Constrained Shortest Path Estimation on the D-Wave 2X:

Accelerating Ionospheric Parameter Estimation Through Quantum Annealing

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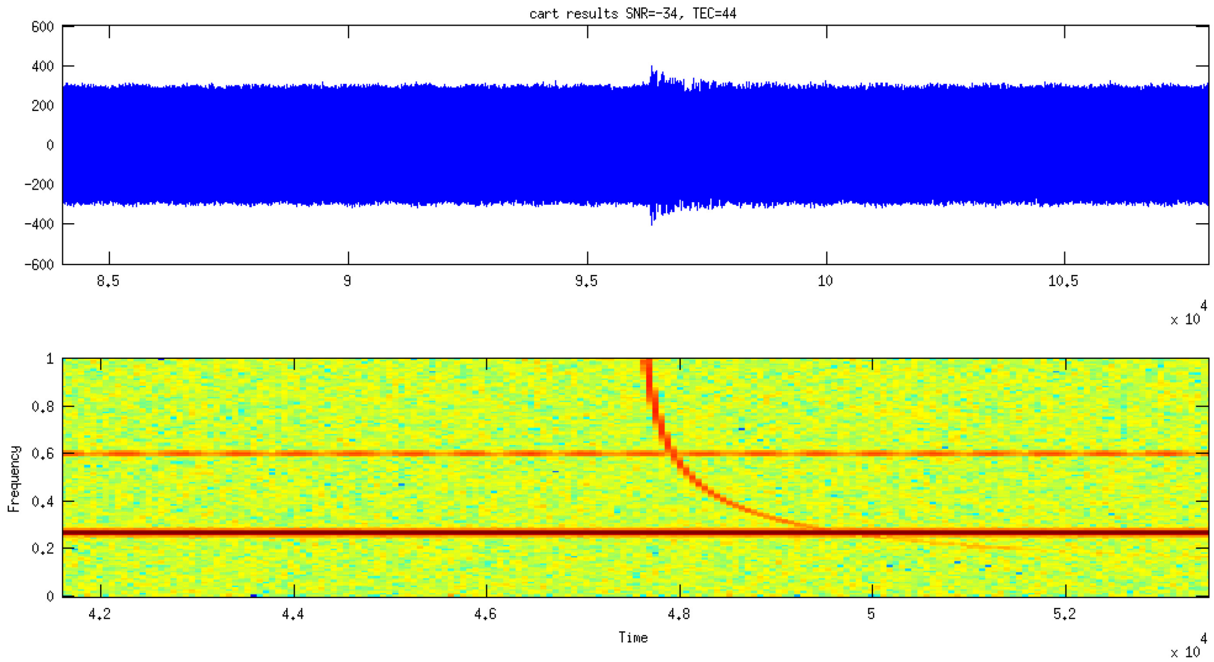
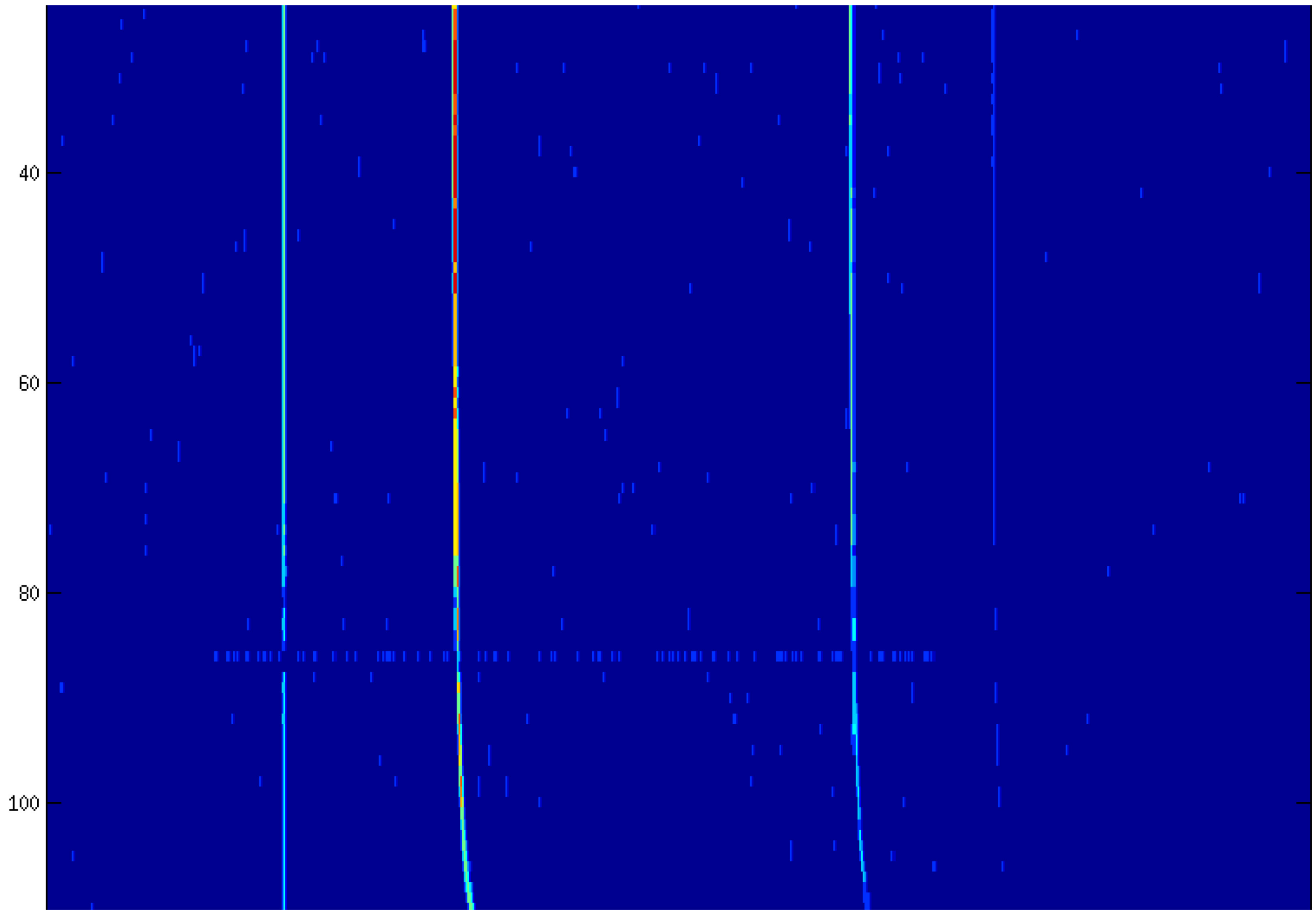
Problem: in heavy RF background noise, find wide band events, then estimate dispersion

- **Large transmitters can overwhelm the wideband events**
- **Convert to frequency/time view**
  - Shape of curve can be used to remove effects of ionosphere
    - Fancy GPS receivers can receive multiple channels to estimate this shape, then remove to improve location accuracy

