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advanced network science initiative (ansi)

Challenges and Successes of Solving Binary Quadratic Programming Benchmarks on the DW2X QPU

Carleton Coffrin, Harsha Nagarajan, Russell Bent

A-1

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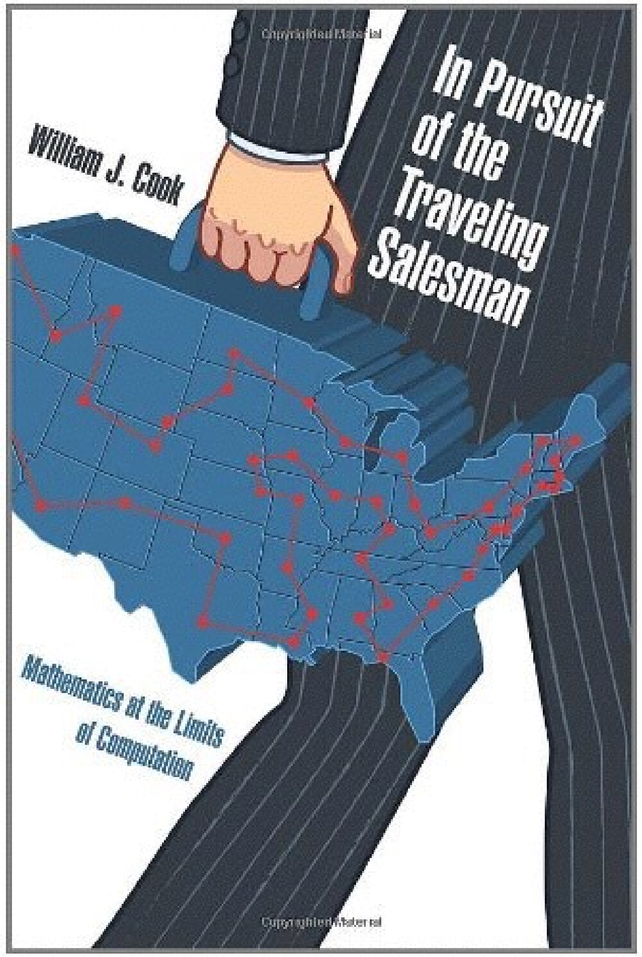
D-Wave for Optimization

Binary-Quadratic Program
(B-QP / QUBO)

NP-Hard Combinatorial Optimization Problem

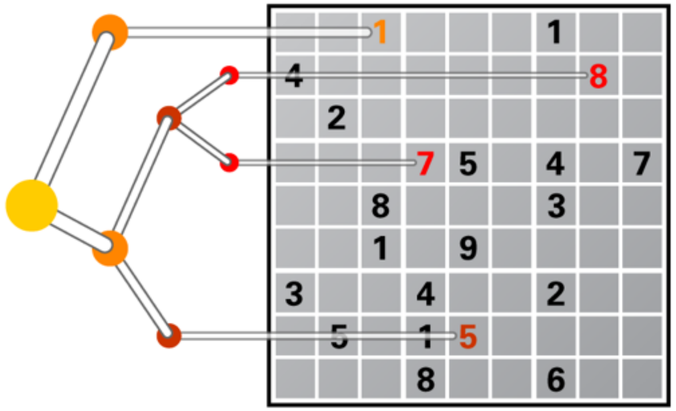
Combinatorial Optimization



Thriving R&D for at least 50 years

500,000x algorithmic speedup from 1993-2013

coursera



Discrete Optimization

http://bob4er.blogspot.com/2015/05/amazing-solver-speedups.html

Open Question

State-of-the-Art
Optimization Tools

D:wave

Open Question

State-of-the-Art Optimization Tools

$<$? $>$

D:wave

Open Question

BIG DEAL!

State-of-the-Art Optimization Tools

$<$? $>$

D:wave

Open Question

BIG DEAL!

State-of-the-Art Optimization Tools $<$? $>$ D:wave

Taking a first step towards answering this question…

The Optimization Landscape

Solvers (i.e. Algorithms)

Benchmarks (i.e. Problems)

The Optimization Landscape

Solvers (i.e. Algorithms)

Benchmarks (i.e. Problems)

The Optimization Landscape

Solvers (i.e. Algorithms)

IBM CPLEX

GUROBI OPTIMIZATION

JACOP

Benchmarks (i.e. Problems)

ACP CSPLIB

cnrs QPLIB

MIPLIB

MINLPLIB

TSPLIB

The Optimization Landscape

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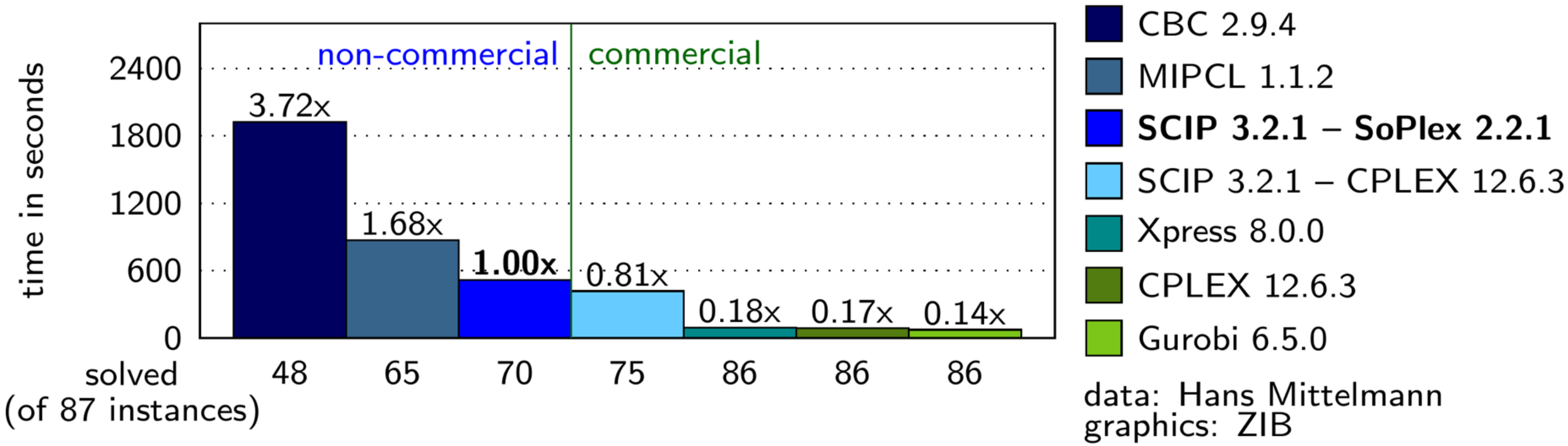
Optimization Science



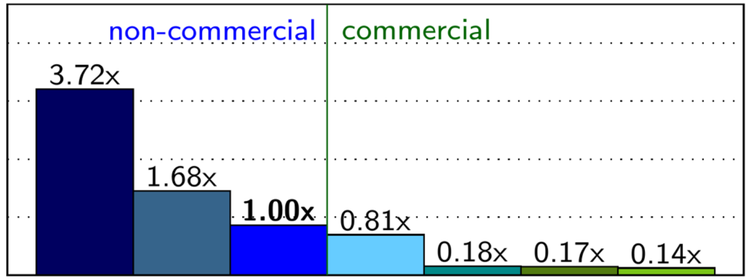
http://plato.asu.edu/ftp/milpc.html

http://scip.zib.de/

Our Observation

- The QPLib benchmark set contains **some** problems that the D-Wave can solve natively! (B-QP/QUBO)
- Lets run the QPLib on the **DW2X**
- Compare to state-of-the-art Solver **Gurobi**

QPLIB



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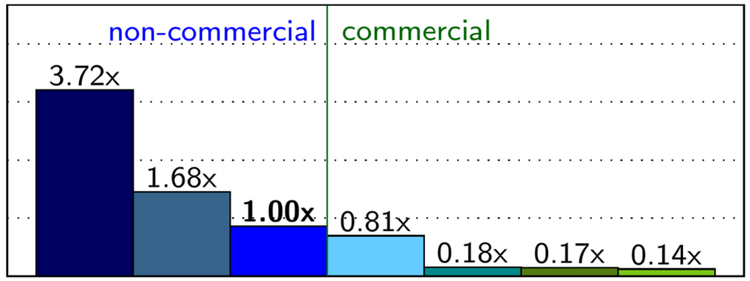

GUROBI OPTIMIZATION **vs** D-Wave

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Overview

- Background / Motivation
- Benchmarking Approach / Challenges
- Three Case Studies
  - QPLib, Max-Clique, Home-brewed
- Open Challenges, Path Forward

