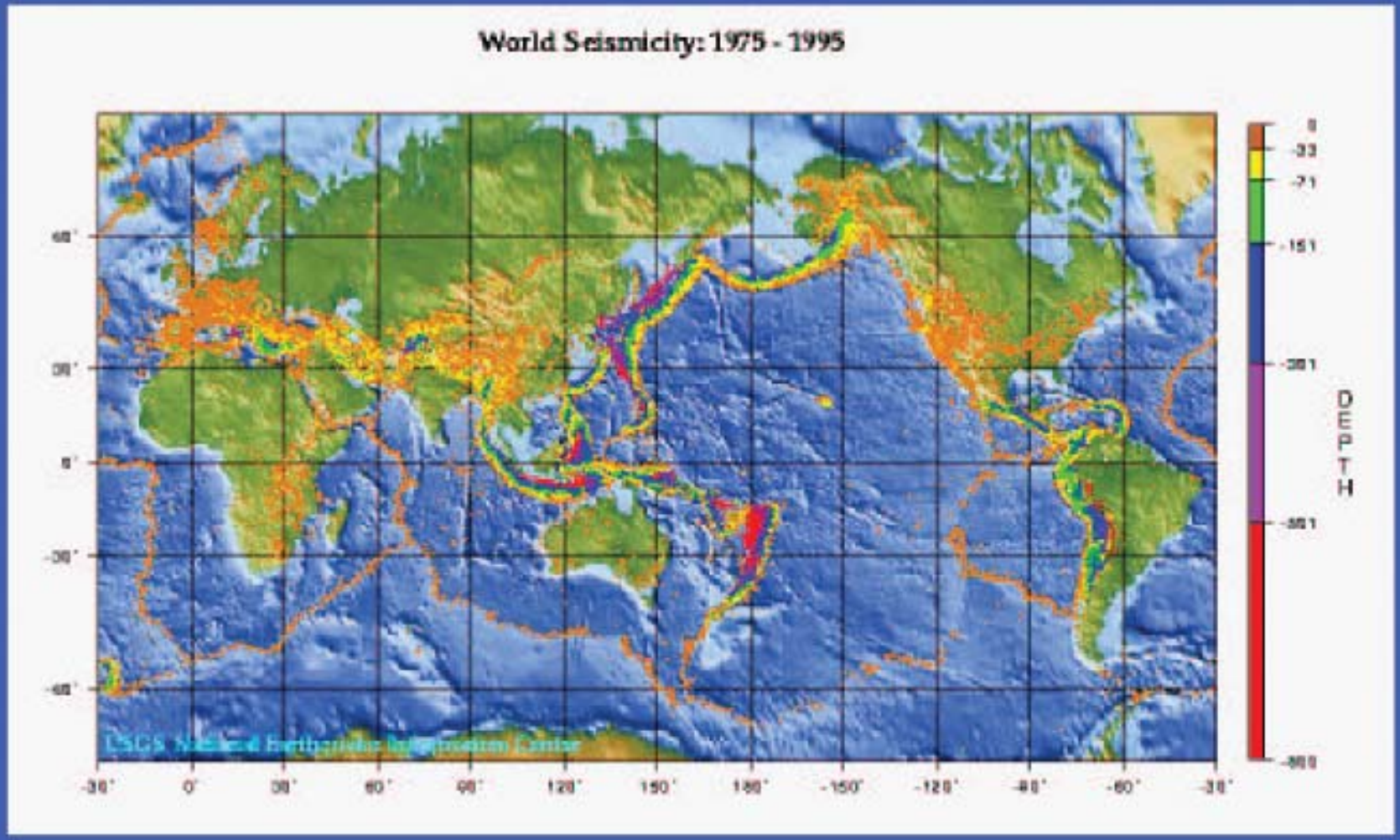
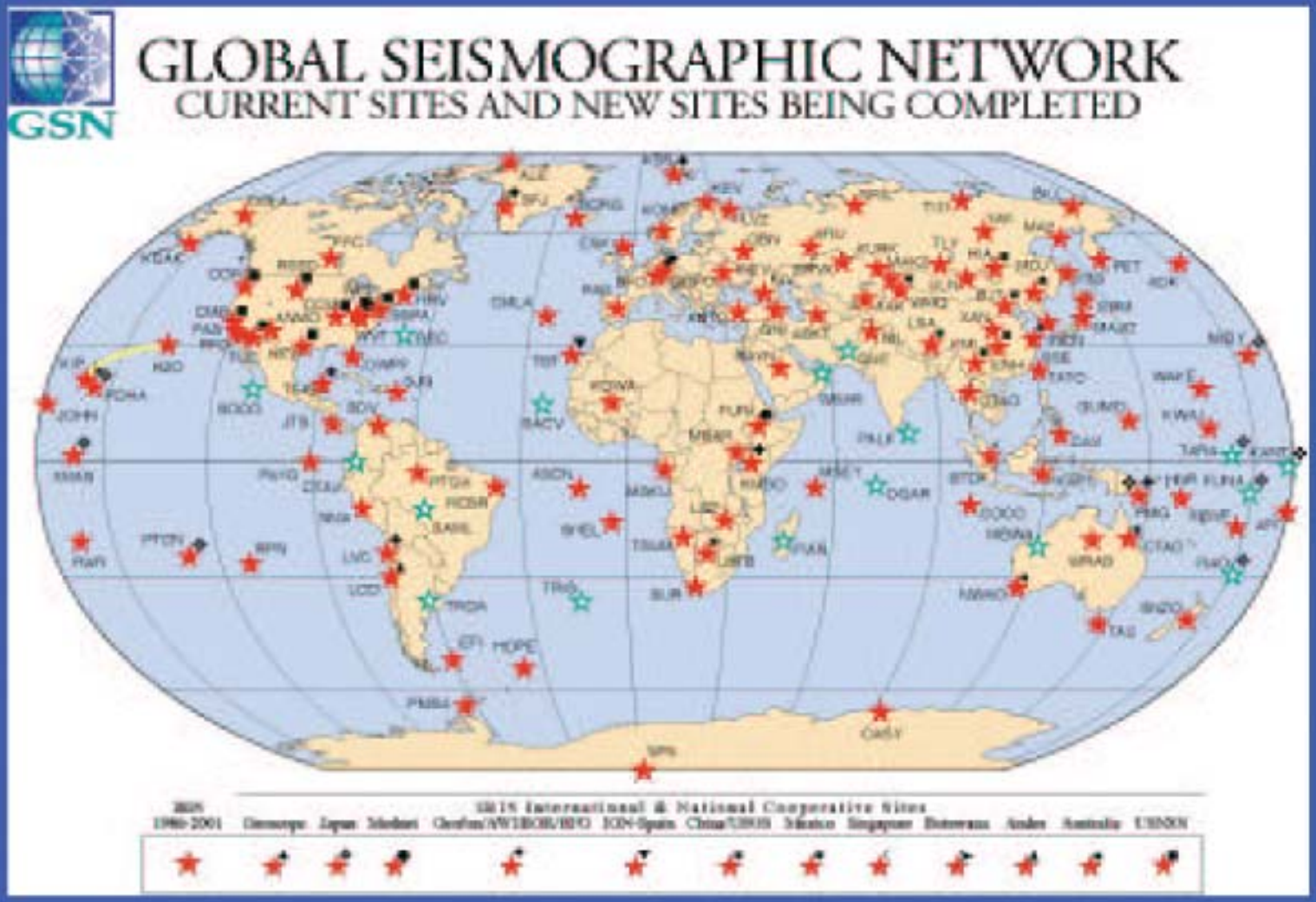
Spectral-Element Simulations of Global Seismic Wave Propagation using the Earth Simulator

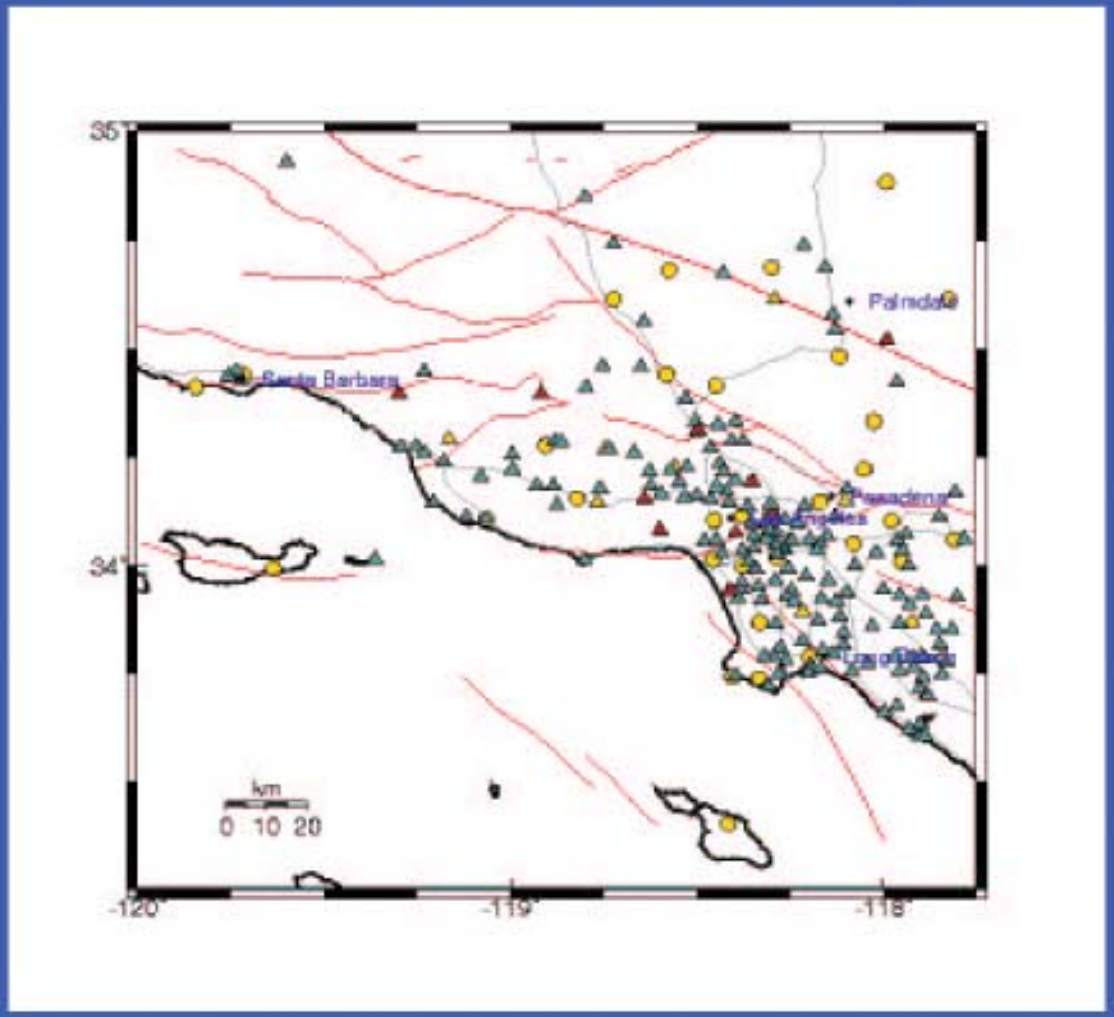
Seiji Tsuboi, Dimitri Komatitsch, Ji Chen & Jeroen Tromp