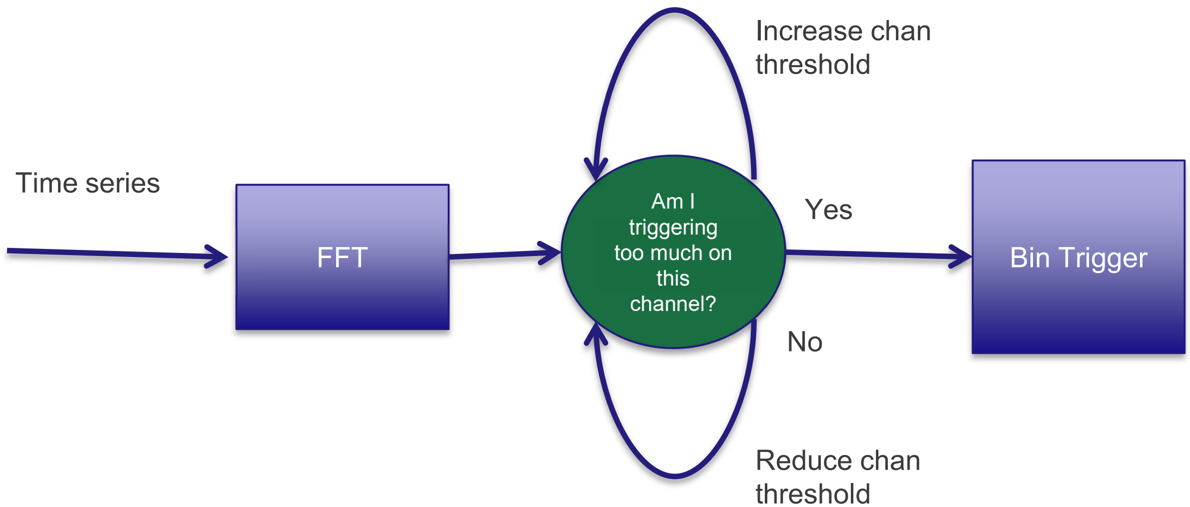


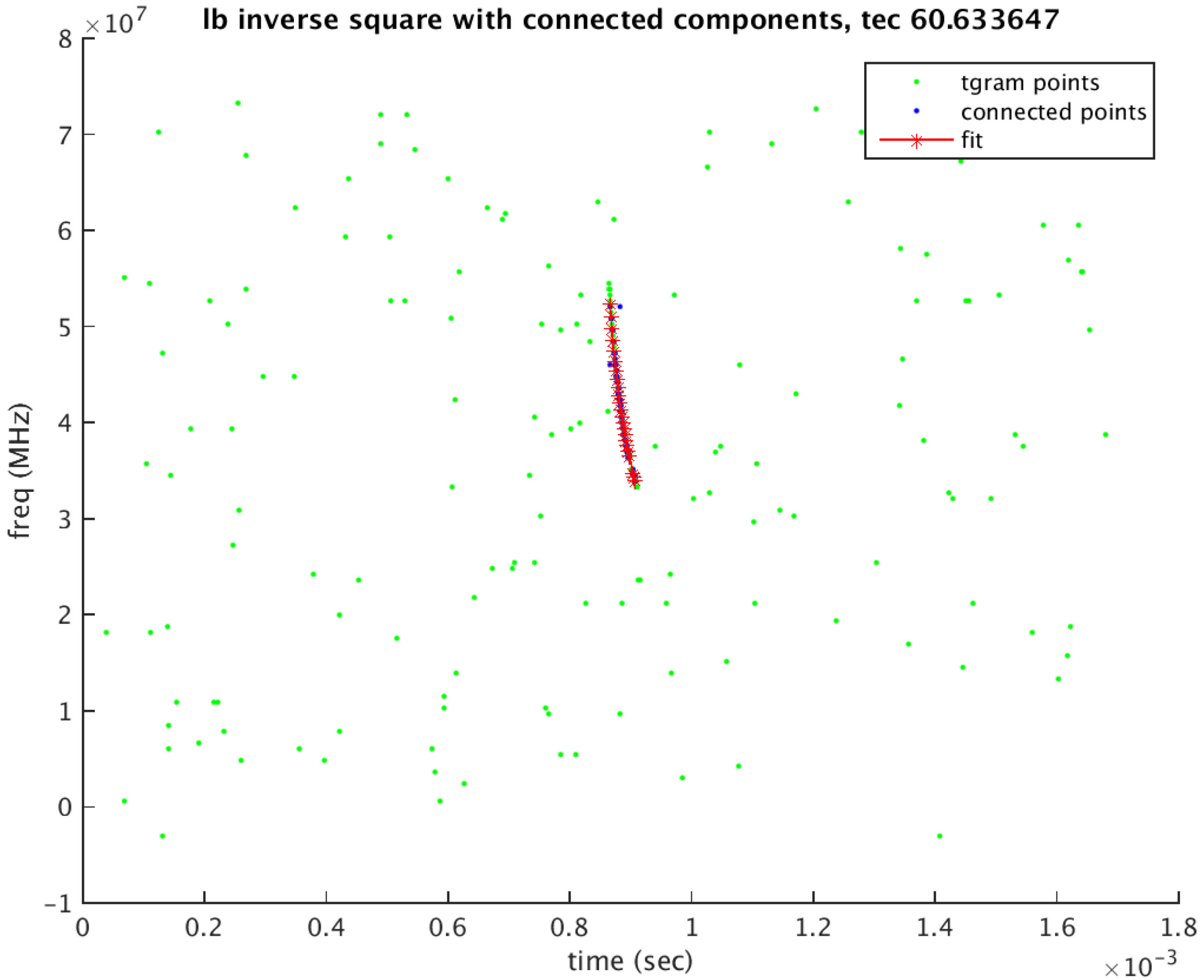
Easy for powerful events, hard for lower power, lower SNR events



Constant Alarm Rate Trigger (CART)

- **FFT-based trigger with per-channel floating thresholds**
  - Currently under test in LEO, planned for GEO




lb inverse square with connected components, tec 60.633647
tgram points
connected points
fit
freq (MHz)
time (sec)
×10^7
×10^-3
8
7
6
5
4
3
2
1
0
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

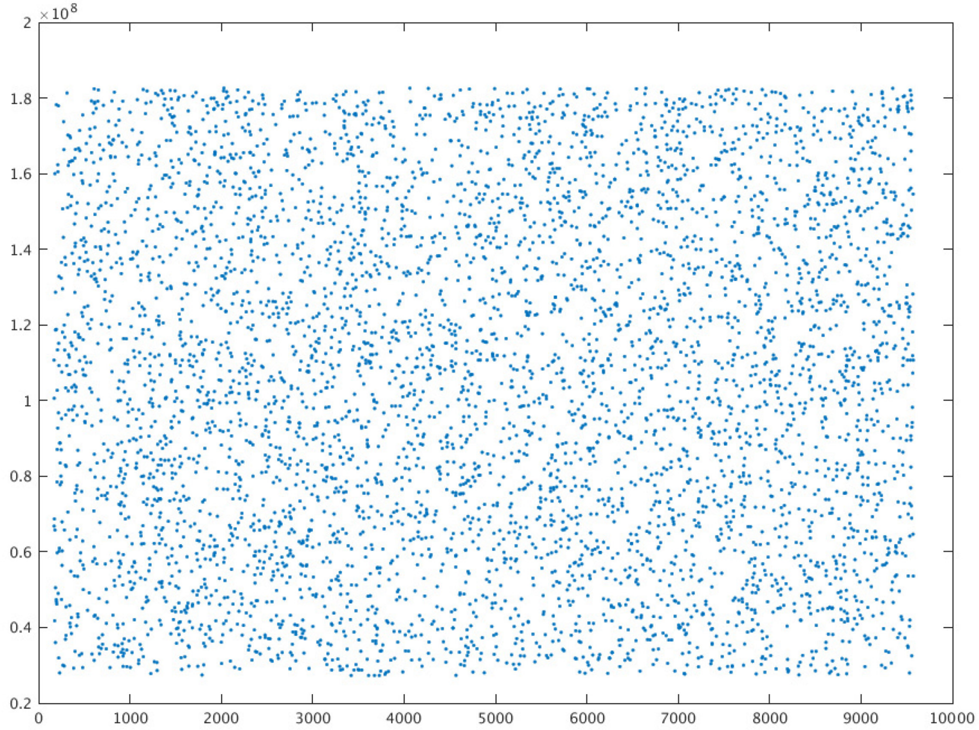
Opportunity

- **Triggergram provides low-bandwidth picture of event shape without computational overhead**
  - Can the event be extracted from this low-bandwidth picture?
  - Can we estimate the dispersion parameters from it?
    - This is important to determining the actual vacuum time of arrival (actual time of event)

Problem

•Depending on the noise settings, the CART filter produces a 'snowy' image. For low SNR events this is particularly a problem. How do you connect the triggergram points together into an event?