D:wave

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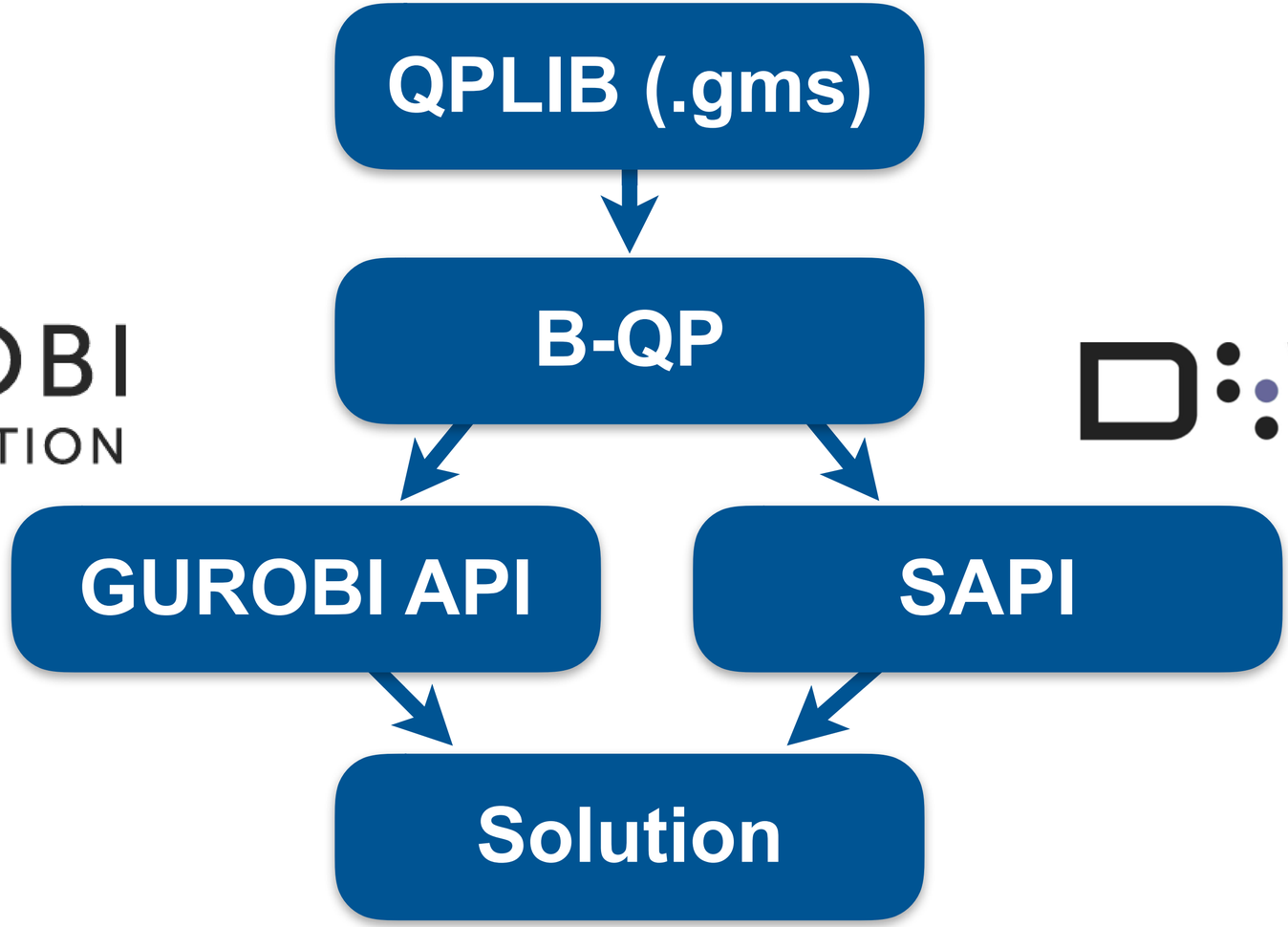
The Approach gms2q.py (Python 2)



GUROBI OPTIMIZATION

D:wave

First Big Question

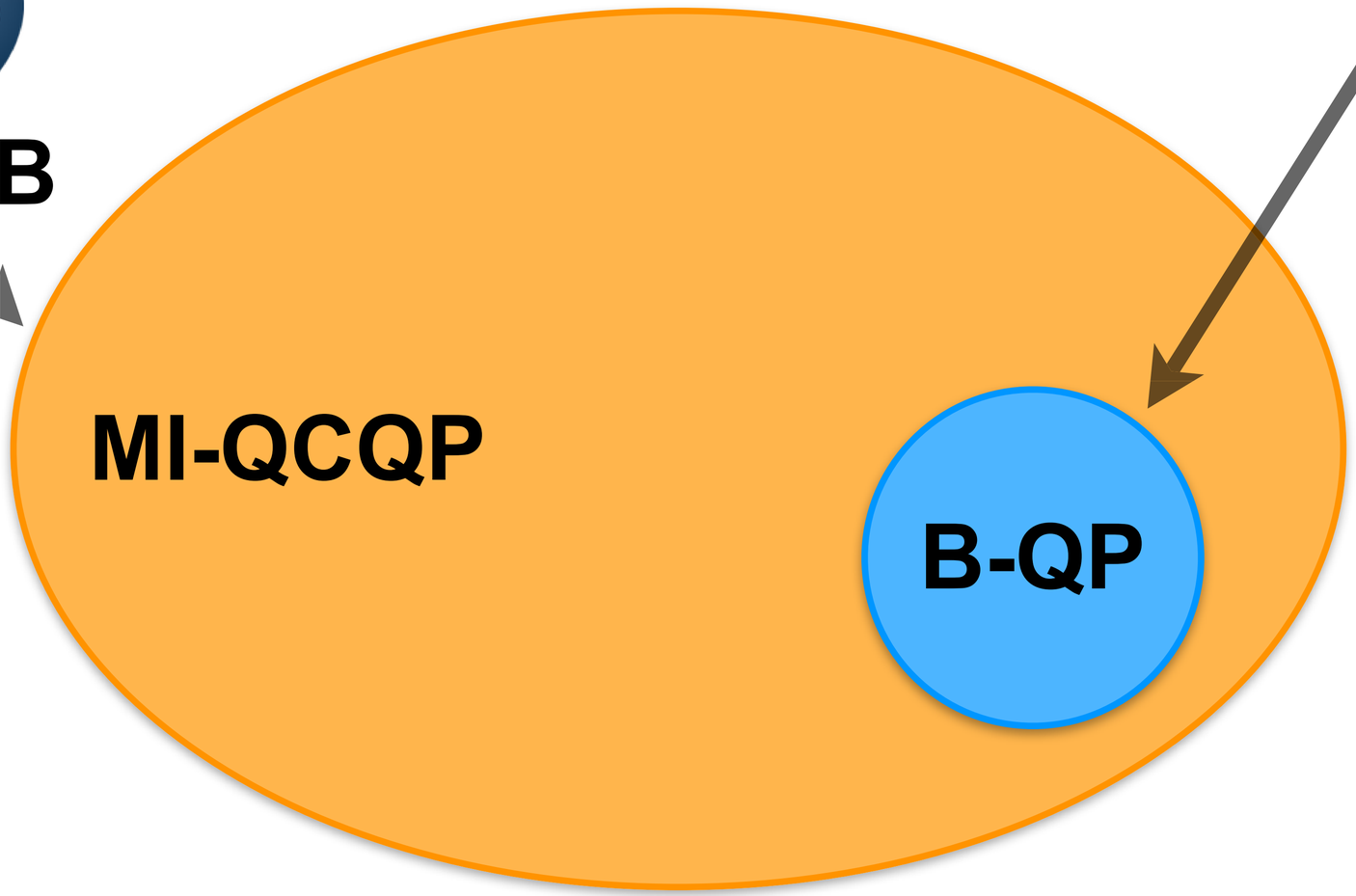
QPLIB

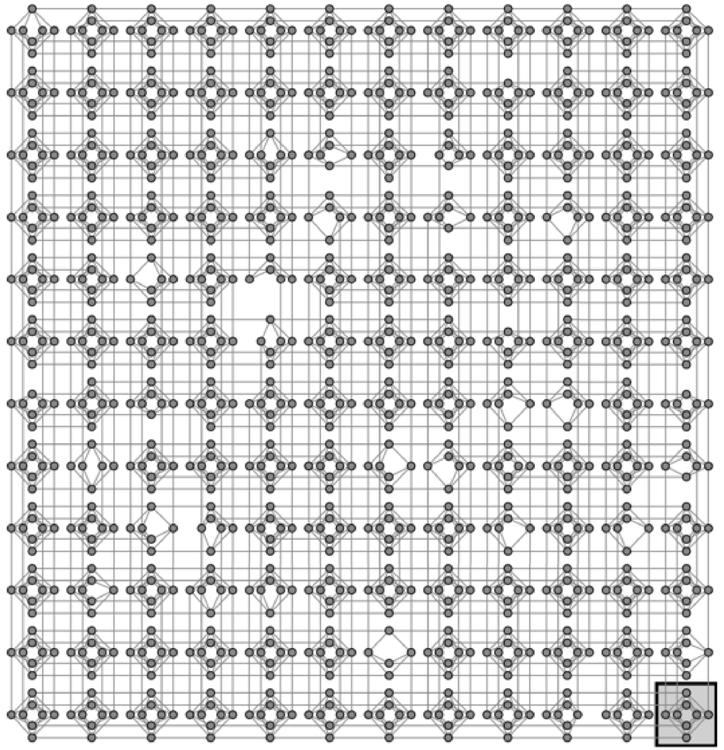
MI-QCQP

First Big Question

QPLIB

D-wave





C_{12} DW2X_SYS4

B-QP vs MI-QCQP

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

s.t.

$$\sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{sij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_{si} x_i \leq 0 \ \forall s \in \mathcal{S}$$

$$x_i \in \{0,1\} \ \forall i \in \mathcal{B} \subseteq \mathcal{N}$$

$$x_i \in \mathbb{N} \ \forall i \in \mathcal{I} \subseteq \mathcal{N}$$

$$x_i \in \mathbb{R} \ \forall i \in \mathcal{R} \subseteq \mathcal{N}$$

QPLIB

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D:wave

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cnrs

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D:wave

QPLIB

B-QP vs MI-QCQP

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s.t.

$$x_i \in \{0,1\} \; \forall i \in \mathcal{N}$$

$$|\mathcal{N}| \leq 1100$$

$$|\mathcal{E}| \leq 3068$$

D:wave

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

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cnrs

QPLIB

B-QP vs MI-QCQP

- 479 cases

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

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QPLIB

B-QP vs MI-QCQP

- 479 cases
- -350 cases, no real or int

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

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QPLIB

B-QP vs MI-QCQP

- 479 cases
- -350 cases, no real or int
- -87 cases, no constraints

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

s.t.