tsuboi@jamstec.go.jp


World Seismicity: 1975 - 1995
DEPTH
0
-33
-71
-151
-301
-501
-800
60°
30°
0°
-30°
-60°
-30°
0°
30°
60°
90°
120°
150°
180°
-150°
-120°
-90°
-60°
-30°

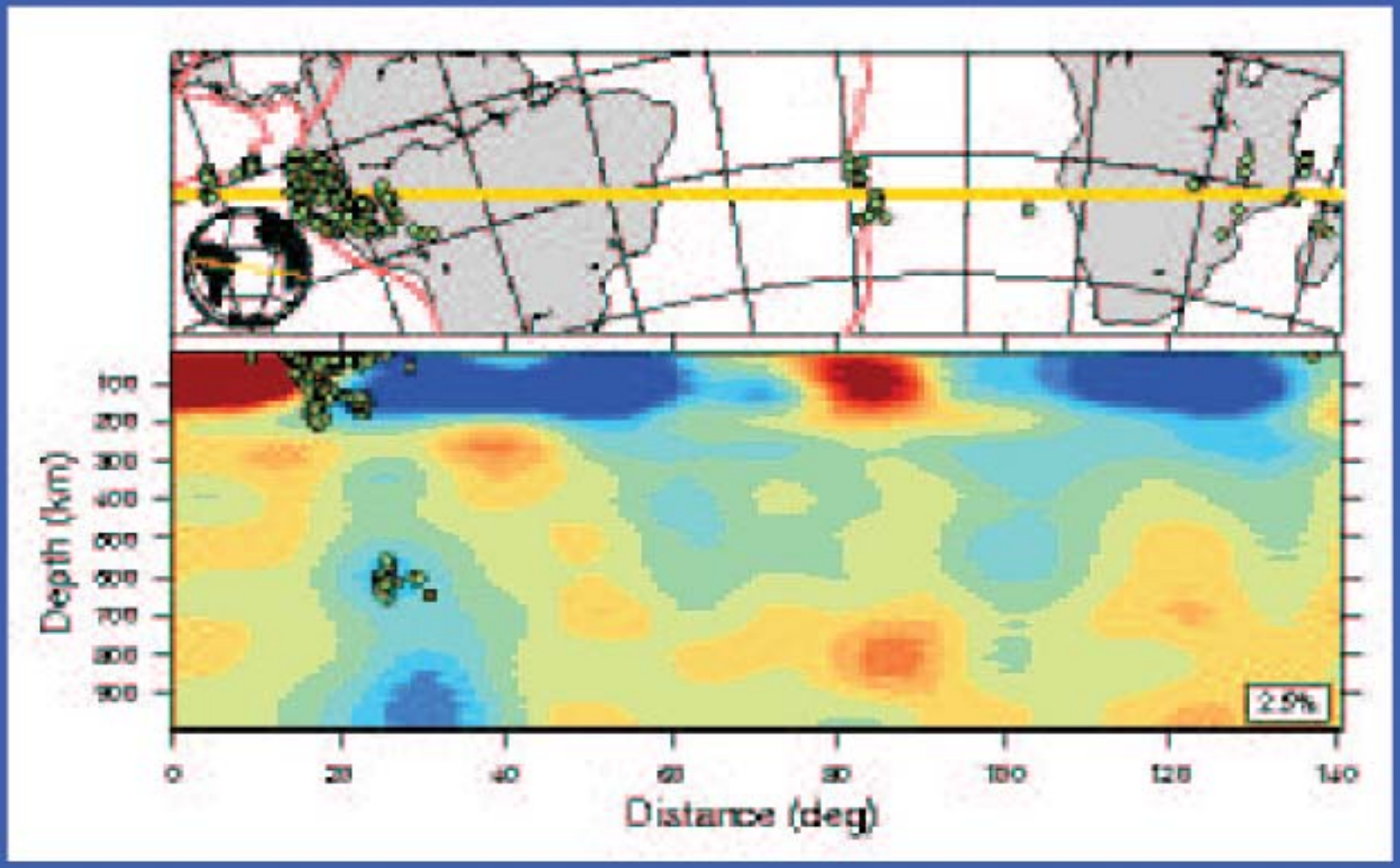

GSN
GLOBAL SEISMOGRAPHIC NETWORK
CURRENT SITES AND NEW SITES BEING COMPLETED

TriNet Stations



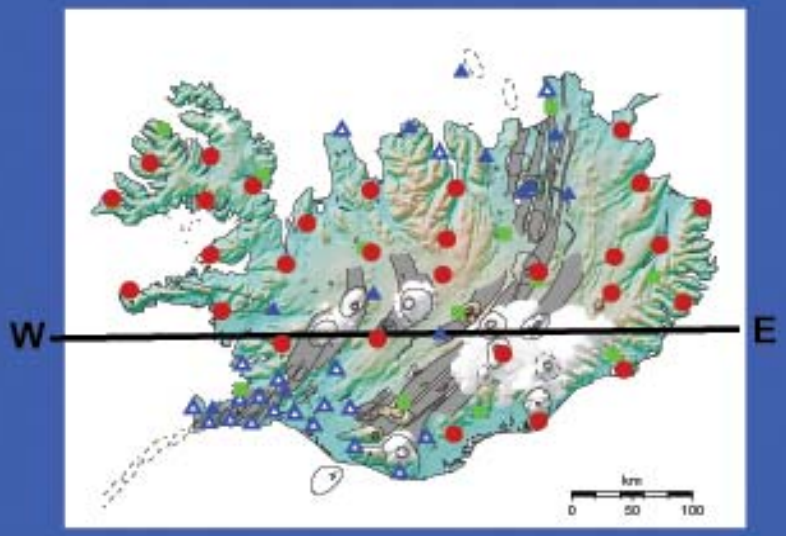
USArray

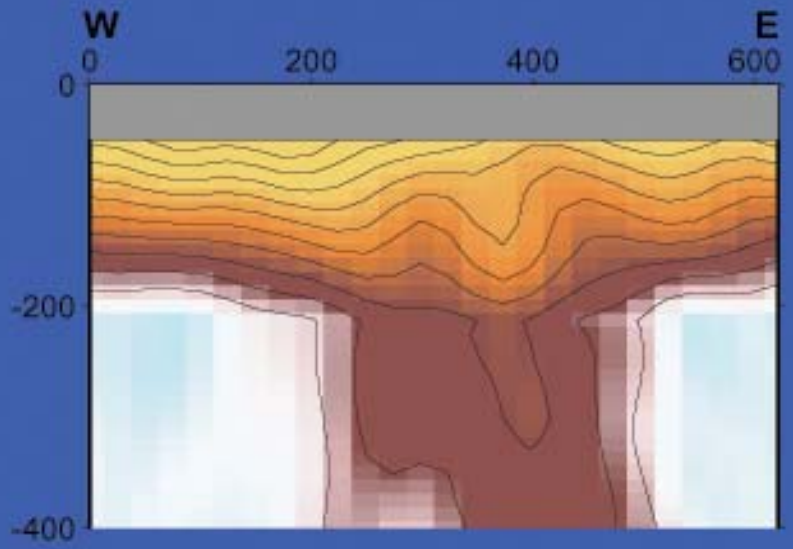
South Atlantic Upper Mantle



Ritsema *et al.* (1999)

The Iceland Plume





Allen (2001)

The Spectral-Element Method

- Developed 15 years ago in Computational Fluid Dynamics
- Accuracy of a pseudospectral method
- Flexibility of a finite-element method

The Challenge of the Globe

- A slow, thin, highly variable crust
- Sharp radial discontinuities
- Fluid-solid boundaries
- Anisotropy
- Attenuation
- Ellipticity, topography & bathymetry
- Rotation
- Self-gravitation

Equations of Motion (Solid)