Find the event

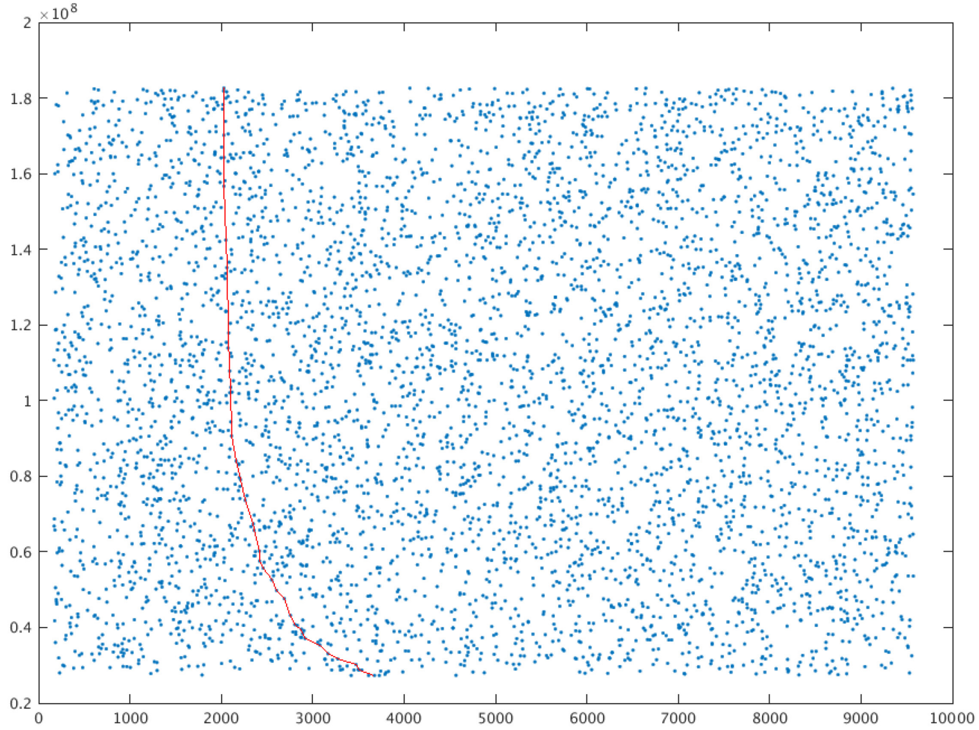


Providing Context

Triggergram provides low-bandwidth picture of event shape without computational overhead

- **But does not provide any context about which points are part of the event**
- **Challenge: connect points together in a high SNR environment into useful information**

Ta-Da!


×10^8
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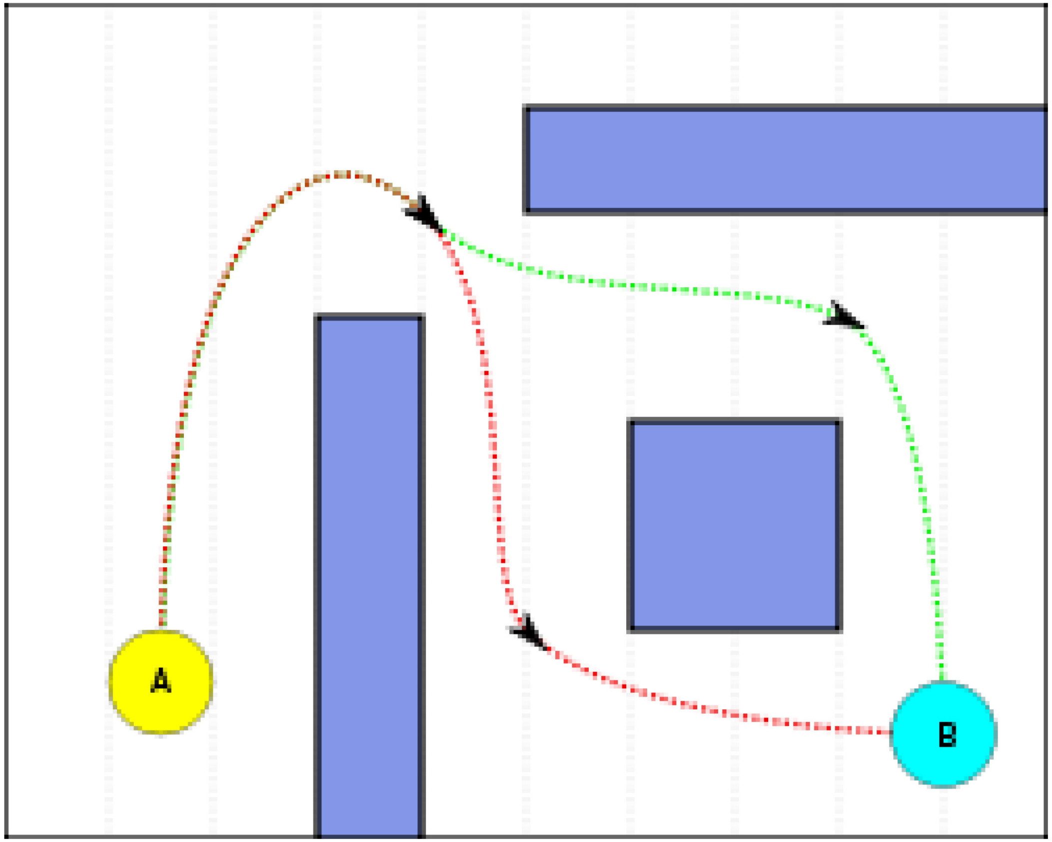
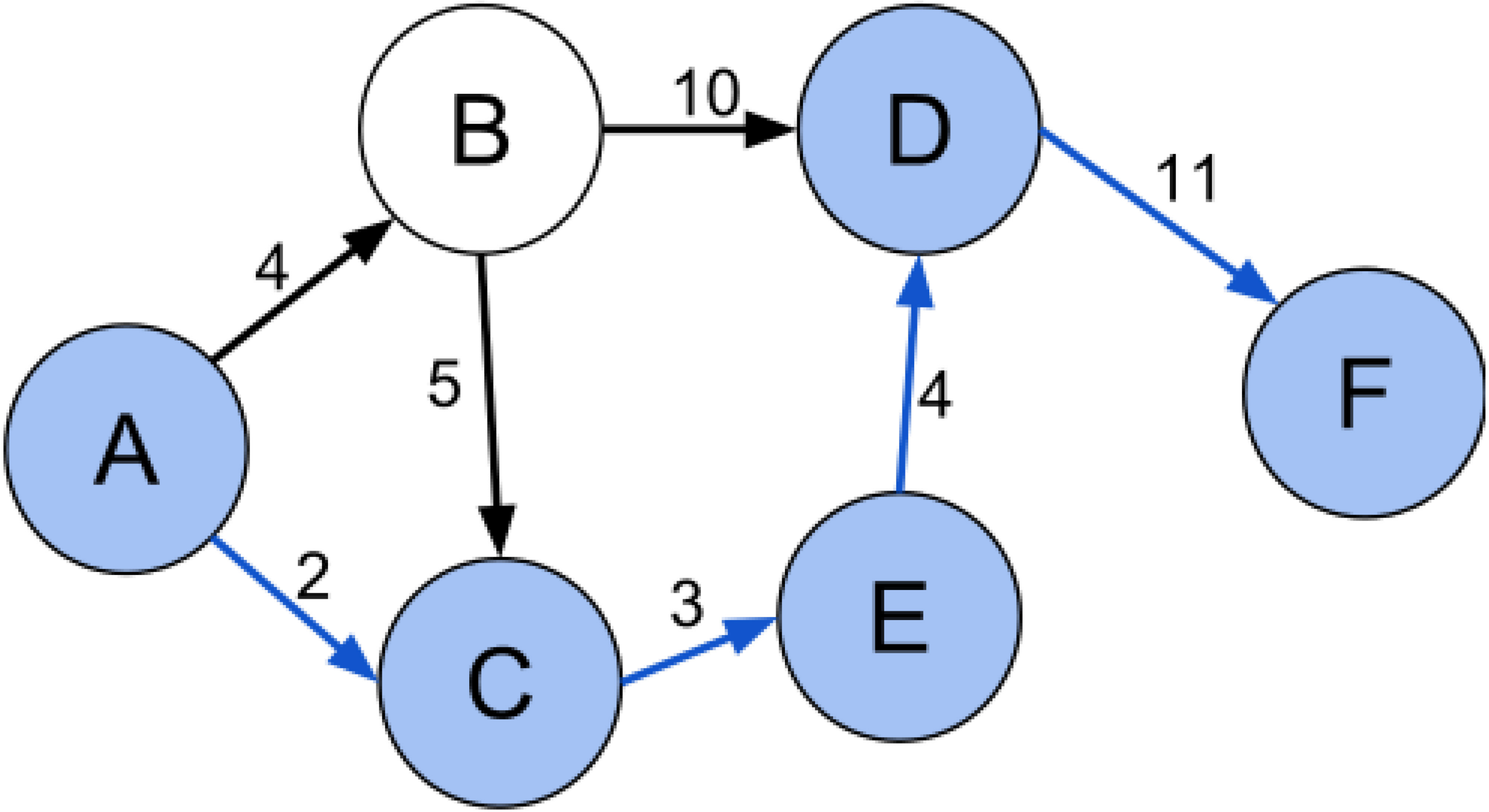
7000
8000
9000
10000