$$\cancel{\sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i \leq 0 \; \forall \mathcal{S} \subseteq \mathcal{E}}$$

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QPLIB

B-QP vs MI-QCQP

- 479 cases
- -350 cases, no real or int
- -87 cases, no constraints
- -23 cases, edge bound

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

s.t.

$$\cancel{\sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i \leq 0 \; \forall \mathcal{S} \subseteq \mathcal{E}}$$

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$$|\mathcal{E}| \leq 3068$$

QPLIB

B-QP vs MI-QCQP

- 479 cases
- -350 cases, no real or int
- -87 cases, no constraints
- -23 cases, edge bound
- -2 cases, variable bound

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

s.t.

$$\cancel{\sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i \leq 0 \; \forall \; \mathcal{S}}$$

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$$|\mathcal{E}| \leq 3068$$

$$|\mathcal{N}| \leq 1100$$

QPLIB

B-QP vs MI-QCQP

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- -350 cases, no real or int
- -87 cases, no constraints
- -23 cases, edge bound
- -2 cases, variable bound
- **17 cases** remain…

$$\min : \sum_{i,j \in \mathcal{E}} c_{ij} x_i x_j + \sum_{i \in \mathcal{N}} c_i x_i$$

s.t.

$$\cancel{\sum_{i,j \in \mathcal{E}} c_{ij} x_i x_j + \sum_{i \in \mathcal{N}} c_i x_i \leq 0 \; \forall \; \subseteq \mathcal{S}}$$

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$$|\mathcal{E}| \leq 3068$$

$$|\mathcal{N}| \leq 1100$$

QPLIB

The B-QP-17

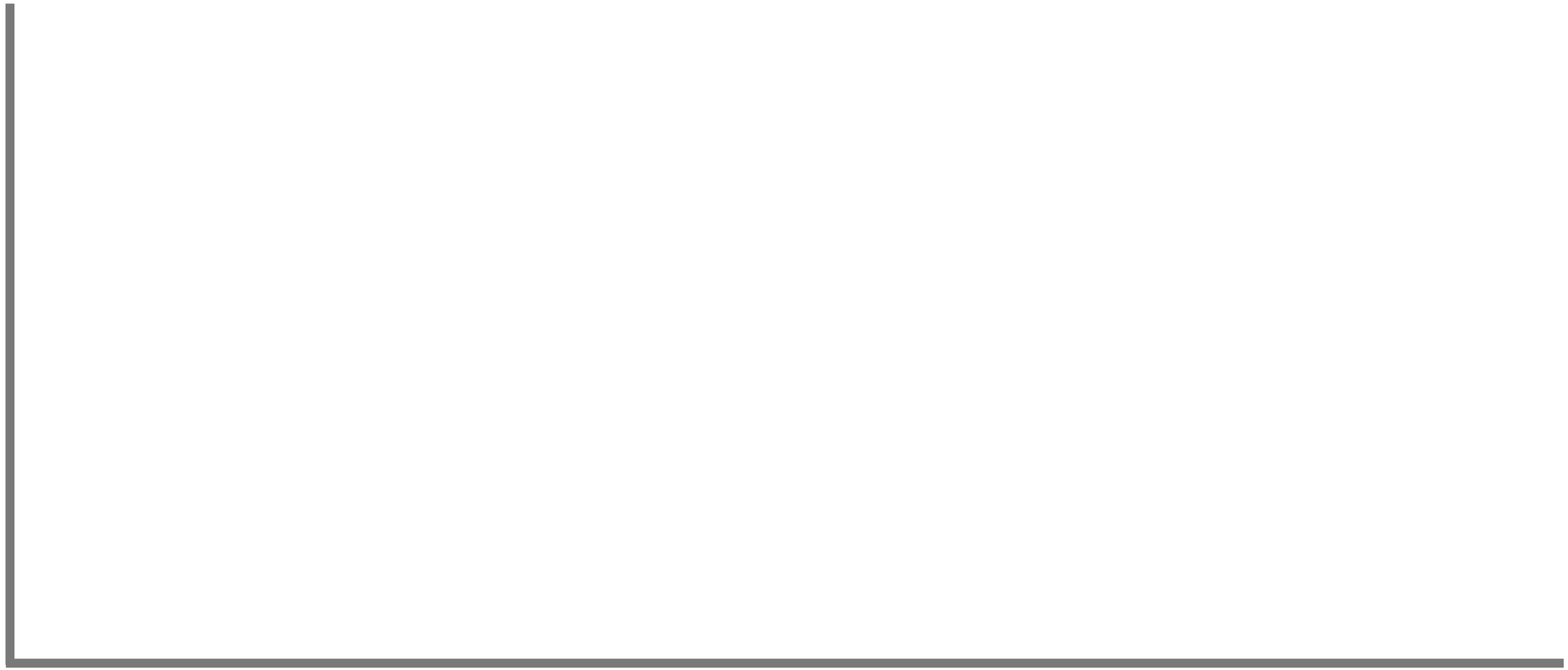
Second Question:
are these “hard” problems?