Differential or *strong* form:

$$\rho\,\partial_t^2 \mathbf{s} = \boldsymbol{\nabla}\cdot\mathbf{T} + \mathbf{f}$$

Integral or *weak* form:

$$\int \rho\,\mathbf{w}\cdot\partial_t^2\mathbf{s}\,\mathrm{d}^3\mathbf{r} = -\int \boldsymbol{\nabla}\mathbf{w} : \mathbf{T}\,\mathrm{d}^3\mathbf{r}$$

$$+\,\mathbf{M} : \boldsymbol{\nabla}\mathbf{w}(\mathbf{r}_\mathrm{s})\,S(t) - \int_{\mathrm{F-S}} \mathbf{w}\cdot\mathbf{T}\cdot\hat{\mathbf{n}}\,\mathrm{d}^2\mathbf{r}$$

Equations of Motion (Fluid)

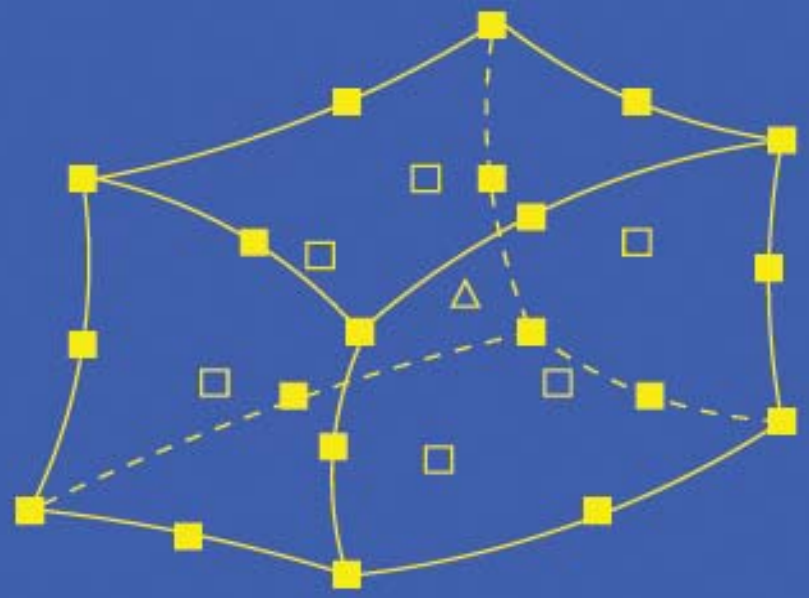
Strong form:

$$\rho\,\partial_t \mathbf{v} = -\boldsymbol{\nabla} p \qquad \partial_t p = -\kappa\,\boldsymbol{\nabla}\cdot\mathbf{v}$$

Define $p = \partial_t \chi$. Weak form:

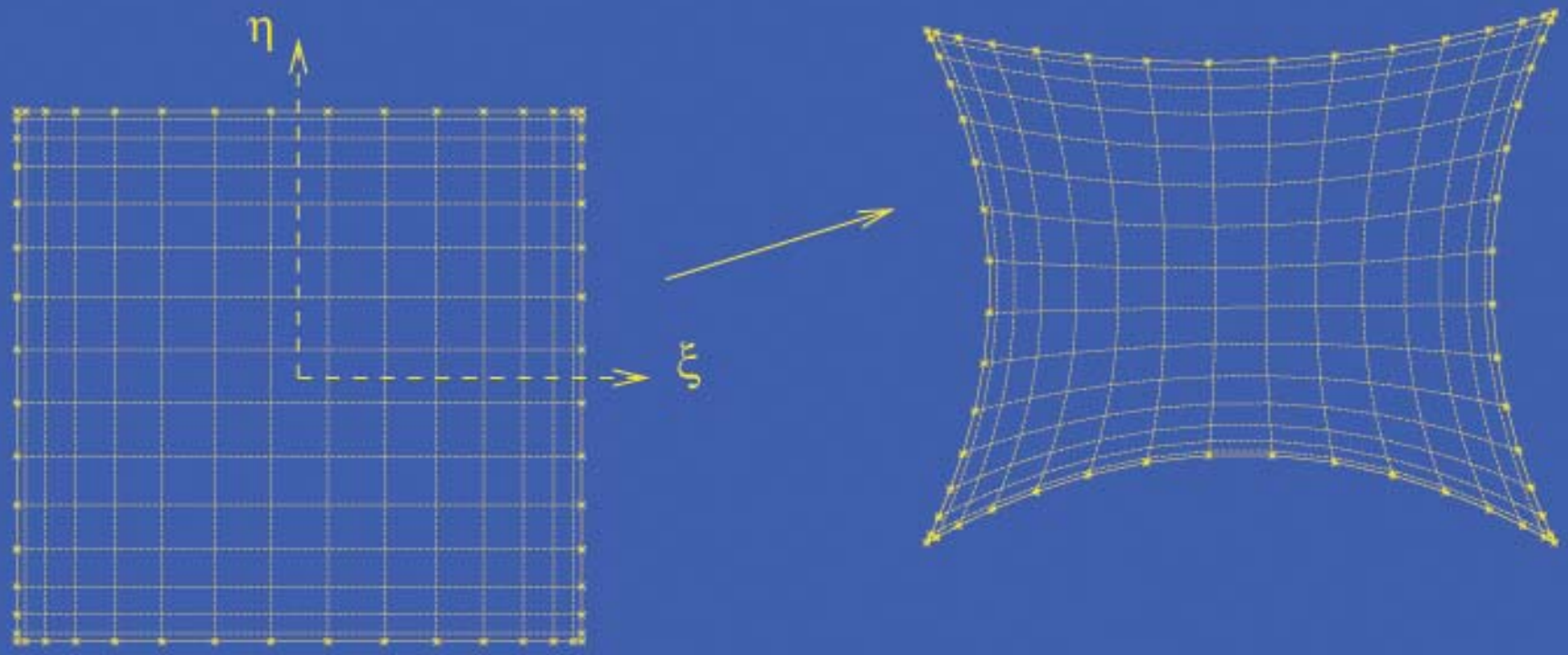
$$\int \kappa^{-1} w \partial_t^2 \chi \,\mathrm{d}^3\mathbf{r} = -\int \rho^{-1} \boldsymbol{\nabla} w \cdot \boldsymbol{\nabla}\chi \,\mathrm{d}^3\mathbf{r} + \int_{\mathrm{F-S}} w\,\hat{\mathbf{n}}\cdot\mathbf{v}\,\mathrm{d}^2\mathbf{r}$$

The 27-Node Element



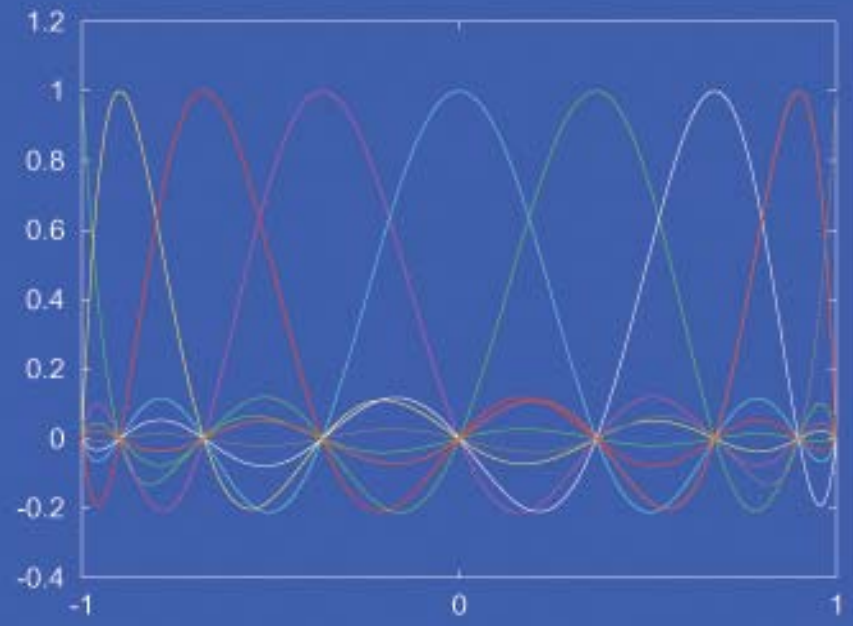
$$\mathbf{x}(\boldsymbol{\xi}) = \sum_{a=1}^{27} N_a(\boldsymbol{\xi})\mathbf{x}_a$$

Gauss-Lobatto-Legendre Points



$$\int f(\mathbf{x})\,\mathrm{d}^2\mathbf{x} = \int_{-1}^{1}\int_{-1}^{1} f(\mathbf{x}(\xi,\eta))\,J_b(\xi,\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta \approx \sum_{\alpha,\beta} \omega_\alpha \omega_\beta f^{\alpha\beta} J_b^{\alpha\beta}$$