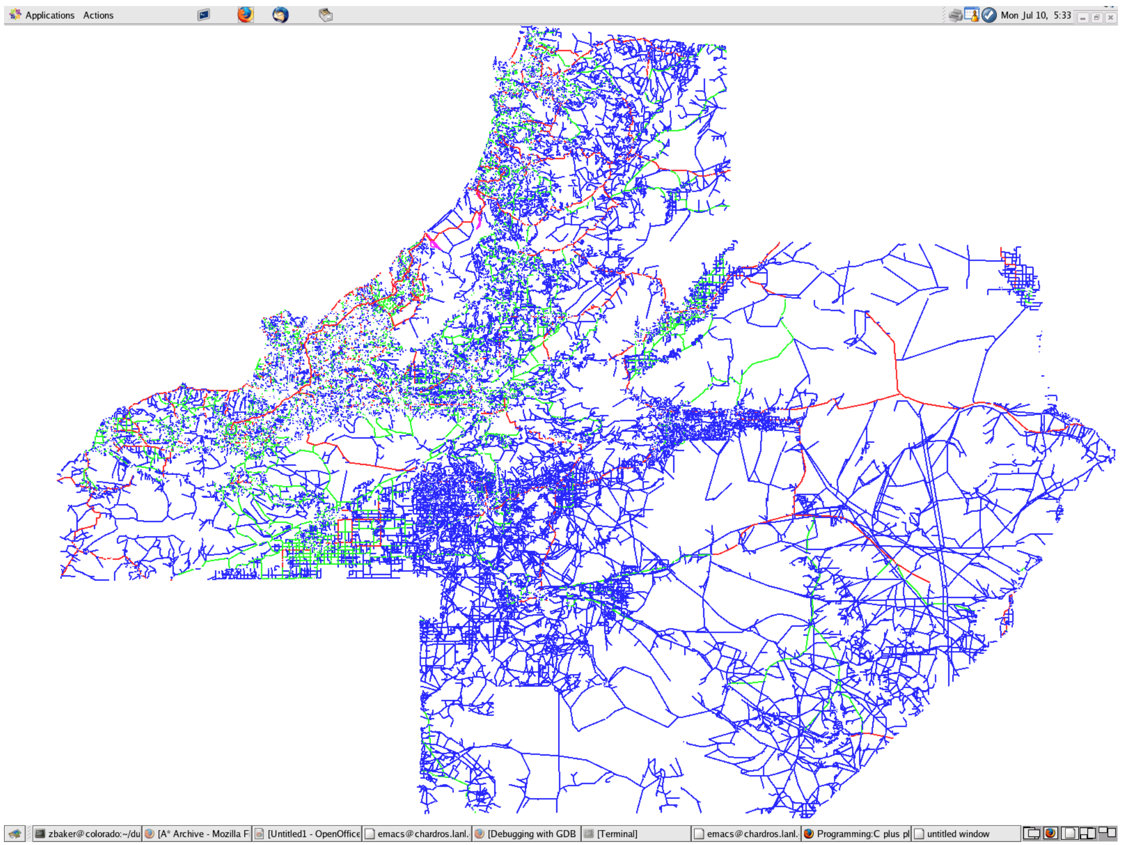
Shortest Path



Graph Theory; Arbitrary graph with arbitrary weights



Weights are based on XY coordinates, traffic



(prior work)

On the Acceleration of Shortest Path Calculations in Transportation Networks[1]

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Abstract

Shortest path algorithms are key elements of many graph problems. They are used in such applications as online direction finding and navigation, and modeling of traffic for large scale simulations of major metropolitan areas. As shortest path algorithm are execution bottlenecks, it is beneficial to move their execution to parallel hardware such as Field-Programmable Gate Arrays (FPGAs).

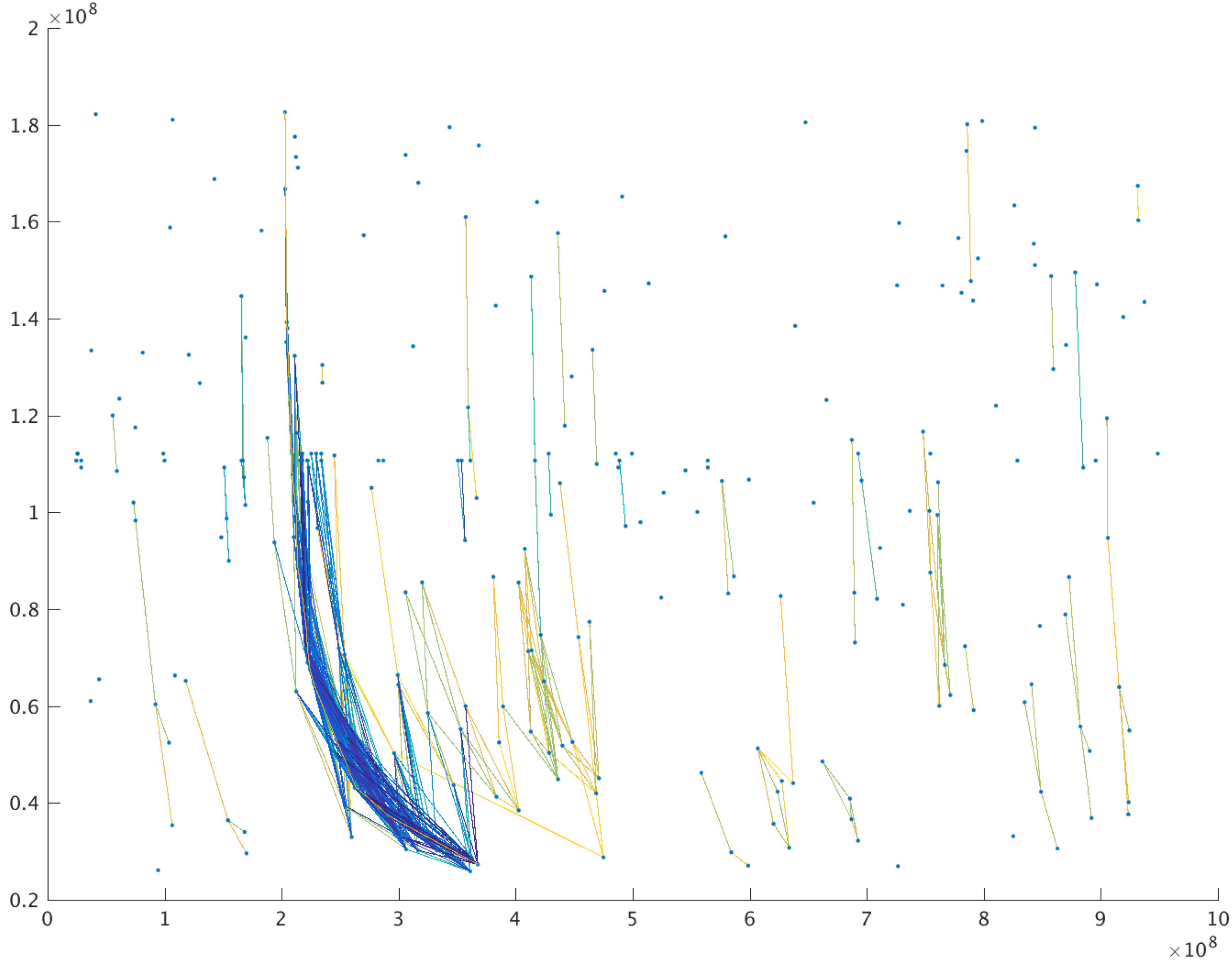
One of the innovations of this approach is the use of a small bubble sort core to produce the extract-min