Classic B-QP Solver

GUROBI OPTIMIZATION

Solution Cost

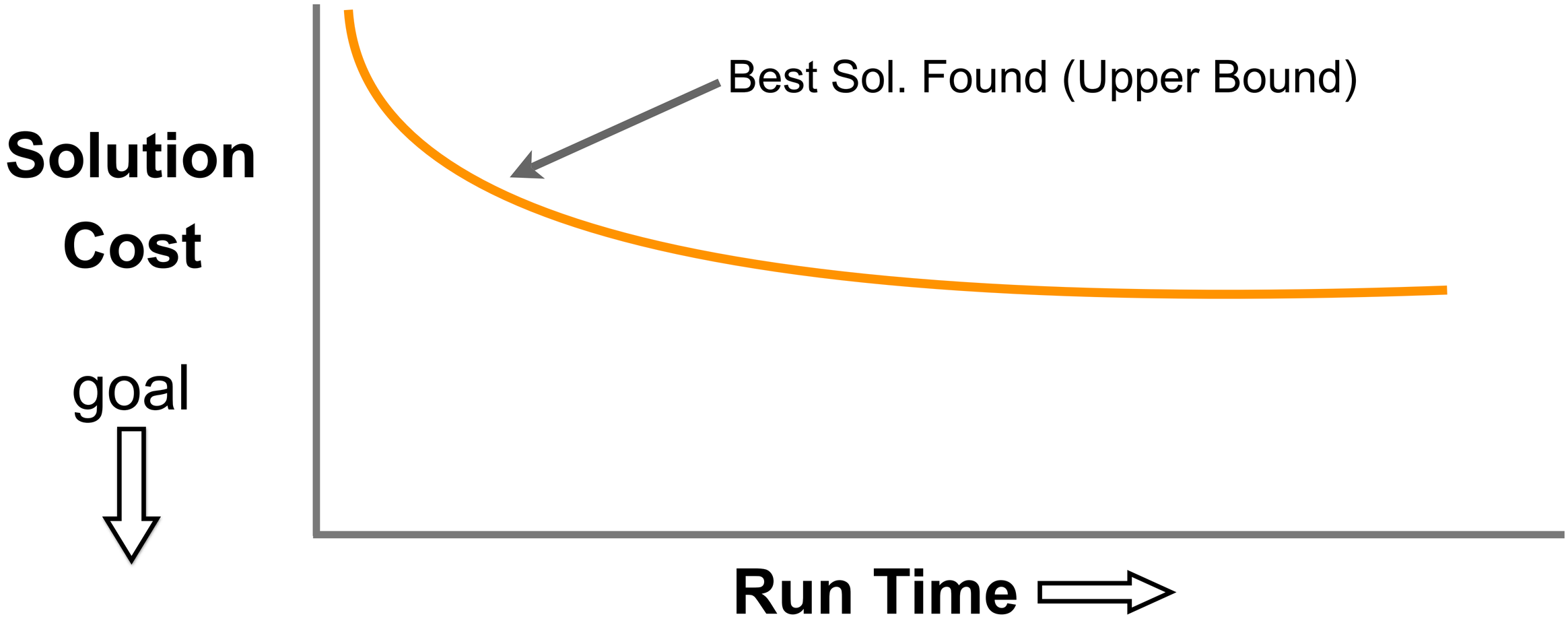
goal



Run Time

Classic B-QP Solver

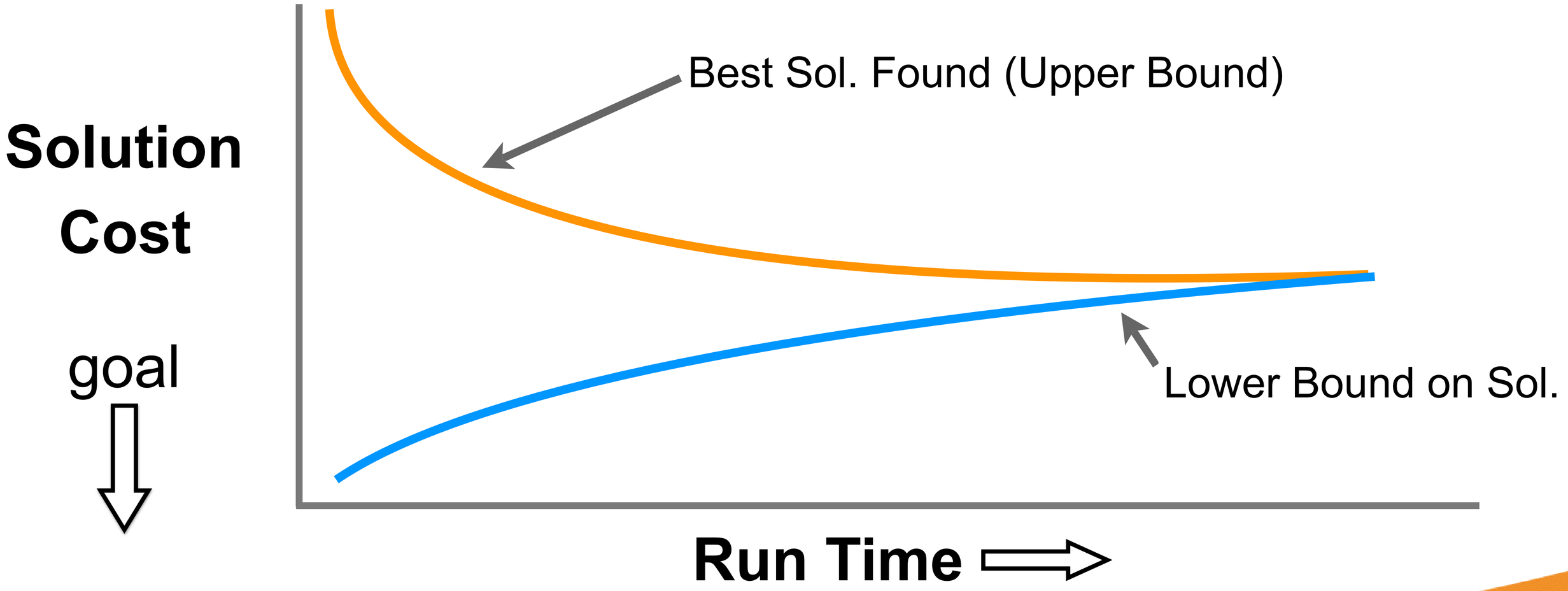
GUROBI OPTIMIZATION



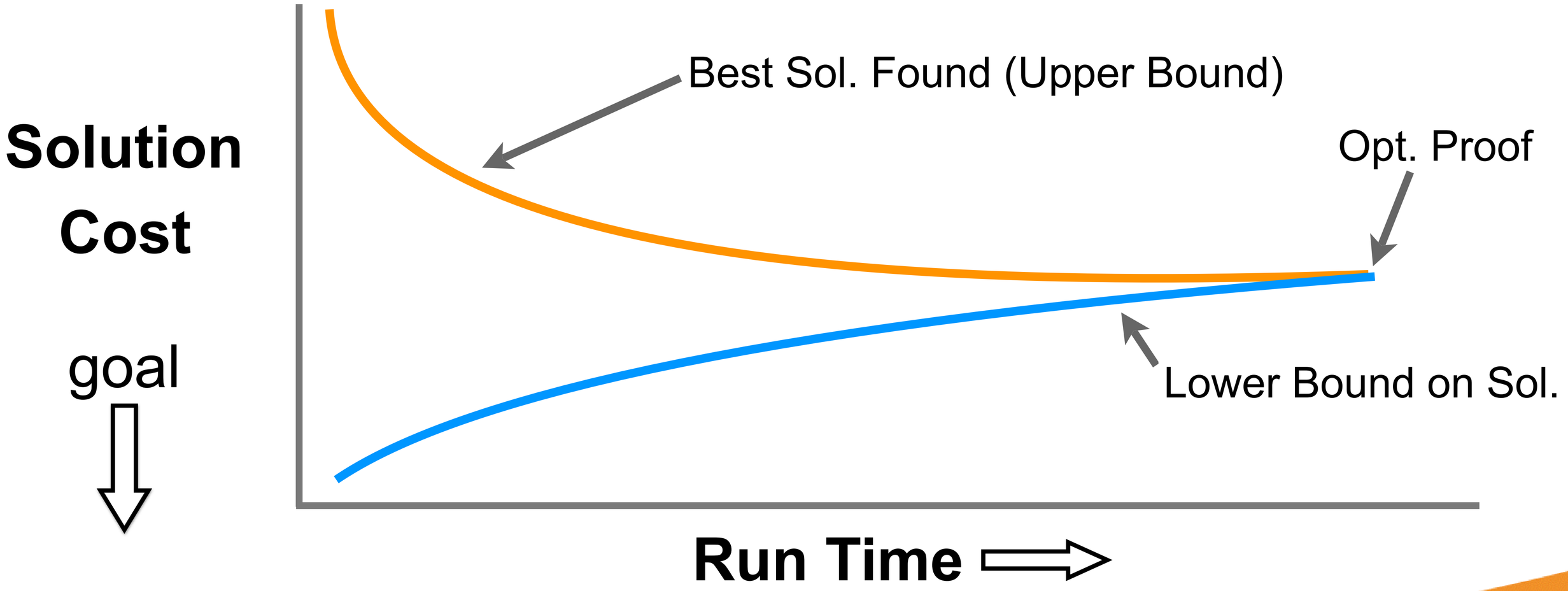
Operated by Los Alamos National Security, LLC for the U.S. Department of Energy's NNSA

Classic B-QP Solver

GUROBI OPTIMIZATION



Classic B-QP Solver



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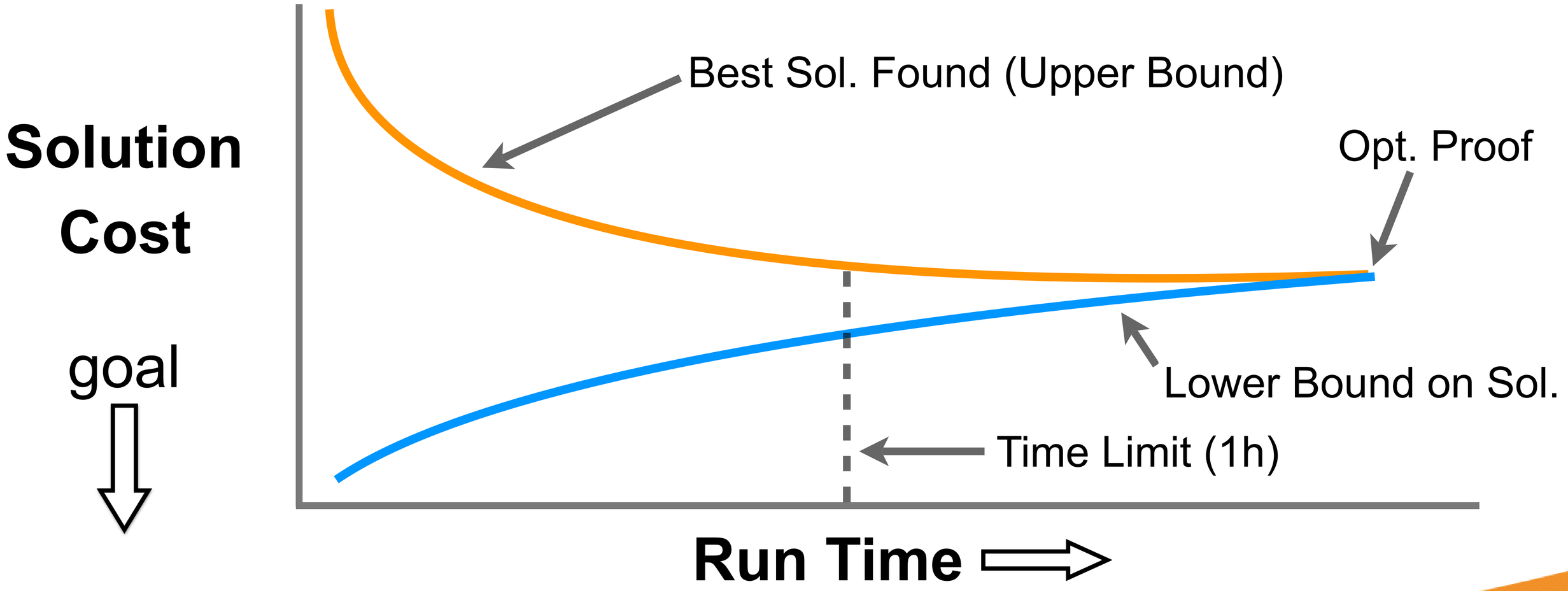
GUROBI OPTIMIZATION



Classic B-QP Solver

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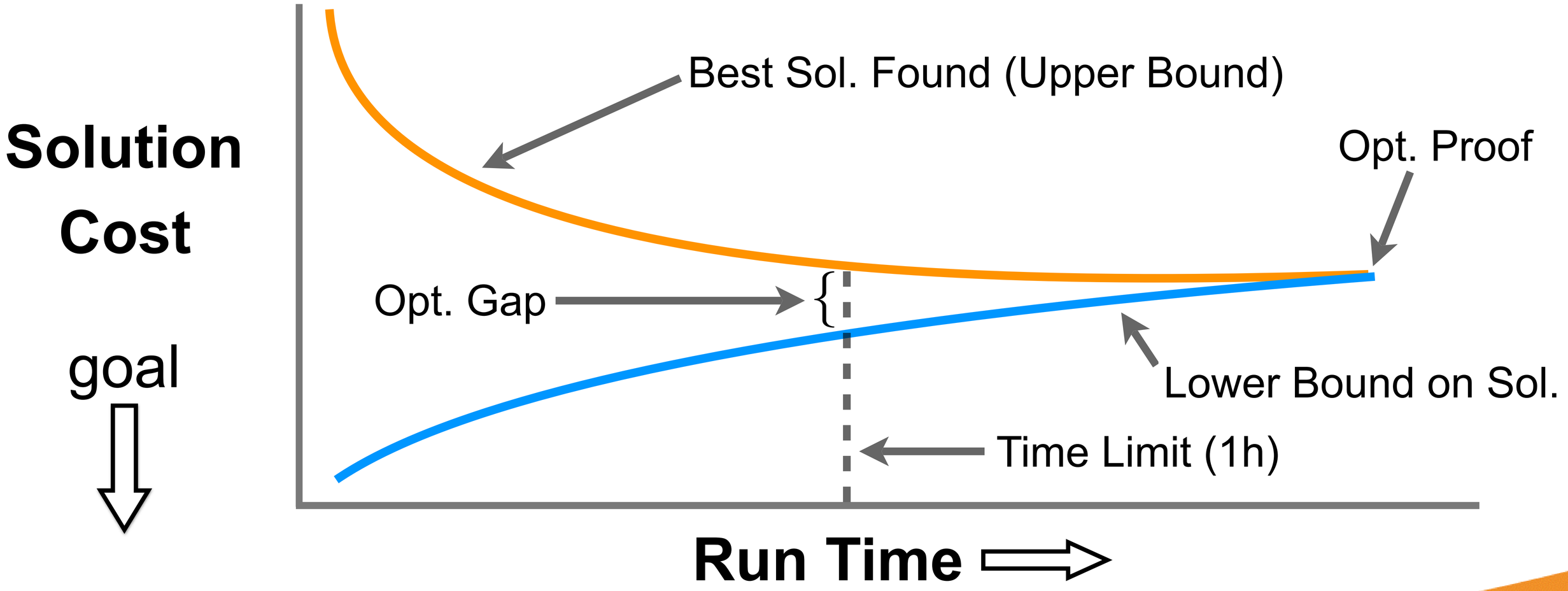