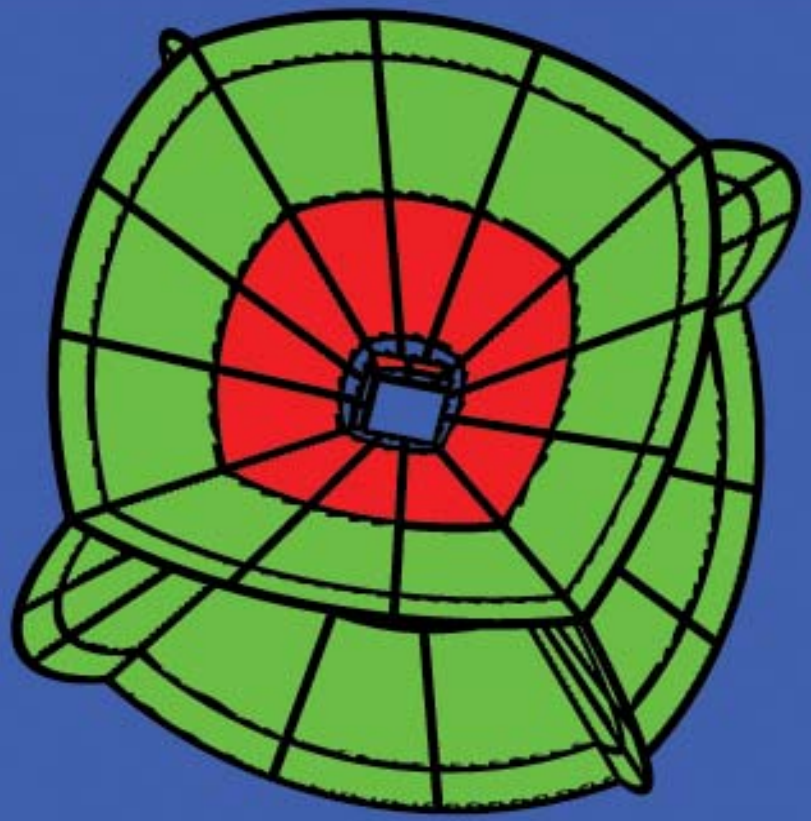
Lagrange Polynomials

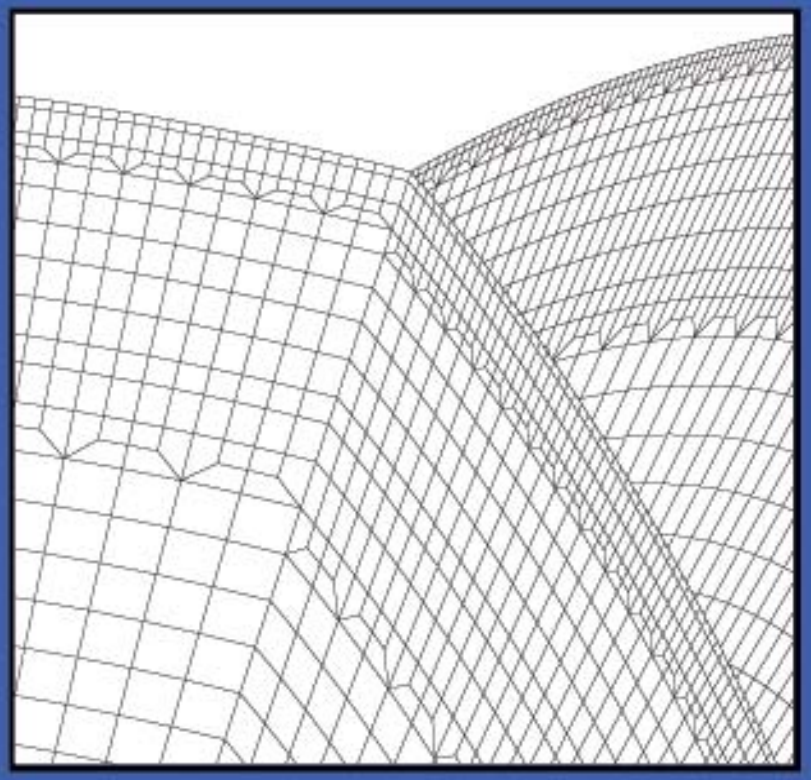


$$\mathbf{s}(\mathbf{x}(\xi,\eta,\zeta),t) \approx \sum_{i=1}^{3} \hat{\mathbf{x}}_i \sum_{\sigma,\tau,\nu} s_i^{\sigma\tau\nu}(t)\ell_\sigma(\xi)\ell_\tau(\eta)\ell_\nu(\zeta)$$

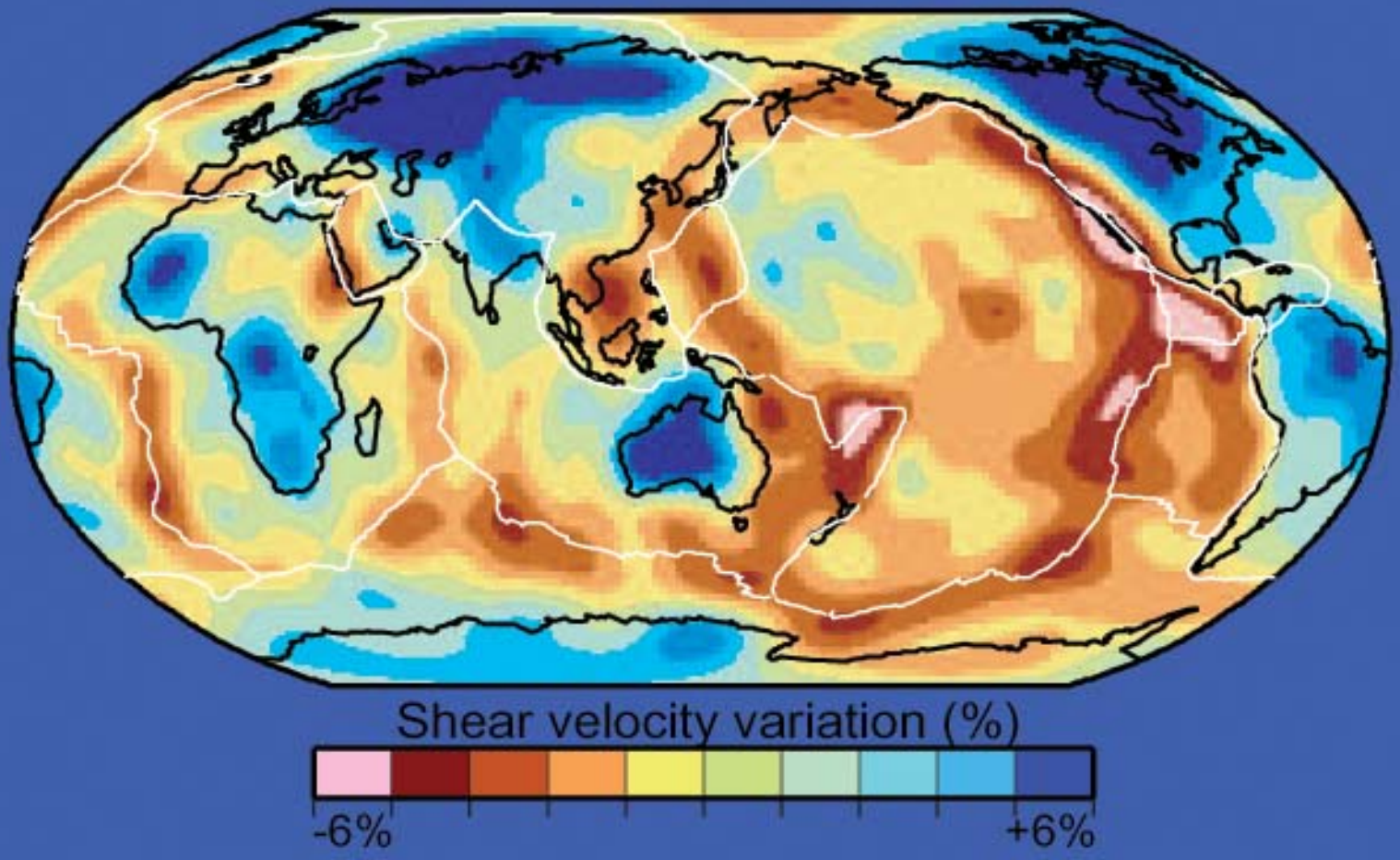
$$\mathbf{w}(\mathbf{x}(\xi,\eta,\zeta)) = \hat{\mathbf{x}}_j\, \ell_\alpha(\xi)\ell_\beta(\eta)\ell_\gamma(\zeta)$$