1 Introduction

Transportation infrastructure is a highly complex system, where hundreds of thousands of vehicles traverse vast road networks. The ability to understand and predict responses of the network to perturbations of the traffic flow is of great value to planners. Exploring the effects of various changes to the infrastructure provides understanding of what to expect when similar changes happen in reality. TranSIMS-LANL is a Los Alamos National Laboratory project aimed at understanding many aspects of infrastructure, including traffic, energy consumption, land use planning, traffic

Physics-based Weighting

- if(tec > 5 && tec < 200)
- t0_weight = abs(t0 - (t0_true/scale_factor))/(t0_true/scale_factor);
- weights(k,j) = (1+t0_weight) * ((trig_a(k) - trig_a(j))^2 + (trig_b(k) - trig_b(j))^2);
- tecmap(k,j) = tec;
- else
- weights(k,j) = Inf;
- end


×10^8
2
1.8
1.6
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1.2
1
0.8
0.6
0.4
0.2
0
1
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3
4
5
6
7
8
9
10
×10^8

Quantum annealing

- **Minor Introduction to Quantum annealing:**
- **Minimize the objective function:**

$$Obj(a_i, b_{ij}; q_i) = \sum_i a_i q_i + \sum_{ij} b_{ij} q_i q_j$$

  - Where q_i is a quantum bit and will settle into one of two final states {0,1}, and a_i and b_{ij} are weights on the interactions

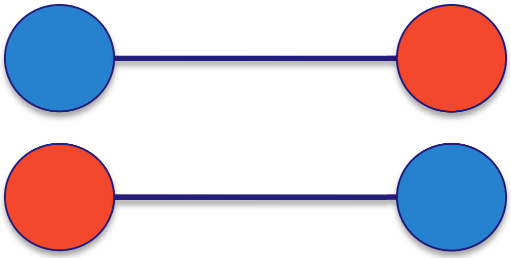
1-of-n mapping

- **Choose one path through the graph**
  - Break path finding into stages
    - Works well for TEC estimation problem, not ideal for general route finding problems
    - Reducing the degree of graph connectivity
      - Allows larger graphs to map to the limited connectivity of the DWAVE system

1-of-2 (not gate)

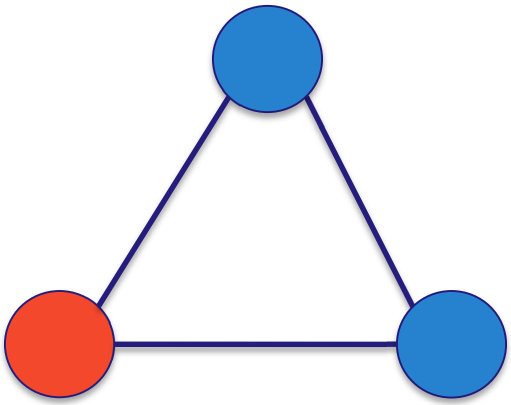
(in)	(out)		Final Energy	In*a0 + out*a1 + in*out*b01	Conclusion
0	1	(good)	<= 0	a1 <= 0	a1 = -1
1	0	(good)	<= 0	a0 <= 0	a0 = -1
0	0	(bad)	>=0	-	-
1	1	(bad)	>=0	a0+a1+b01 >= 0	b01 = 2

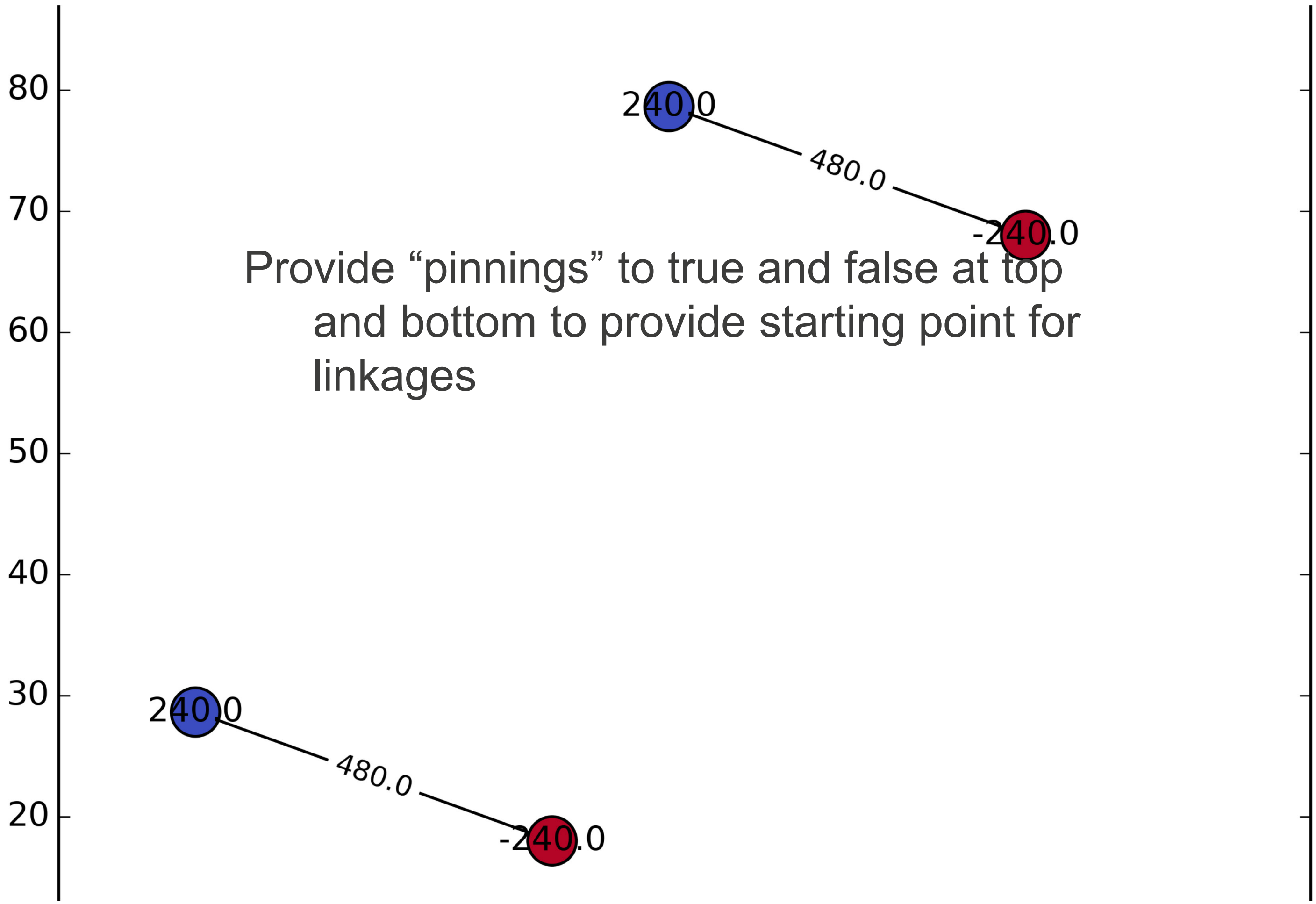
Equally likely:



1-of-3

in2	in1	in0		Final Energy	a0 + a1 + a2 + b01 + b02 + b12	Conclusion
0	0	0	(bad)	>=0	-	-
0	0	1	(good)	<= 0	a0 <= 0	a0 = -1
0	1	0	(good)	<= 0	a1 <= 0	a1 = -1
0	1	1	(bad)	>=0	a0+a1+b01 >= 0	b01 = 2
1	0	0	(good)	<= 0	a2 <= 0	a2 = -1
1	0	1	(bad)	>=0	a2 + a0 + b02 >= 0	b02 = 2
1	1	0	(bad)	>=0	a2 + a1 + b12 >= 0	b12 = 2
1	1	1	(bad)	>=0	a0 + a1 + a2 + b01 + b02 + b12 >= 0	b01 = 2

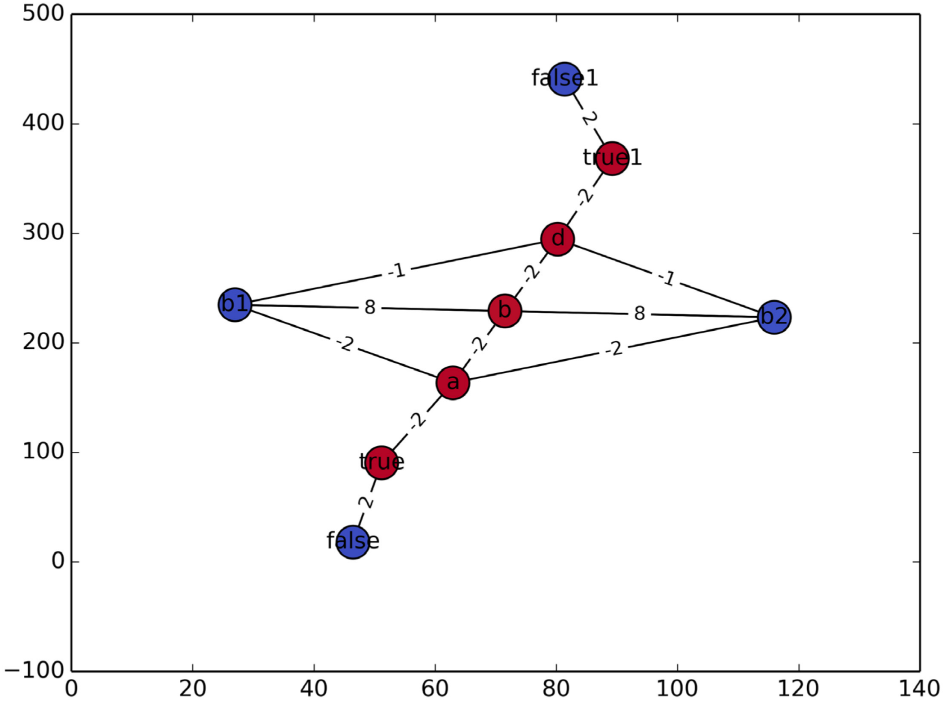




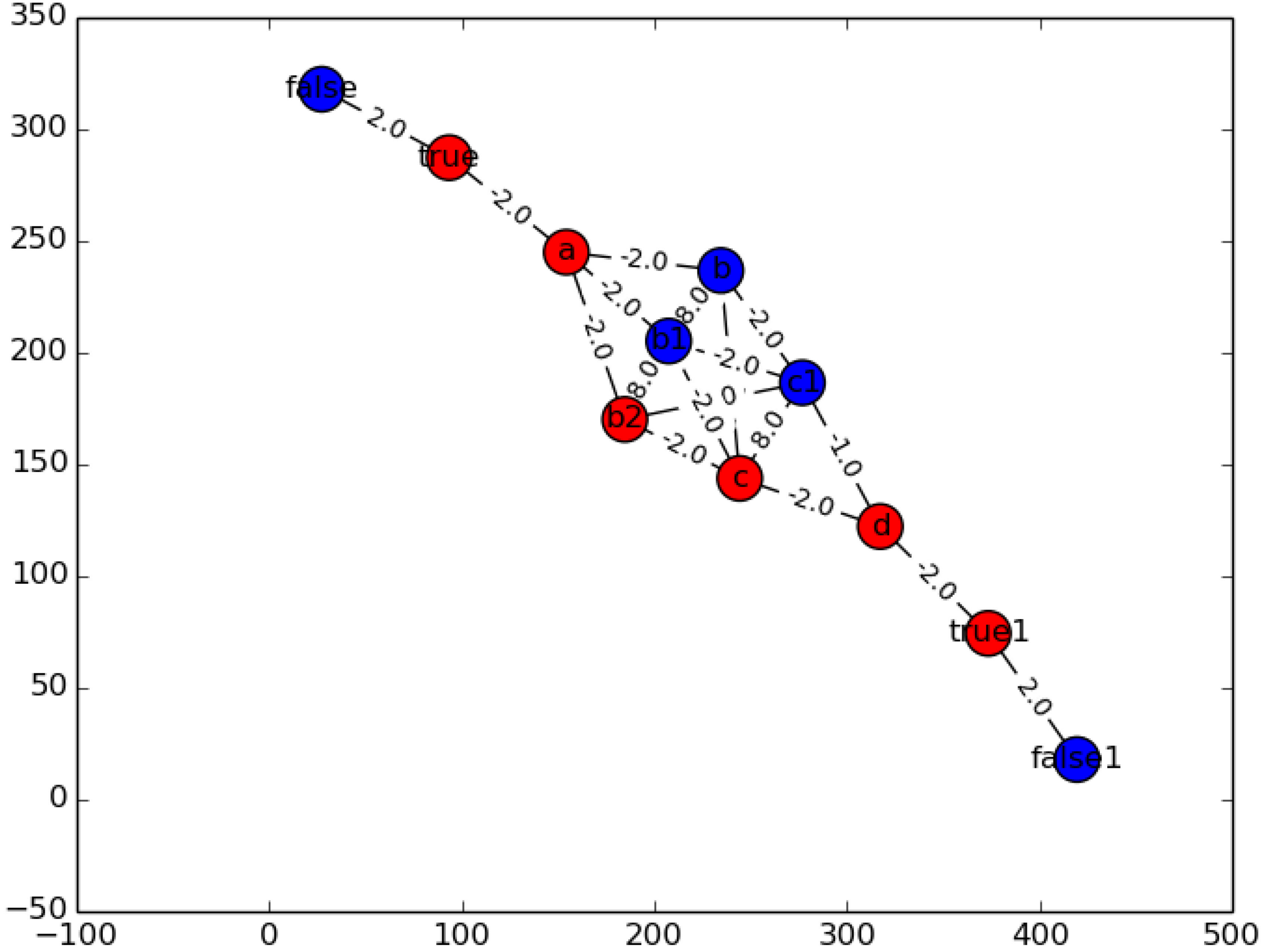
Provide “pinnings” to true and false at top and bottom to provide starting point for linkages

- **Choose one path through the graph**
  - Break path finding into stages
  - Works well for TEC estimation problem, not ideal for general route finding problem
  - Reducing the degree of graph connectivity
  - Allows larger graphs to map to the limited connectivity of the D-Wave 2X system