The B-QP-17

GUROBI OPTIMIZATION

Case	$\|\mathcal{N}\|$	$\|\mathcal{E}\|$	gurobi Best Sol.	gurobi Opt. Gap	gurobi Time
qplib_3876	28	48	-24	0%	<1
qplib_3607	66	120	-68	0%	<1
qplib_5727	225	450	-15051133	0%	<1
qplib_3756	153	288	-160	0%	1.2
qplib_5755	400	800	-24838942	0%	1.5
qplib_3821	190	360	-192	0%	15
qplib_3565	276	528	-282	0%	93
qplib_3705	378	728	-384	0%	202
qplib_3745	325	624	-334	0%	753
qplib_3506	496	960	-478	1%	T.L.
qplib_3738	435	840	-422	1%	T.L.
qplib_3877	630	1224	-602	1%	T.L.
qplib_3706	703	1368	-682	2%	T.L.
qplib_3642	1035	2024	-1030	3%	T.L.
qplib_3650	946	1848	-918	3%	T.L.
qplib_5889	250	3045	-40358	15%	T.L.
qplib_5909	250	3015	-33587	28%	T.L.

The B-QP-17

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Hard Problems!

The B-QP-17

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Hard Problems!

D-Wave

B-QP

↓

SAPI

↓

Solution

Preparing a D-Wave Problem

B-QP

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

s.t.

$x_i \in \{0, 1\} \; \forall i \in \mathcal{N}$

$\mathcal{E} \subseteq \mathcal{C}_{12}$

Ising

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} \sigma_i \sigma_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i \sigma_i$$

s.t.

$\sigma_i \in \{-1, 1\} \; \forall i \in \mathcal{N}$

$\mathcal{E} \subseteq \mathcal{C}_{12}$

Preparing a D-Wave Problem

$$\sigma_i = 2x_i - 1$$

B-QP

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

s.t.

$$x_i \in \{0, 1\} \; \forall i \in \mathcal{N}$$

$$\mathcal{E} \subseteq \mathcal{C}_{12}$$

Ising

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} \sigma_i \sigma_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i \sigma_i$$

s.t.

$$\sigma_i \in \{-1, 1\} \; \forall i \in \mathcal{N}$$

$$\mathcal{E} \subseteq \mathcal{C}_{12}$$

Preparing a D-Wave Problem

B-QP

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

s.t.

$x_i \in \{0, 1\} \; \forall i \in \mathcal{N}$

$\mathcal{E} \subseteq \mathcal{C}_{12}$

$$\sigma_i = 2x_i - 1$$

$$x_i = \frac{\sigma_i + 1}{2}$$

Ising

$$\min : \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} \sigma_i \sigma_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i \sigma_i$$

s.t.

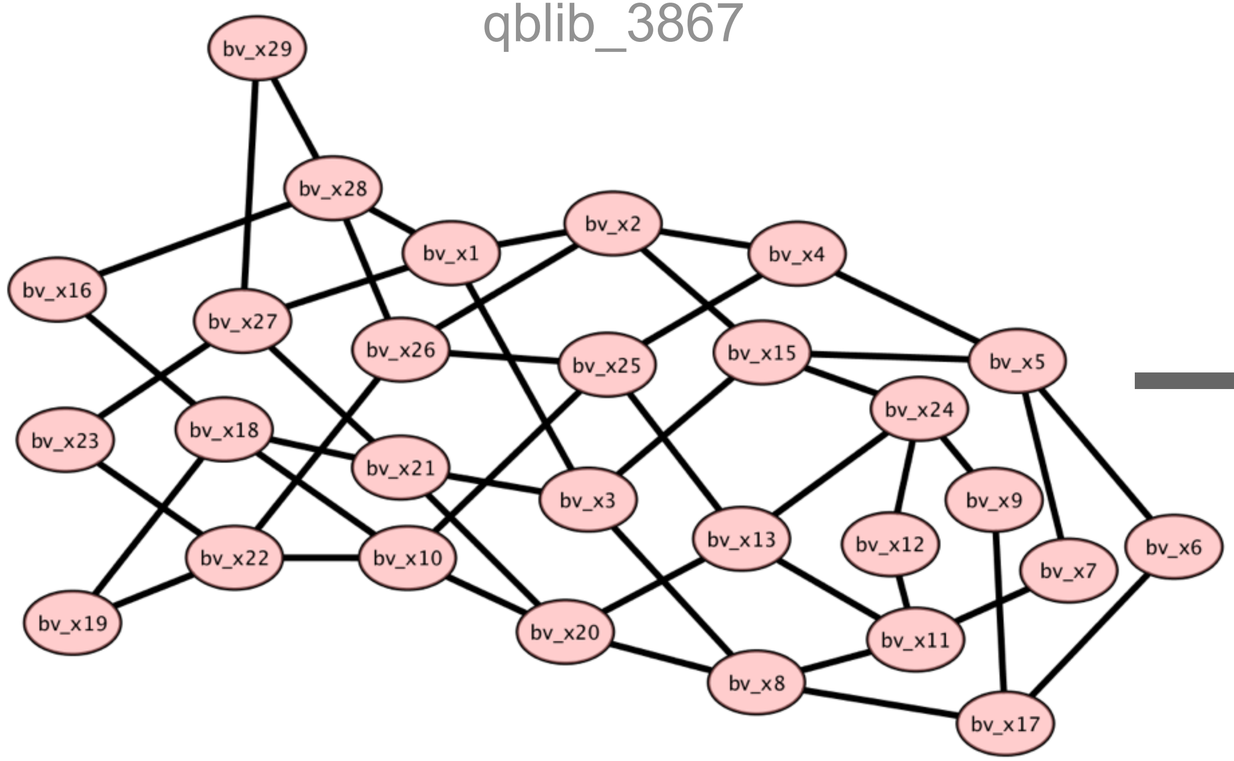
$\sigma_i \in \{-1, 1\} \; \forall i \in \mathcal{N}$