Mesh



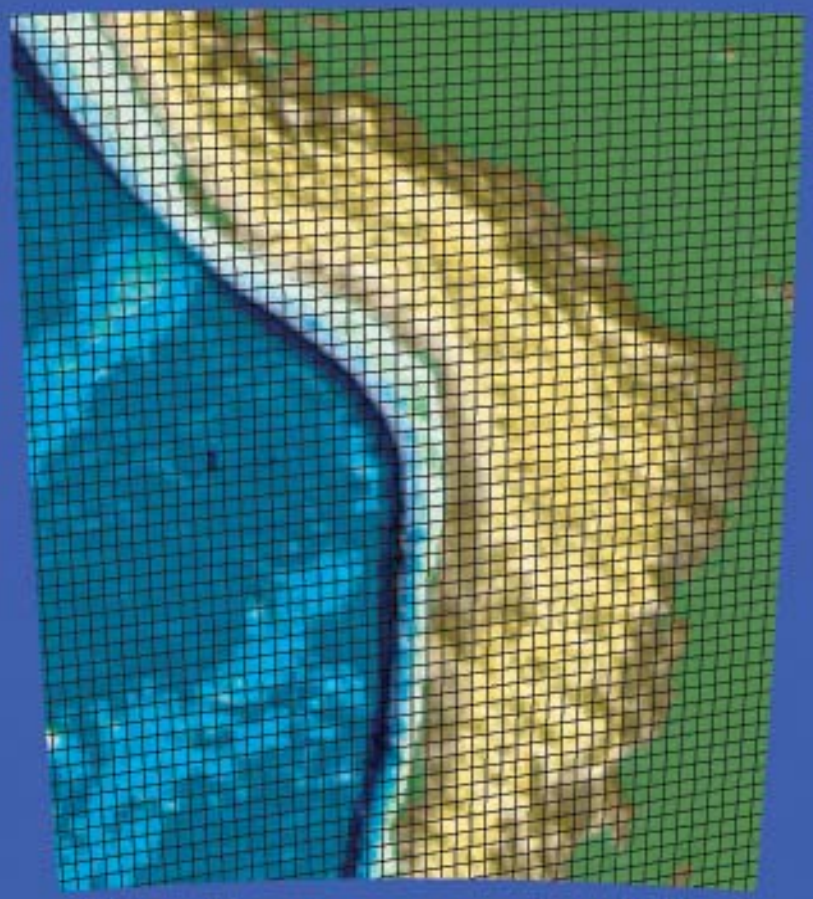


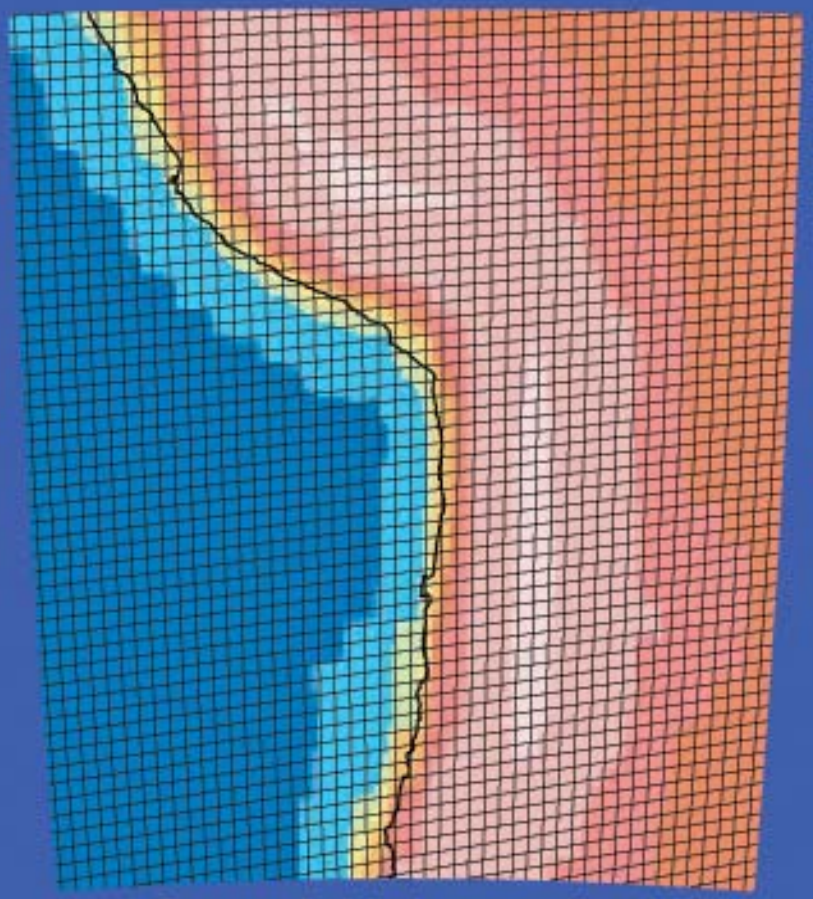
Mantle Model



Ritsema, Van Heijst & Woodhouse (1999)

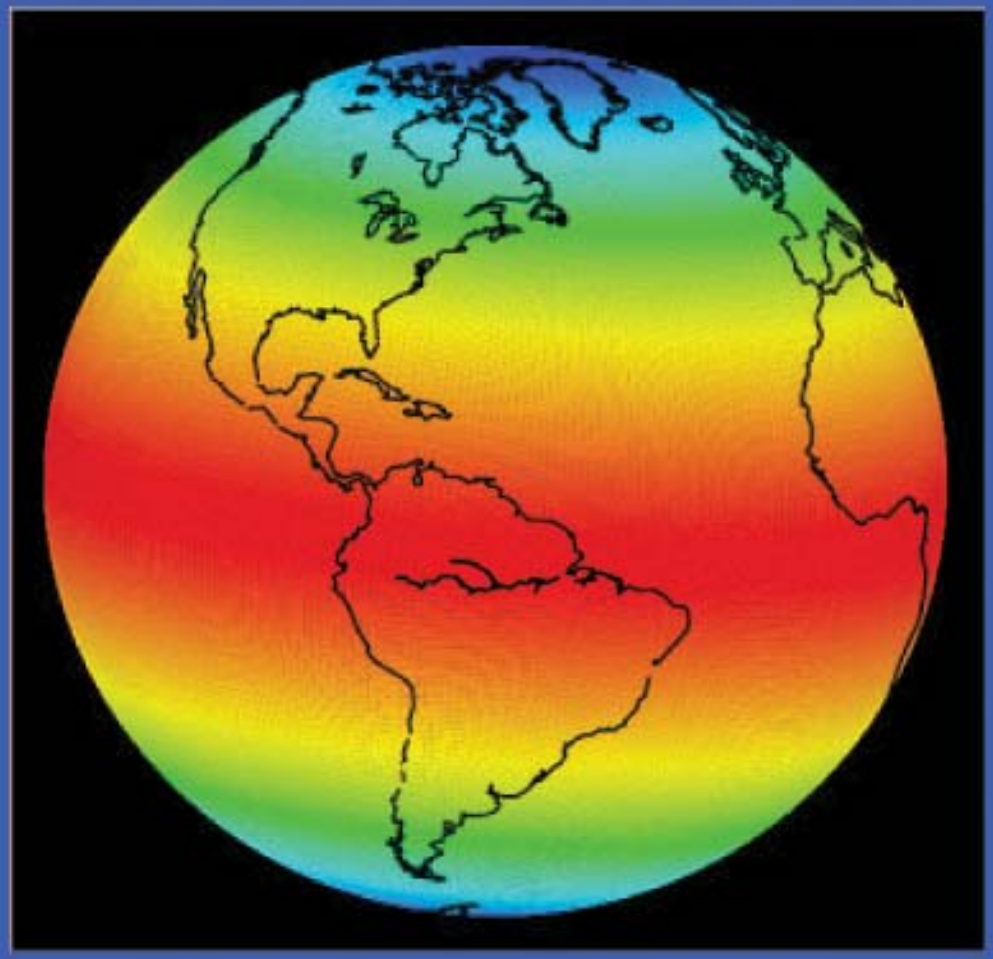
Topography & Crustal Model



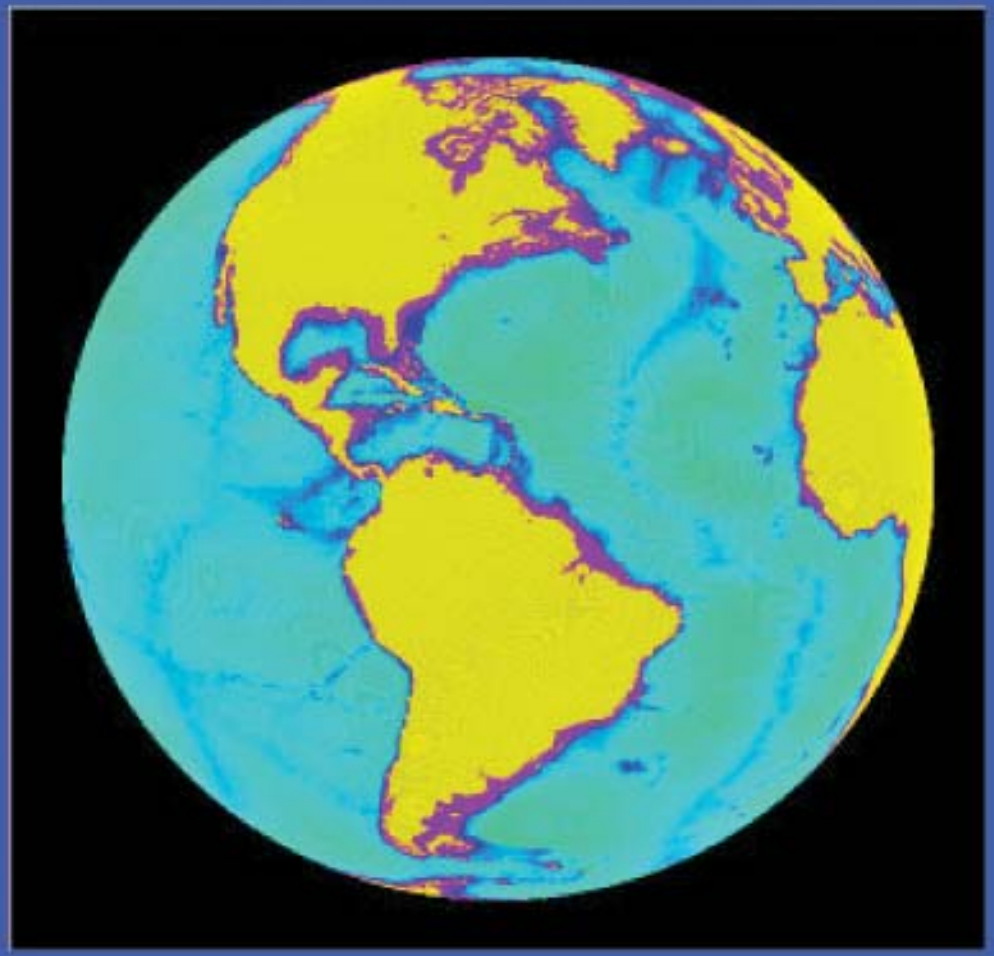


Bassin *et al.* (2000)