Linkages are chosen to minimize overall energy



In this graph, the center node ('b') is weighted favorably compared to the outside nodes ('b1' and 'b2'), and there is only one solution


false
true
a
b
b1
c1
b2
c
d
true1
false1
-2.0
8.0
-1.0
350
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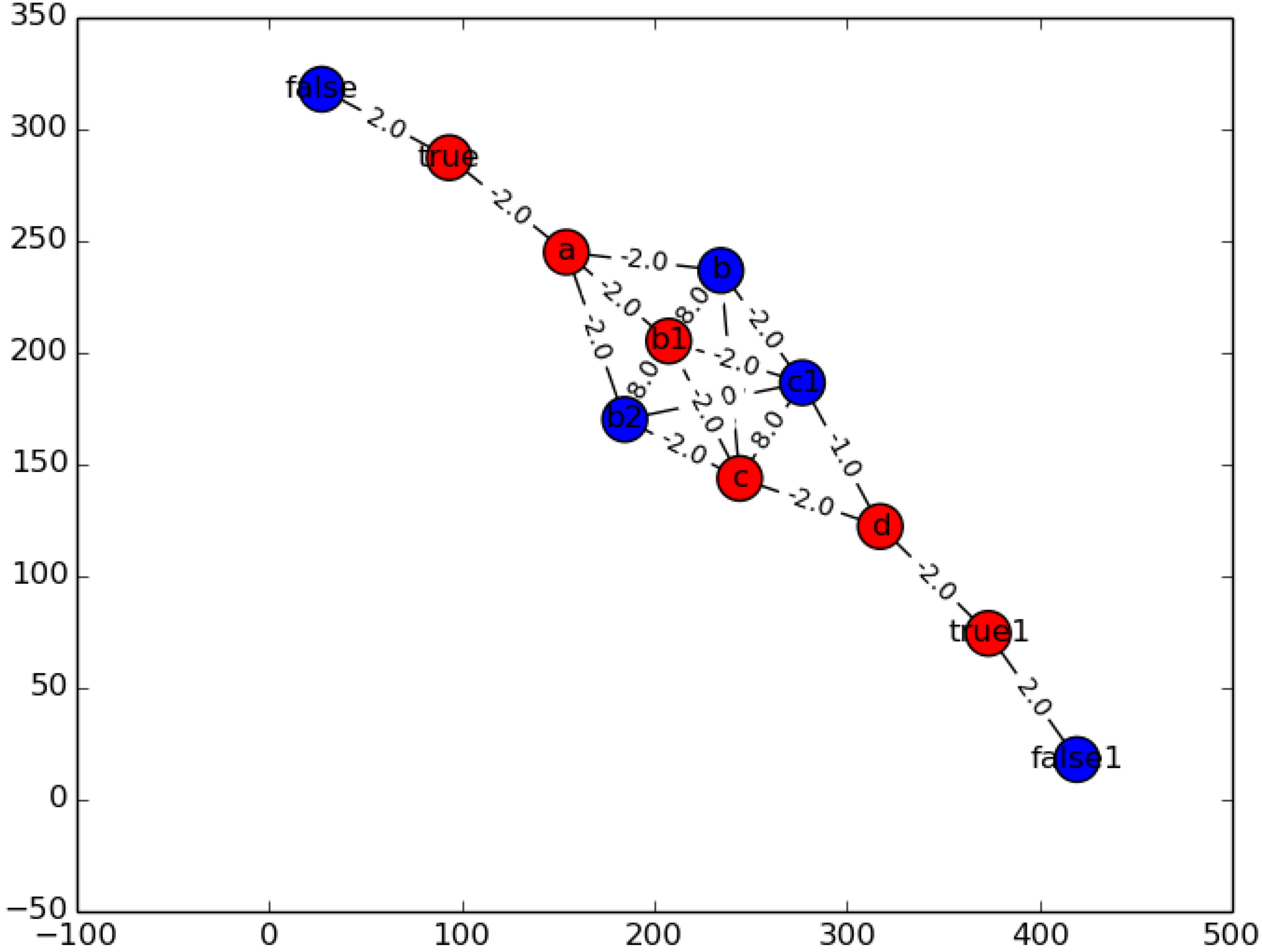
250
200
150
100
50
0
−50
−100
0
100
200
300
400
500


false
true
a
b
b1
c1
b2
c
d
true1
false1
-2.0
-1.0
8.0
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−50
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400
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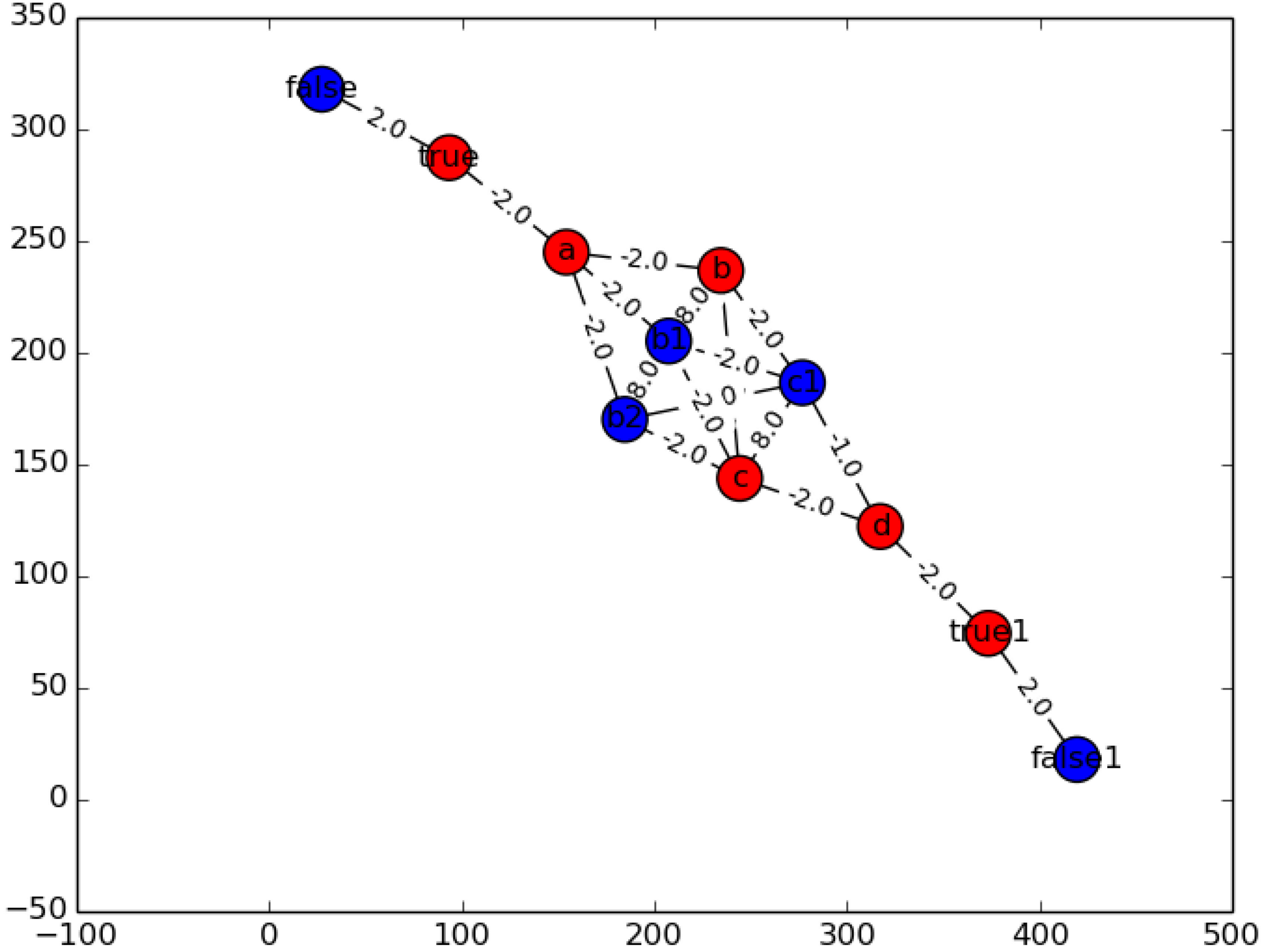