$\mathcal{E} \subseteq \mathcal{C}_{12}$

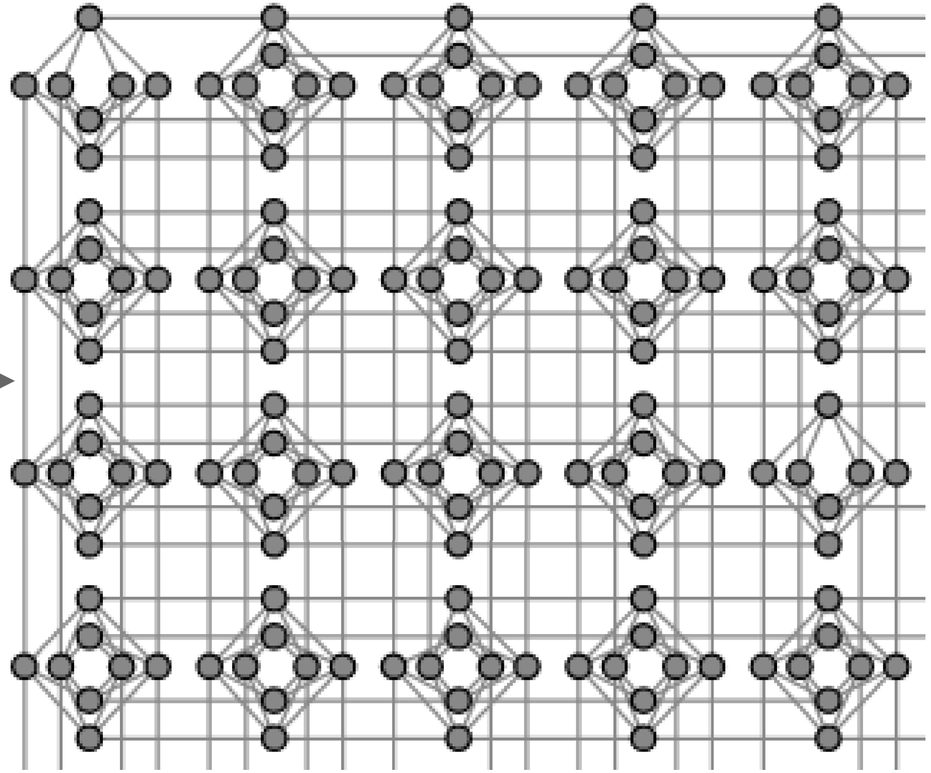
Preparing a D-Wave Problem

Source Graph

qblib_3867



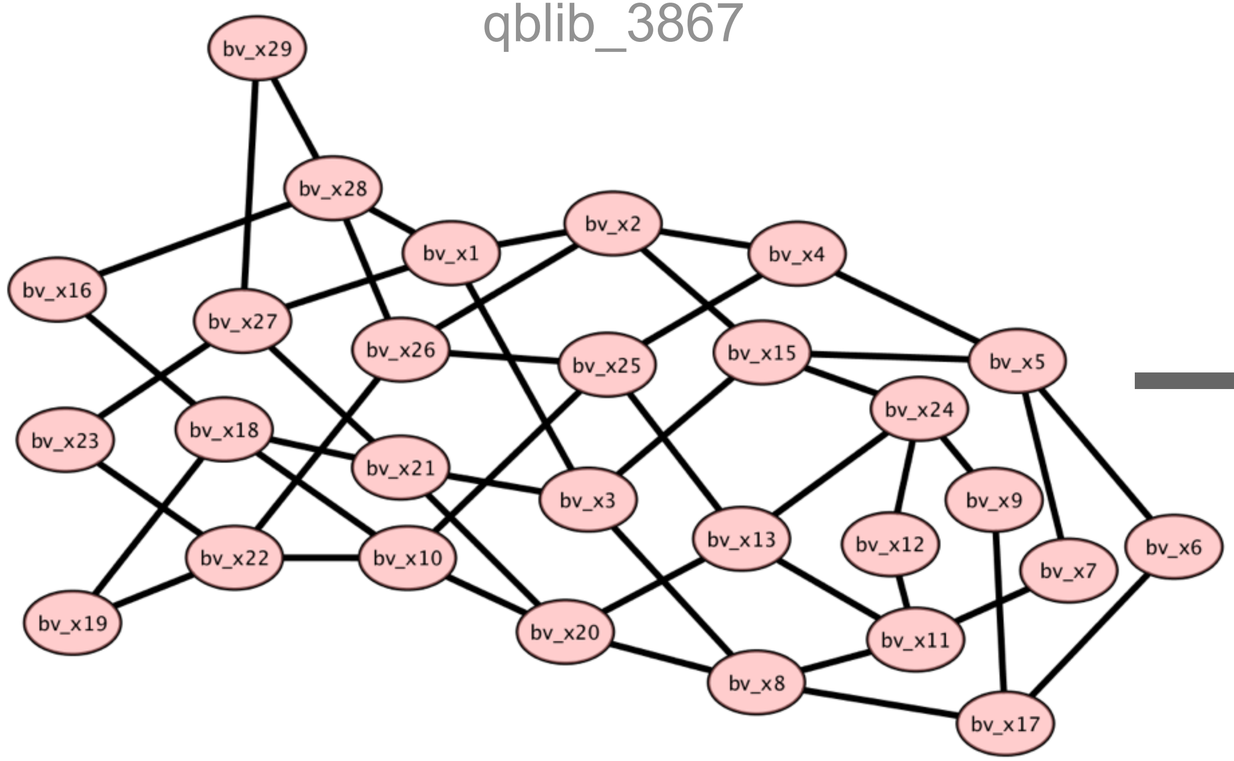
Target Graph



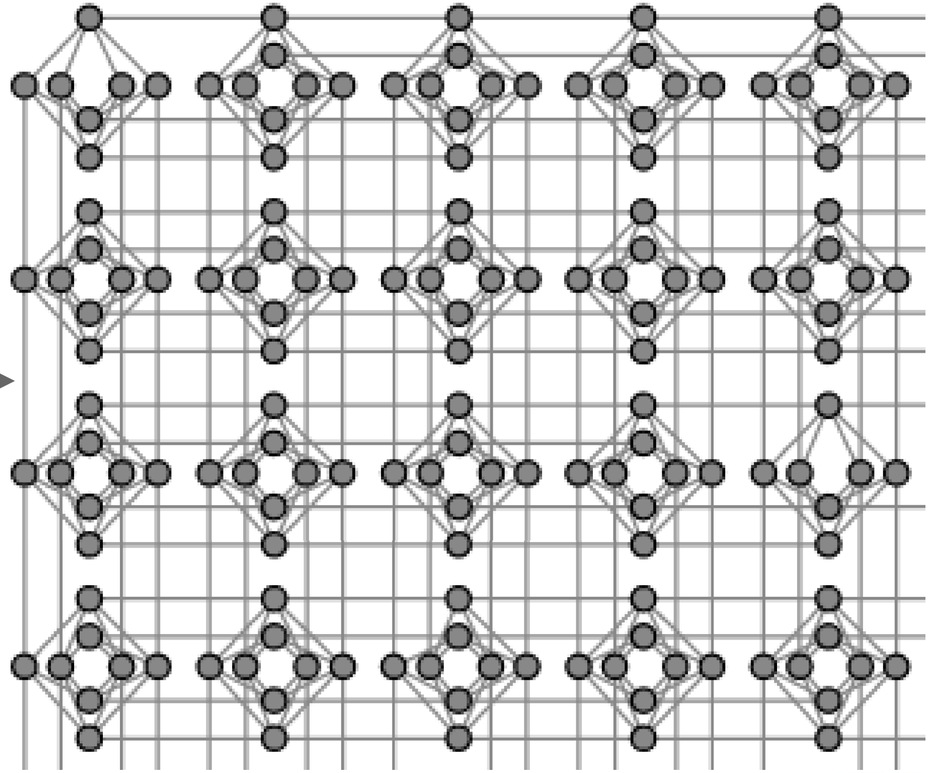
Preparing a D-Wave Problem

Source Graph

qblib_3867



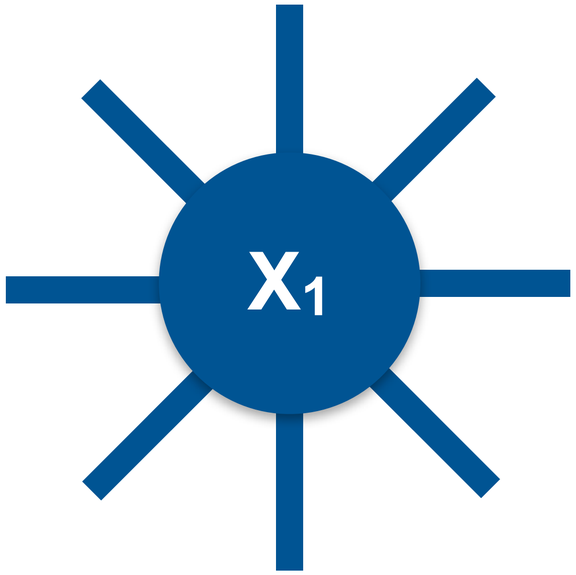
Target Graph



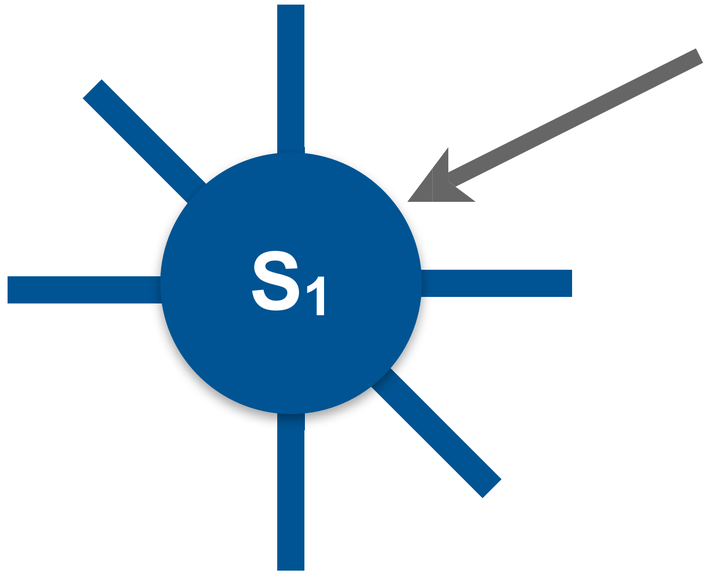
Warning: NP-Hard

Preparing a D-Wave Problem

Source Graph

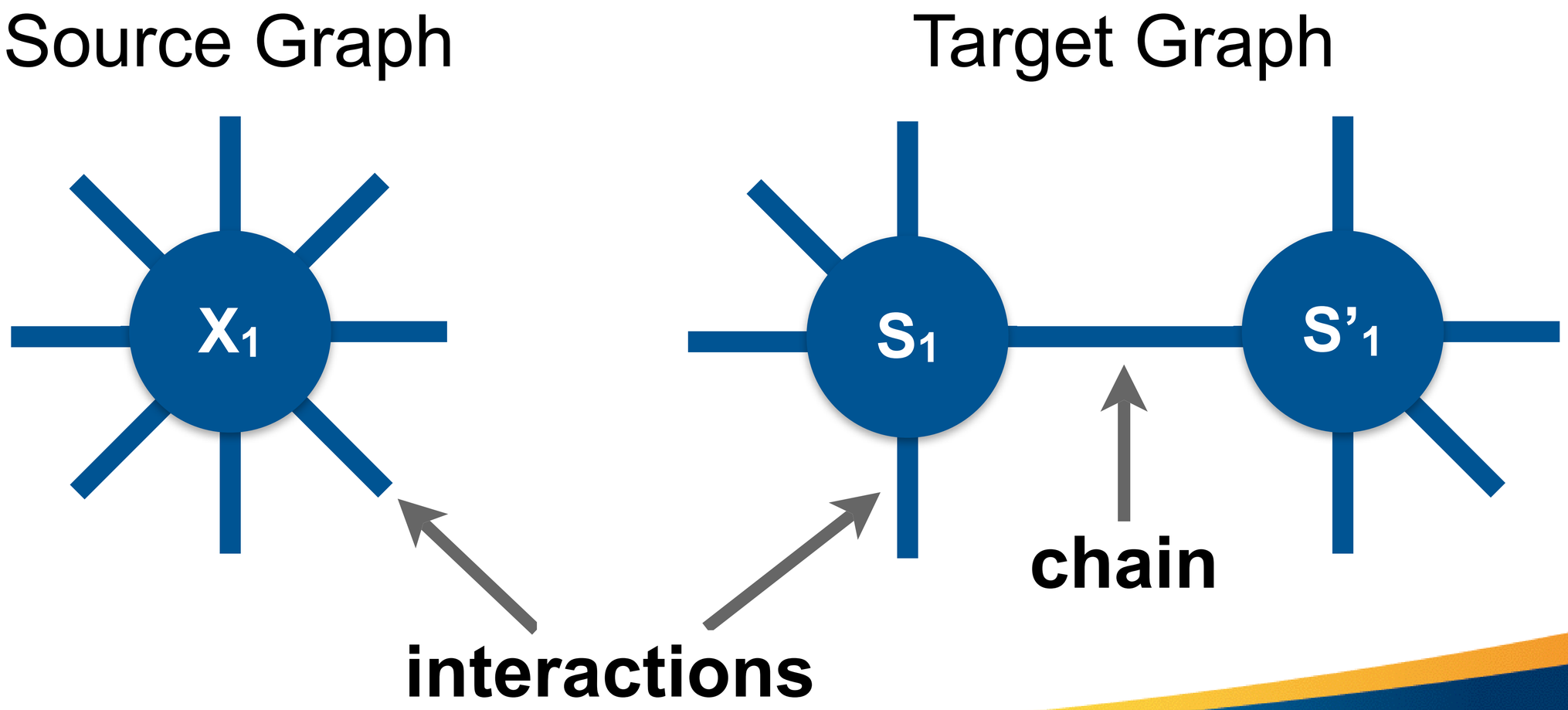


Target Graph



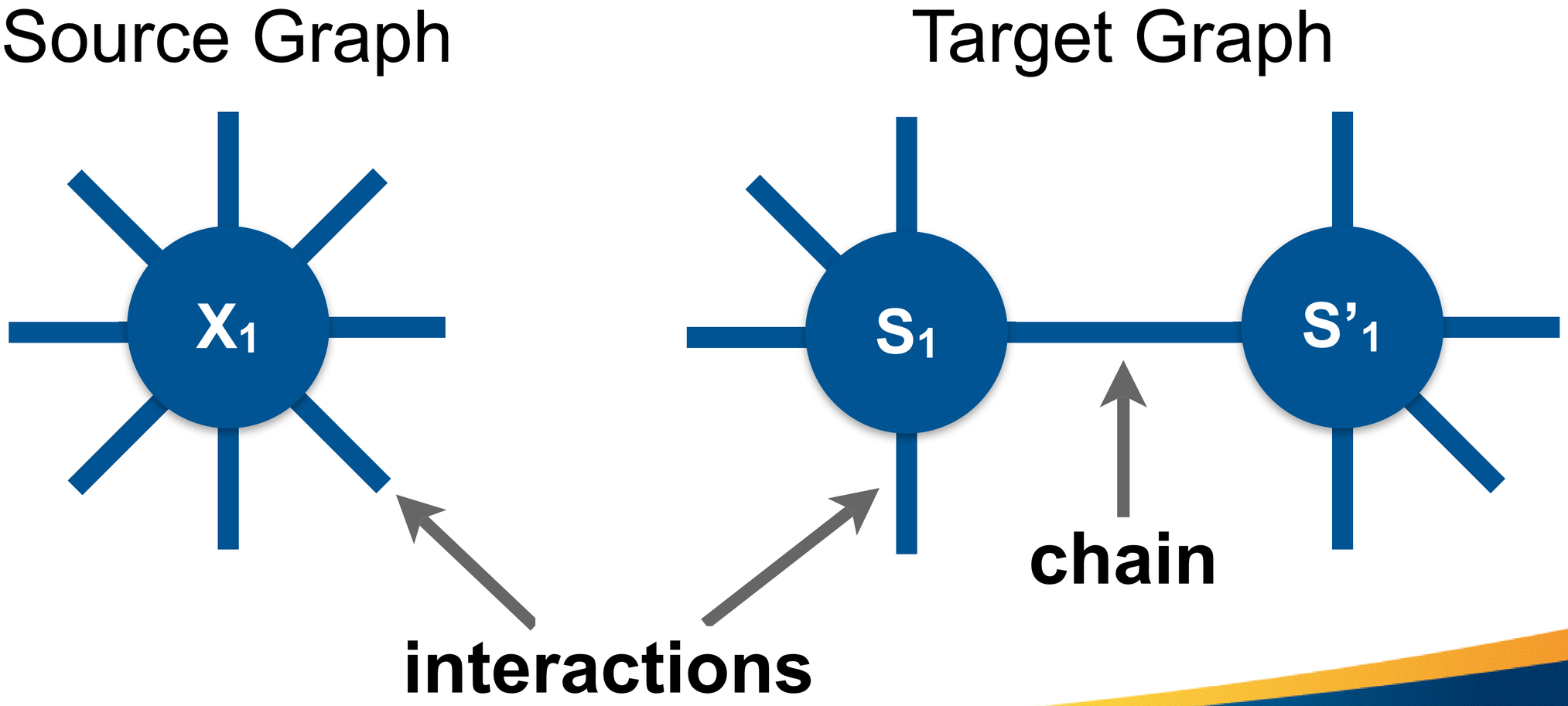
only 6 edges!

Preparing a D-Wave Problem



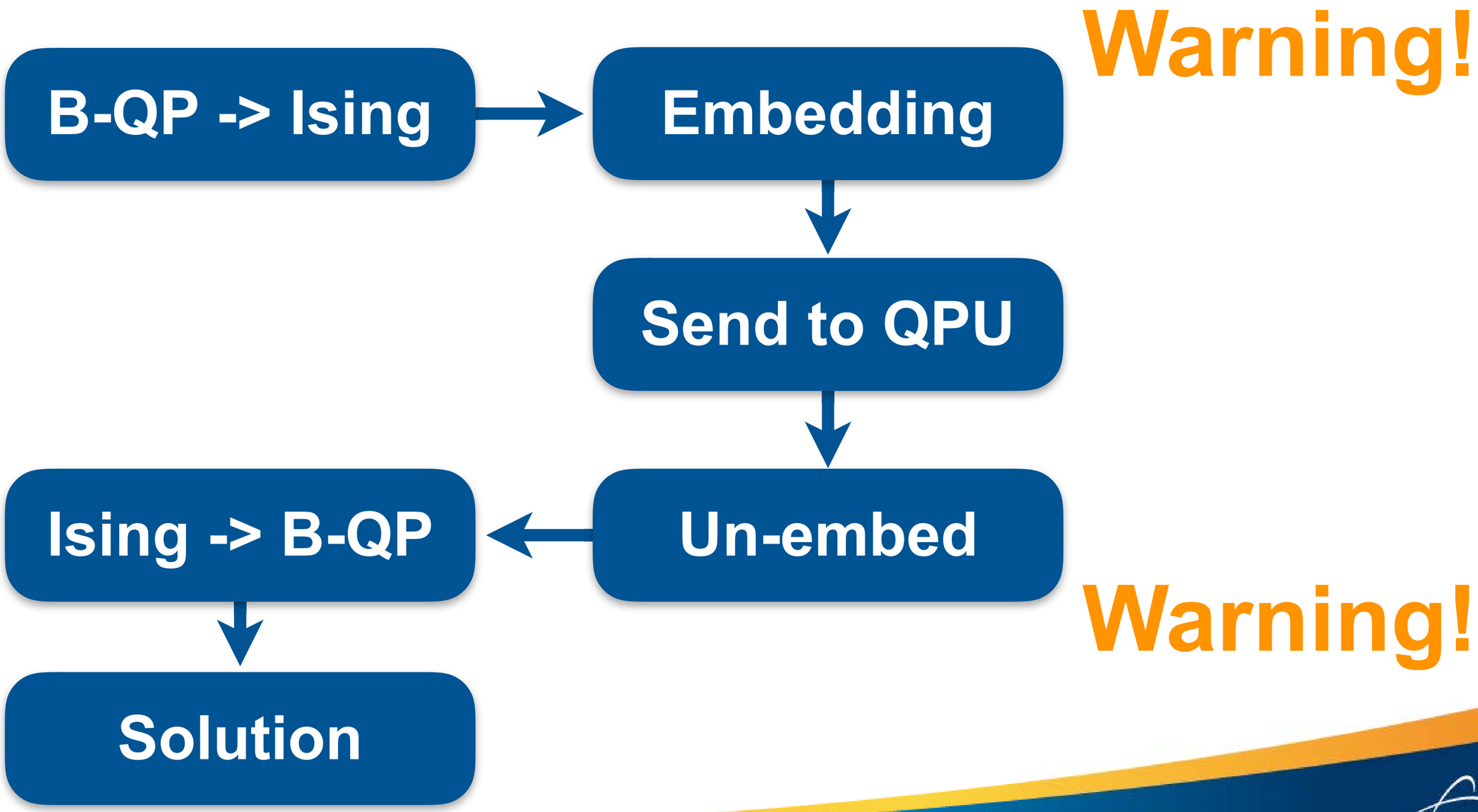
Preparing a D-Wave Problem

Warning: broken chains = infeasible solution



Preparing a D-Wave Problem

SAPI



The QP-17

Case	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{E}\rvert$	gurobi Best Sol.	gurobi Opt. Gap	gurobi Time
qplib_3876	28	48	-24	0%	<1
qplib_3607	66	120	-68	0%	<1
qplib_5727	225	450	-15051133	0%	<1
qplib_3756	153	288	-160	0%	1.2
qplib_5755	400	800	-24838942	0%	1.5
qplib_3821	190	360	-192	0%	15
qplib_3565	276	528	-282	0%	93
qplib_3705	378	728	-384	0%	202
qplib_3745	325	624	-334	0%	753
qplib_3506	496	960	-478	1%	T.L.
qplib_3738	435	840	-422	1%	T.L.
qplib_3877	630	1224	-602	1%	T.L.
qplib_3706	703	1368	-682	2%	T.L.
qplib_3642	1035	2024	-1030	3%	T.L.
qplib_3650	946	1848	-918	3%	T.L.
qplib_5889	250	3045	-40358	15%	T.L.
qplib_5909	250	3015	-33587	28%	T.L.