Ellipticity



Oceans



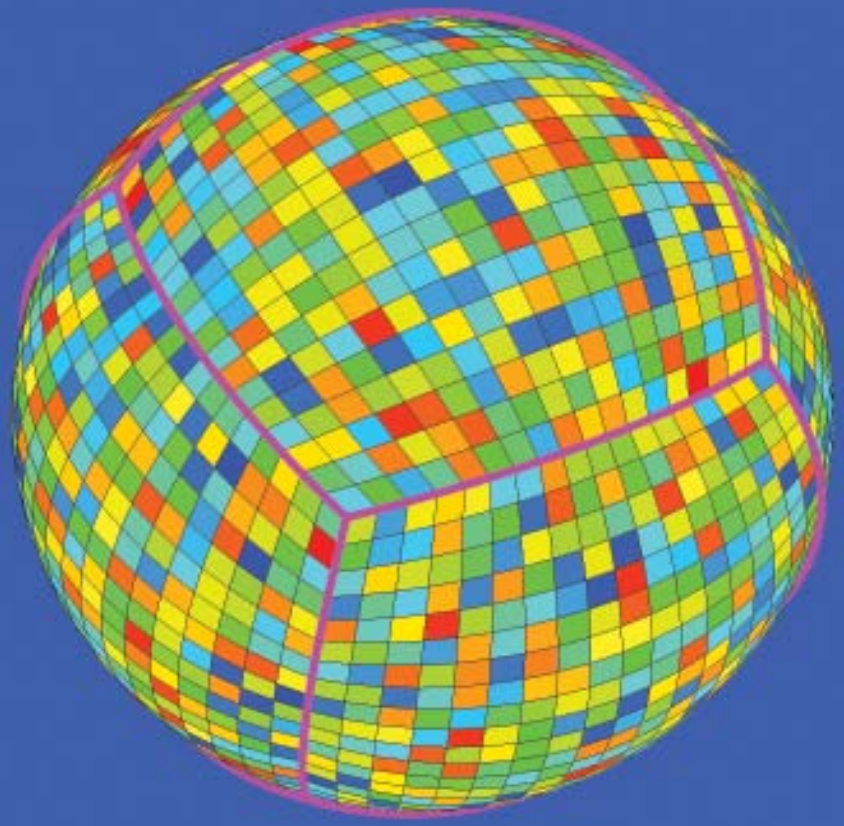
NOAA (1988)

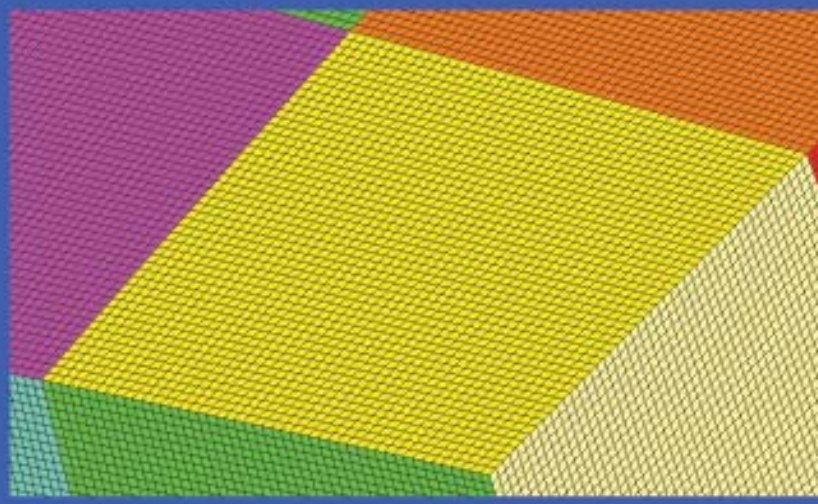
The Earth Simulator

- 640 nodes
- 16 GB of memory per node, 10 TB of distributed memory
- 8 processors per node, 5120 total