350
300
250
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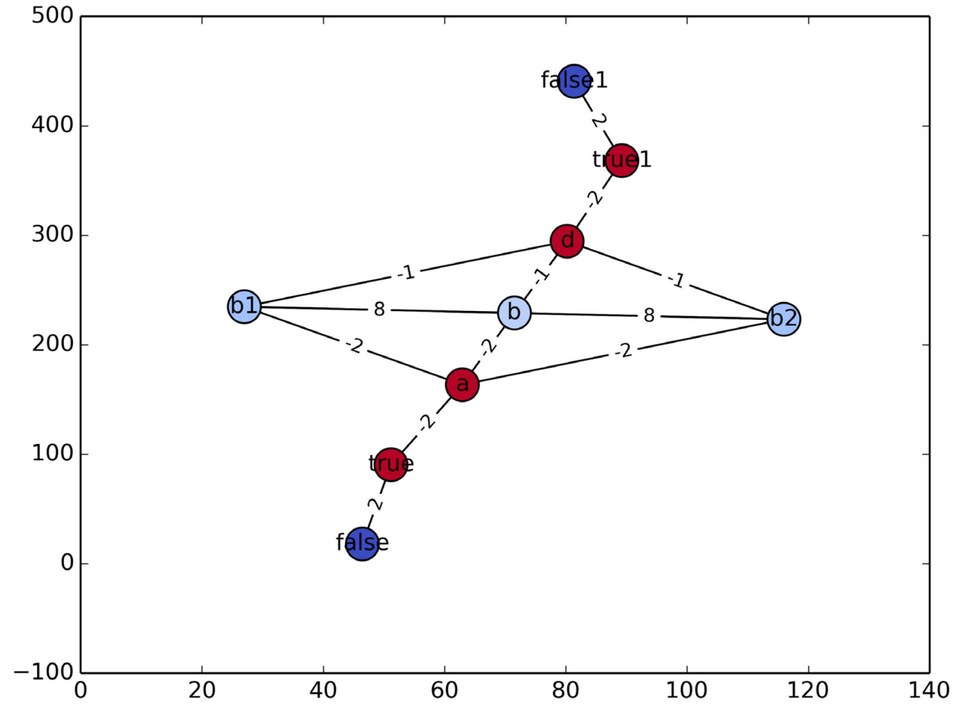
150
100
50
0
−50
−100
0
100
200
300
400
500
false
true
a
b
b1
b2
c1
c
d
true1
false1
-2.0
-1.0
8.0

Capturing Probability in the Graph

In this graph, the nodes b, b1, b2 are weighted equally, causing the solutions to be chosen equally



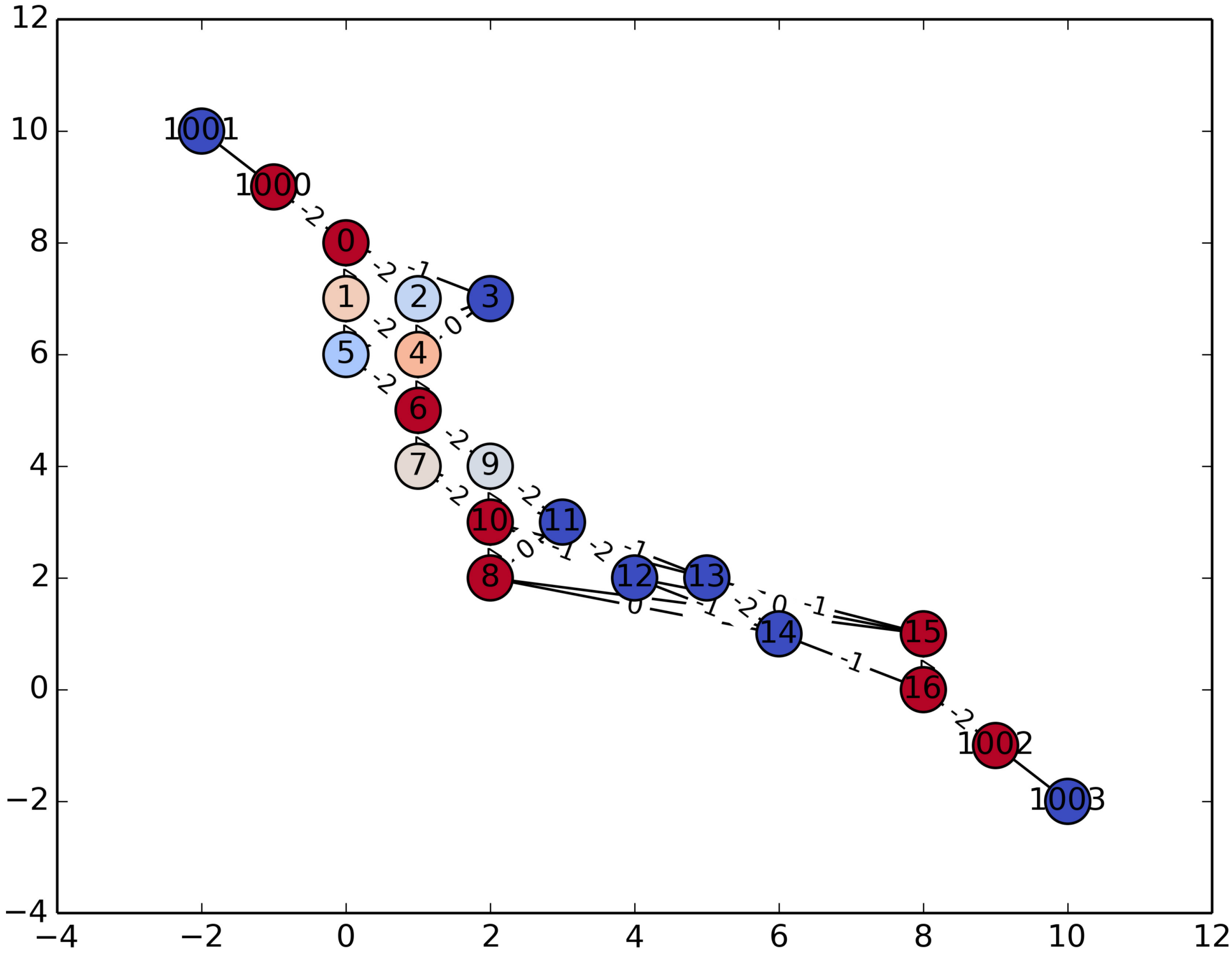
Real distributions of solutions

Actual distributions from D-Wave 2X (DW2X_SYS4):

num_reads = 100
solution 0 energy -2024.0 num 31 (b)
solution 1 energy -2024.0 num 38 (b1)
solution 2 energy -2024.0 num 30 (b2)
Solution 3 energy -1010.0 num 1 (a = False (invalid)

num_reads = 1000
solution 0 energy -2024.0 num 246 (b)
solution 1 energy -2024.0 num 308 (b1)
solution 2 energy -2024.0 num 409 (b2)
Solution 3 energy -1502.0 num 21 (b1 chain broken)

More Complicated



Observations

- **Behavior on simulator, Burnaby and LANL systems consistent**
  - Real machines suffer from low dynamic range
  - Simulator an acceptable approach for small-scale development and test of ideas
    - If it works on the real machine (small problems), it will work on the simulator (qubit limited)
    - If it works on the simulator, it will probably require some re-scaling for the real thing
    - Others have seen very different results with real system, and unacceptably slow behavior for larger problems

Observations, cont

- **The system anneals to the lowest energy state**
  - The system does not care about your problem
    - The system would rather break your problem than not find the lowest state!
- **Chain strength, pinnings, and 1-of-n tensions most critical to producing valid outputs**
    - Pin strength > sum (weights of shortest path)
    - Chain strength > sum (weights of shortest path)
- **More dynamic range can be achieved by replicating qubits that require high weights**
  - In particular, the pinnings at both ends
  - Custom embedding may be required to push chains onto the high weight qubits

Observations, cont.

- **Can a large problem be broken into many pieces?**
  - Easily with choke points in the graph
    - Single nodes that the path must go through
  - Otherwise, iteratively remove unlikely paths in order to introduce choke points/splits

Conclusion

- **Successful mapping of a practical shortest-path problem to the quantum annealer**
  - Plausible backend for a signal processing system
- **D-Wave 2X provides enough connectivity and predictability to be useful**
  - For this approach to this problem, probably difficult to beat a classical approach
  - Also unlikely to fly on a satellite
- **Useful exercise in mapping arbitrary problems**