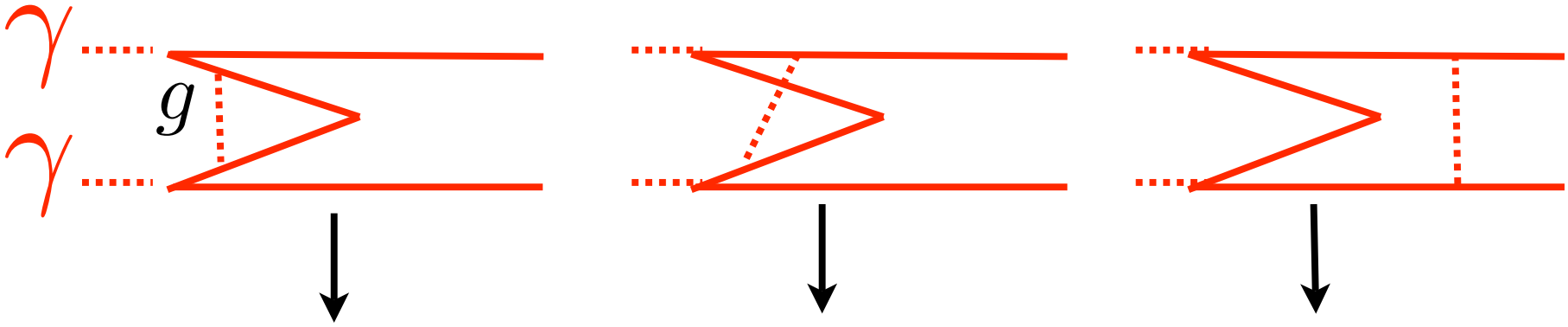

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EFT_1 $C_2 = C_2 + 0 + 1/\varepsilon_{UV} + 1/\varepsilon_{IR}$ ✗

EFT_2 $C_2 = C_2 + 0 + 0$ ✓

EFT_2 is an Effective Theory with Glauber modes, and it is the Right one!

Matching calculation

Summary of the matching calculation

Matching calculation

Summary of the matching calculation

- In active-active and spectator-active topologies putting the Glauber mode or not into SCET doesn't make any difference
- In spectator-spectator topology, the presence of Glauber mode makes a non-trivial contribution to the Drell-Yan amplitude and only including Glaubers into effective theory we get the right answer for the matching coefficient C_2
- Our results are in no conflict with Collins, Soper, Sterman's "pinch" analysis of Drell-Yan loop integrals
- Taking into account the zero-bins, or the overlaps between the different modes was crucial for our analysis

Conclusions

- We completed a one-loop matching calculation for the operator O_2 with special partonic final states
- SCET needs to be expanded for with Glauber mode
- Understanding the cancellation of Glauber gluons in the Drell-Yan cross-section using Effective Theory hasn't been achieved(yet)