Collaboration with Seiji Tsuboi

Earth Simulator

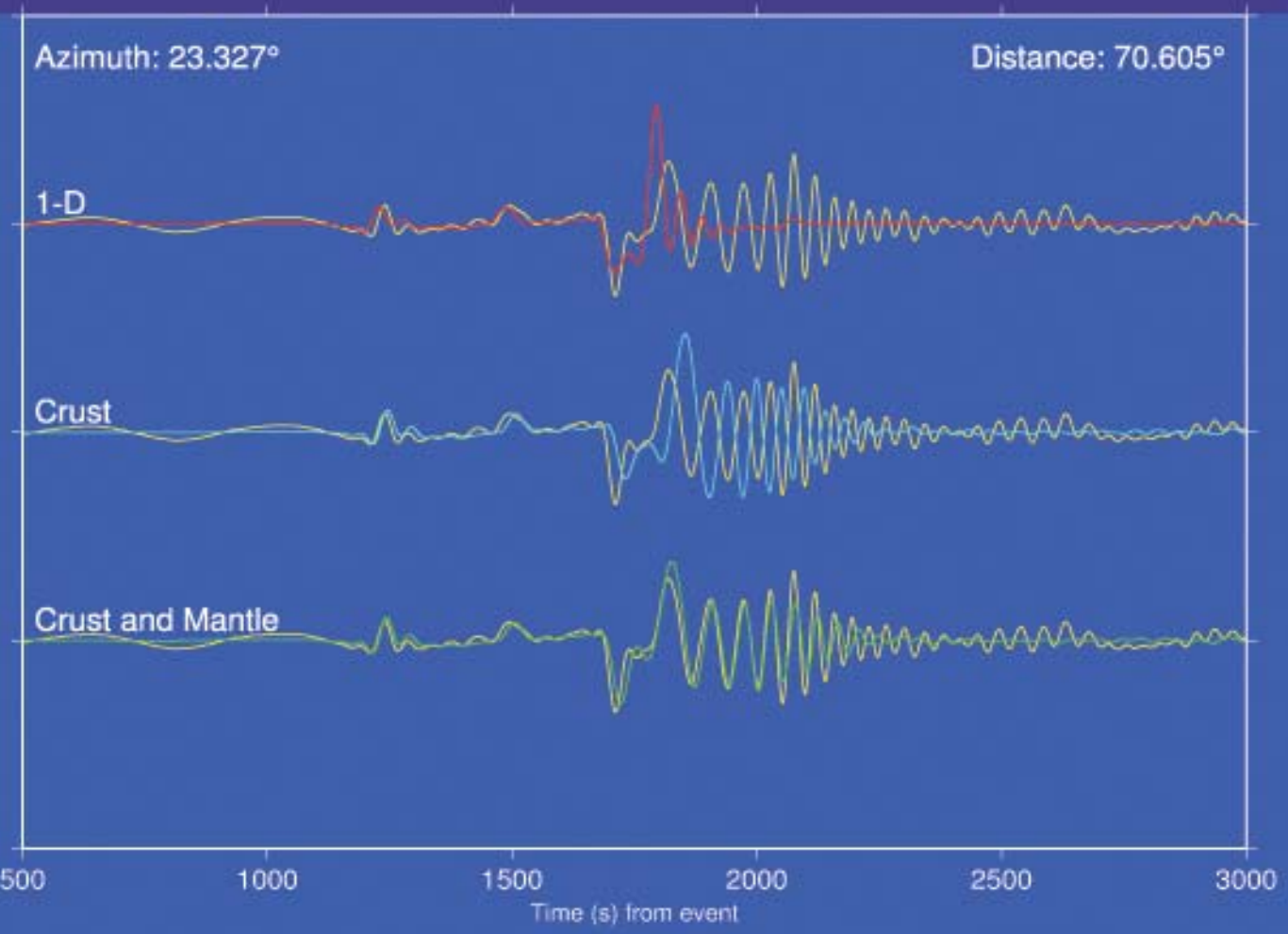




1944 processors, 3.4 TB memory

Relative Importance of Crust and Mantle Structure

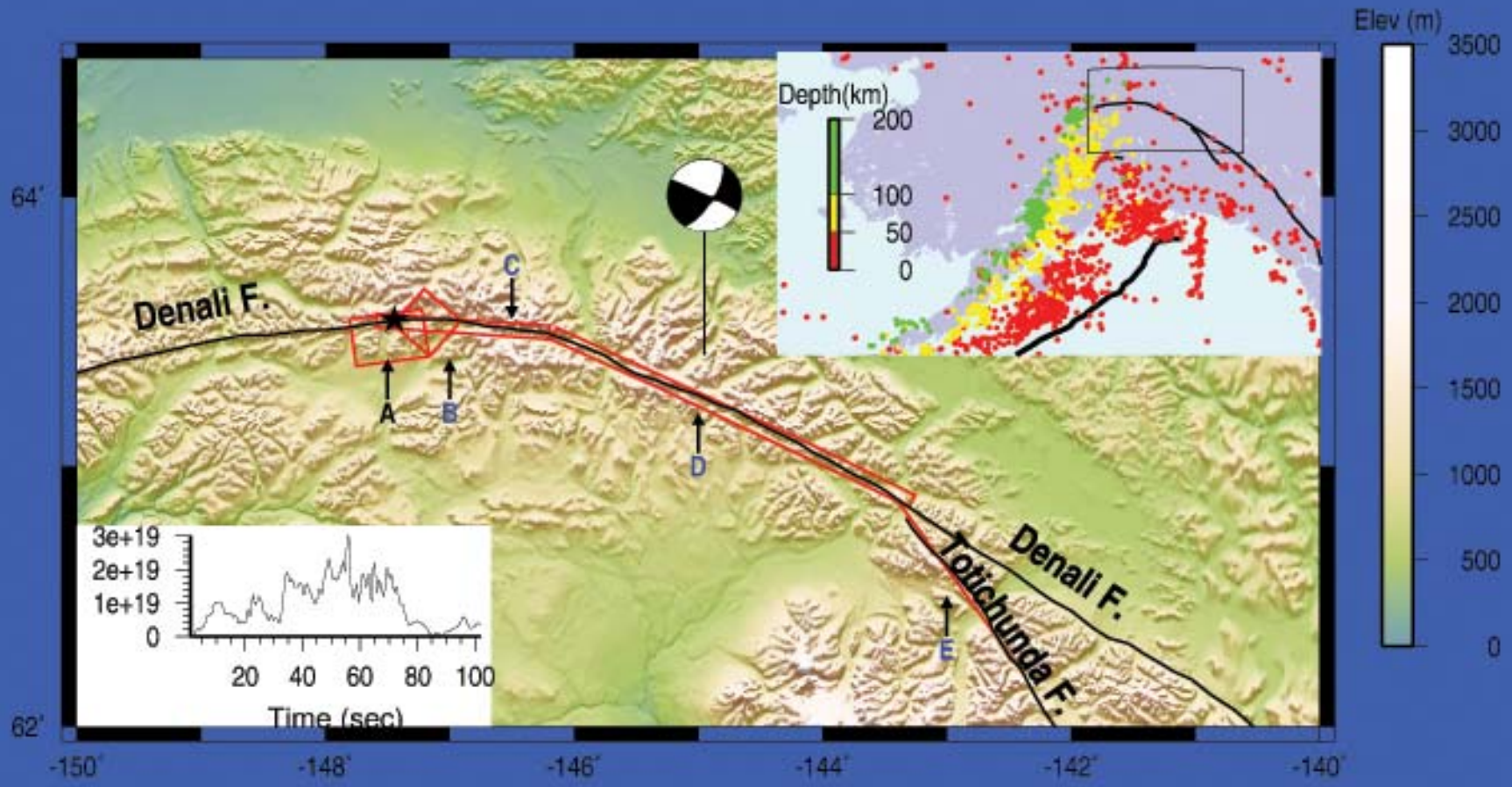
India Station: BILL LHT



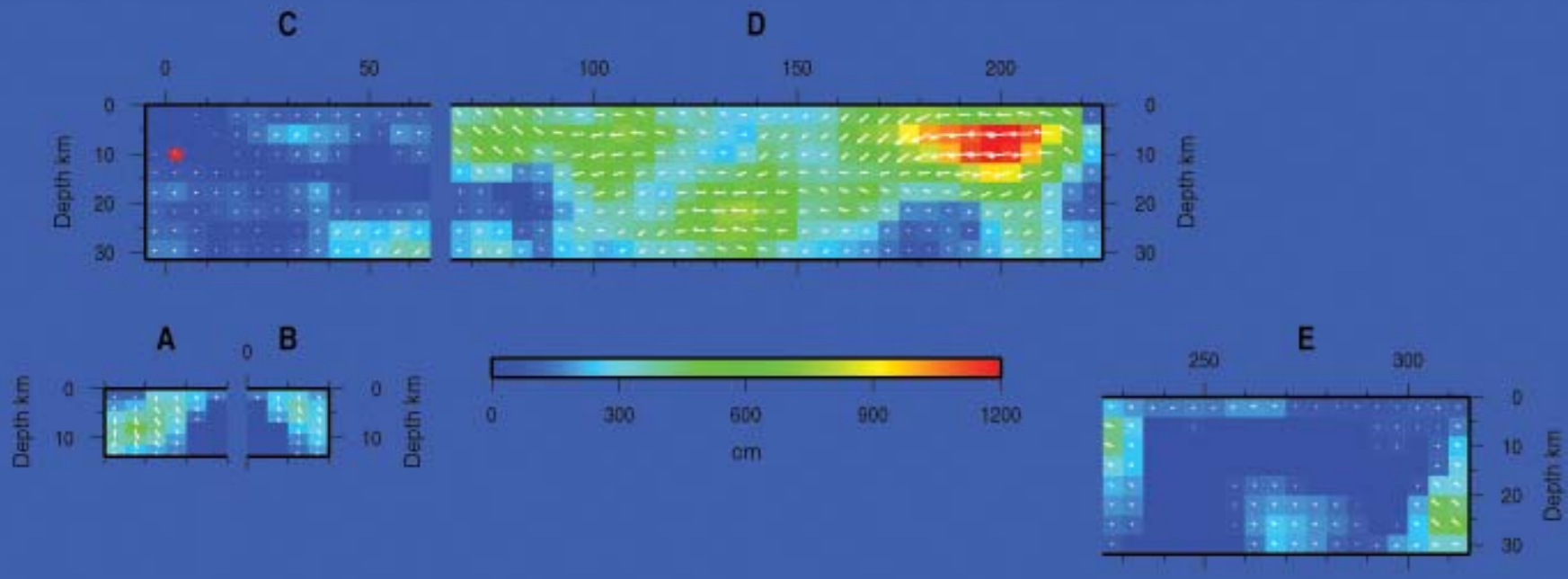
Hjörleifsdóttir, Kanamori & Tromp (2002)

Source Directivity

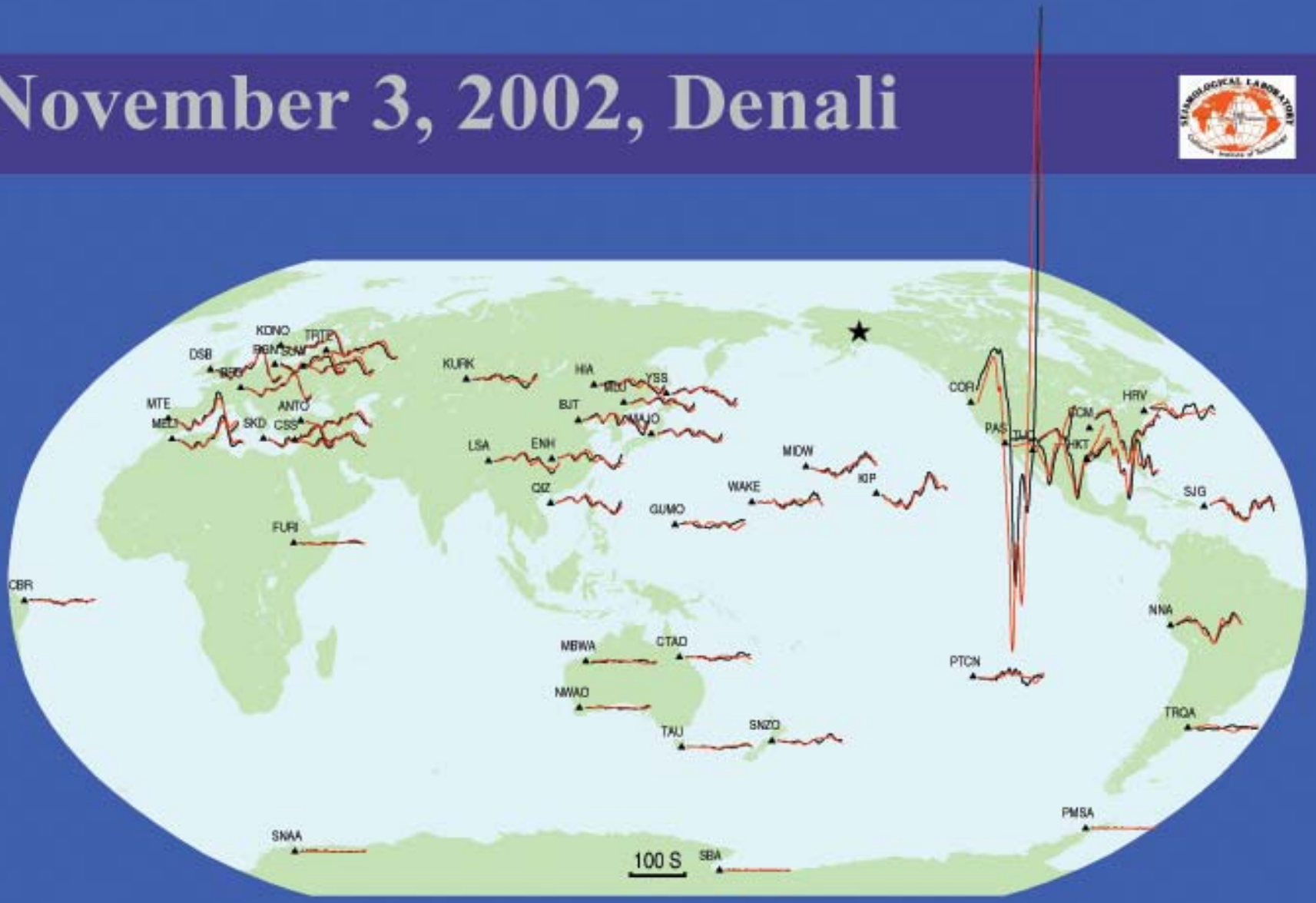
November 3, 2002, Denali



November 3, 2002, Denali

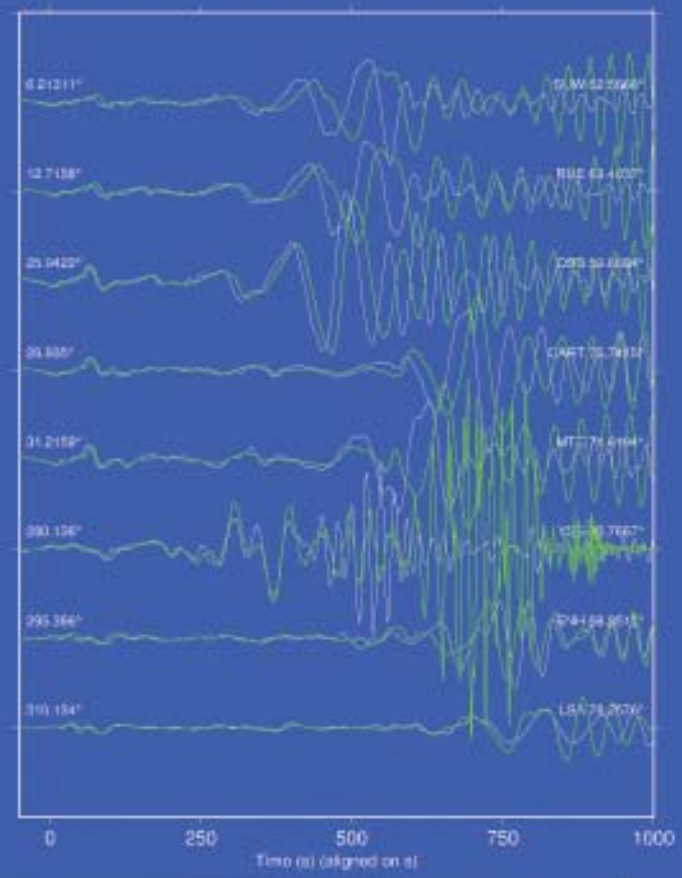


November 3, 2002, Denali

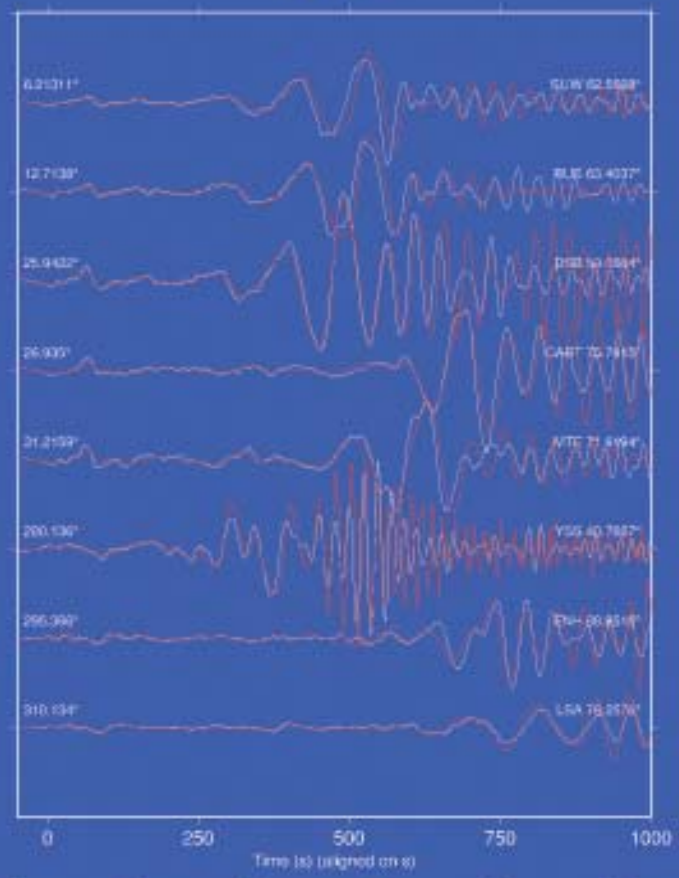


November 3, 2002, Denali

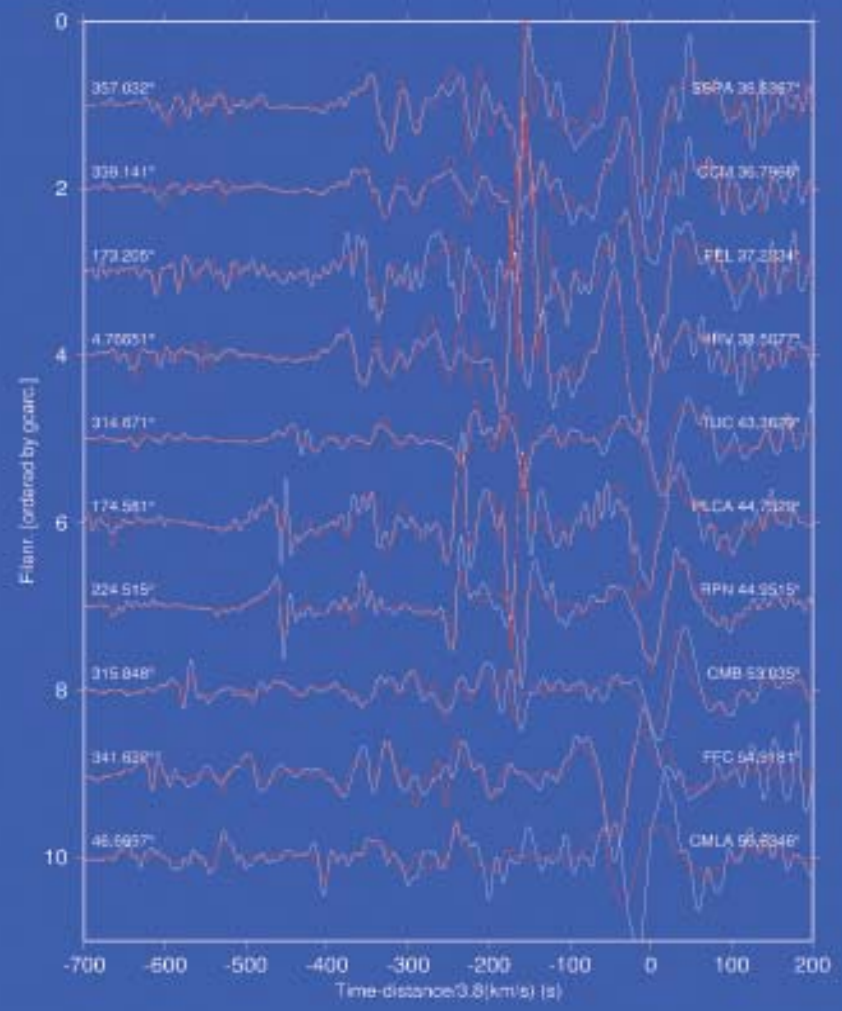
PREM



3D

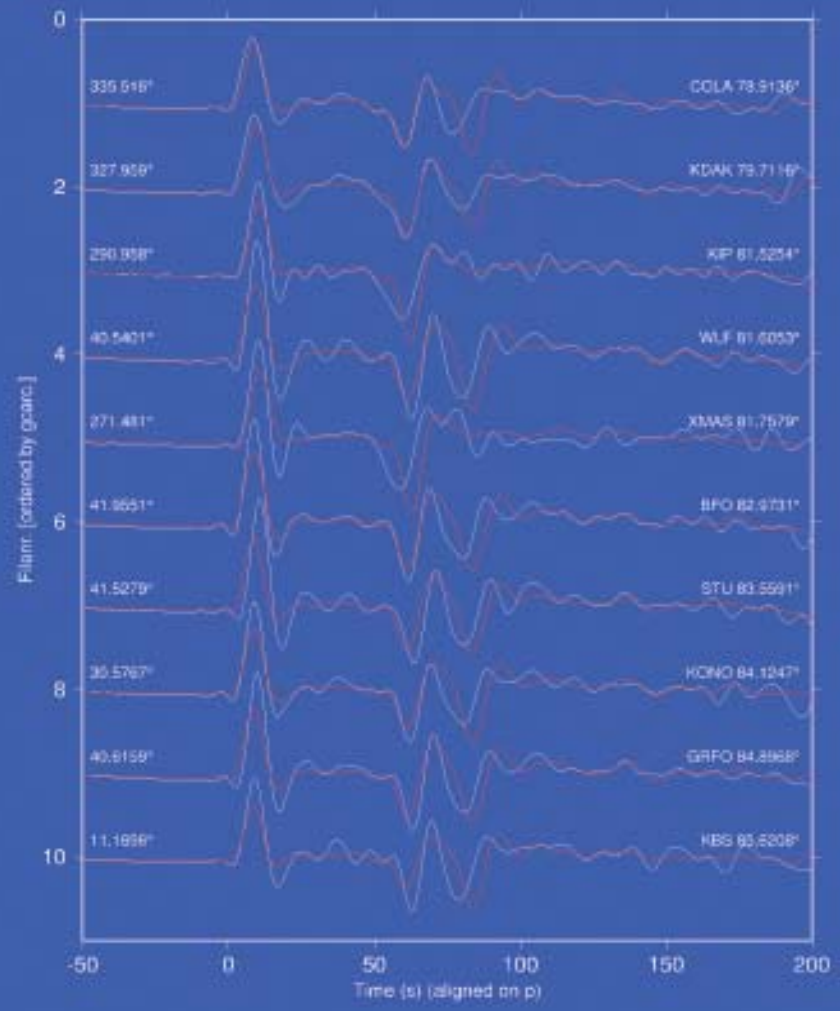


Earth Simulator



Columbia 1997 (Rayleigh wave, vertical)

Earth Simulator



Columbia 1997 (P, pP, and sP, vertical)

The World's Fastest Computer

Courtesy T. Sterling (CACR, Caltech)

Conclusions

- Importance of the crustal model.
- Finite-source models for large events.
- Global simulations accurate up to 5 seconds now feasible on the EarthSimulator.