F.E.

Failed Embed

T.L.

Time Limit

(1 hour)

The QP-17

Case	$\|\mathcal{N}\|$	$\|\mathcal{E}\|$	gurobi Best Sol.	gurobi Opt. Gap	gurobi Time	dwave Best Sol.	dwave Best	dwave Inf.	dwave Samples	dwave Time
qplib_3876	28	48	-24	0%	<1	-24	8574	352	10000	0+4
qplib_3607	66	120	-68	0%	<1	-68	1720	4424	10000	1+4
qplib_5727	225	450	-15051133	0%	<1	F.E.	-	-	-	T.L.
qplib_3756	153	288	-160	0%	1.2	-160	234	8451	10000	57+5
qplib_5755	400	800	-24838942	0%	1.5	F.E.	-	-	-	T.L.
qplib_3821	190	360	-192	0%	15	F.E.	-	-	-	T.L.
qplib_3565	276	528	-282	0%	93	F.E.	-	-	-	T.L.
qplib_3705	378	728	-384	0%	202	F.E.	-	-	-	T.L.
qplib_3745	325	624	-334	0%	753	F.E.	-	-	-	T.L.
qplib_3506	496	960	-478	1%	T.L.	F.E.	-	-	-	T.L.
qplib_3738	435	840	-422	1%	T.L.	F.E.	-	-	-	T.L.
qplib_3877	630	1224	-602	1%	T.L.	F.E.	-	-	-	T.L.
qplib_3706	703	1368	-682	2%	T.L.	F.E.	-	-	-	T.L.
qplib_3642	1035	2024	-1030	3%	T.L.	F.E.	-	-	-	T.L.
qplib_3650	946	1848	-918	3%	T.L.	F.E.	-	-	-	T.L.
qplib_5889	250	3045	-40358	15%	T.L.	F.E.*	-	-	-	T.L.
qplib_5909	250	3015	-33587	28%	T.L.	F.E.*	-	-	-	T.L.

F.E.

Failed Embed

T.L.

Time Limit

(1 hour)

The QP-17

Case	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{E}\rvert$	gurobi Best Sol.	gurobi Opt. Gap	gurobi Time	dwave Best Sol.	dwave Best	dwave Inf.	dwave Samples	dwave Time
qplib_3876	28	48	-24	0%	<1	-24	8574	352	10000	0+4
qplib_3607	66	120	-68	0%	<1	-68	1720	4424	10000	1+4
qplib_5727	225	450	-15051133	0%	<1	F.E.	-	-	-	T.L.
qplib_3756	153	288	-160	0%	1.2	-160	234	8451	10000	57+5
qplib_5755	400	800	-24838942	0%	1.5	F.E.	-	-	-	T.L.
qplib_3821	190	360	-192	0%	15	F.E.	-	-	-	T.L.
qplib_3565	276	528	-282	0%	93	F.E.	-	-	-	T.L.
qplib_3705	378	728	-384	0%	202	F.E.	-	-	-	T.L.
qplib_3745	325	624	-334	0%	753	F.E.	-	-	-	T.L.
qplib_3506	496	960	-478	1%	T.L.	F.E.	-	-	-	T.L.
qplib_3738	435	840	-422	1%	T.L.	F.E.	-	-	-	T.L.
qplib_3877	630	1224	-602	1%	T.L.	F.E.	-	-	-	T.L.
qplib_3706	703	1368	-682	2%	T.L.	F.E.	-	-	-	T.L.
qplib_3642	1035	2024	-1030	3%	T.L.	F.E.	-	-	-	T.L.
qplib_3650	946	1848	-918	3%	T.L.	F.E.	-	-	-	T.L.
qplib_5889	250	3045	-40358	15%	T.L.	F.E.*	-	-	-	T.L.
qplib_5909	250	3015	-33587	28%	T.L.	F.E.*	-	-	-	T.L.

F.E.

Failed Embed

T.L.

Time Limit

(1 hour)

UNCLASSIFIED

If you fail…

If you fail…

Try, Again!

DIMACS Max-Clique Cases

dimacs benchmark set

This is a selected set of instances form the Second DIMACS Implementation Challenge (1992-1993).
In the following table, the clique number ω(G) corresponds to the global optimum or to the lower bound as indicated by the
For some instances the lower bound has been confirmed to coincide with the global optimum. For such instances, the cliqu
The exact algorithms that confirmed the bound are reported later on this page.

instance	ω(g)	best known	nodes	edges	graph degrees		best degrees	
					median	iqr	median	iqr
C125.9	34*	34	125	6 963	112.0	(5.00)	114.5	(4.75)
C250.9	44*	44	250	27 984	224.0	(6.00)	227.0	(5.00)
C500.9	≥ 57	57	500	112 332	449.0	(9.00)	455.0	(9.00)
C1000.9	≥ 68	68	1 000	450 079	900.0	(13.00)	907.0	(11.25)
C2000.9	≥ 80	80	2 000	1 799 532	1 800.0	(18.00)	1 803.0	(15.25)
DSJC1000_5	15	15	1 000	499 652	500.0	(20.00)	503.0	(23.00)

Why Max-Clique?

- Formulate on the Compliment Graph (super **sparse**, i.e. easy to embed!)
- **Combinatorial** Problem (minimal accuracy issues)
- Easy to **generate** problems (scale in size)

DIMACS Max-Clique Cases

			gurobi			dwave				
Case	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{E}\rvert$	Best Sol.	Opt. Gap	Time	Best Sol.	Best	Inf.	Samples	Time
C015_9	15	12	-11	0%	<1	-11	9073	1	10000	0+3
C020_9	20	17	-14	0%	<1	-14	8370	85	10000	0+3
C030_9	30	44	-16	0%	<1	-16	5651	123	10000	0+3
C040_9	40	77	-18	0%	<1	-18	3865	316	10000	0+4
C050_9	50	108	-24	0%	<1	-24	16	1254	10000	0+4
C060_9	60	158	-25	0%	<1	-25	22	5465	10000	0+5
C070_9	70	215	-27	0%	<1	-26	1	9855	10000	4+5
C080_9	80	306	-29	0%	<1	F.E.	-	-	-	T.L.
C090_9	90	407	-29	0%	1.0	F.E.	-	-	-	T.L.
C100_9	100	508	-30	0%	2.0	F.E.	-	-	-	T.L.
C110_9	110	615	-32	0%	5.1	F.E.	-	-	-	T.L.
C120_9	120	729	-32	0%	45	F.E.	-	-	-	T.L.
C125_9	125	787	-34	0%	55	F.E.	-	-	-	T.L.
C250_9	250	3141	-43	40%	T.L.	F.E.*	-	-	-	T.L.

F.E.
Failed Embed

T.L.
Time Limit (1 hour)

DIMACS Max-Clique Cases

			gurobi			dwave				
Case	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{E}\rvert$	Best Sol.	Opt. Gap	Time	Best Sol.	Best	Inf.	Samples	Time
C015_9	15	12	-11	0%	<1	-11	9073	1	10000	0+3
C020_9	20	17	-14	0%	<1	-14	8370	85	10000	0+3
C030_9	30	44	-16	0%	<1	-16	5651	123	10000	0+3
C040_9	40	77	-18	0%	<1	-18	3865	316	10000	0+4
C050_9	50	108	-24	0%	<1	-24	16	1254	10000	0+4
C060_9	60	158	-25	0%	<1	-25	22	5465	10000	0+5
C070_9	70	215	-27	0%	<1	-26	1	9855	10000	4+5
C080_9	80	306	-29	0%	<1	F.E.	-	-	-	T.L.
C090_9	90	407	-29	0%	1.0	F.E.	-	-	-	T.L.
C100_9	100	508	-30	0%	2.0	F.E.	-	-	-	T.L.
C110_9	110	615	-32	0%	5.1	F.E.	-	-	-	T.L.
C120_9	120	729	-32	0%	45	F.E.	-	-	-	T.L.
C125_9	125	787	-34	0%	55	F.E.	-	-	-	T.L.
C250_9	250	3141	-43	40%	T.L.	F.E.*	-	-	-	T.L.

F.E.

Failed Embed

T.L.

Time Limit (1 hour)

DIMACS Max-Clique Cases

			gurobi			dwave				
Case	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{E}\rvert$	Best Sol.	Opt. Gap	Time	Best Sol.	Best	Inf.	Samples	Time
C015_9	15	12	-11	0%	<1	-11	9073	1	10000	0+3
C020_9	20	17	-14	0%	<1	-14	8370	85	10000	0+3
C030_9	30	44	-16	0%	<1	-16	5651	123	10000	0+3
C040_9	40	77	-18	0%	<1	-18	3865	316	10000	0+4
C050_9	50	108	-24	0%	<1	-24	16	1254	10000	0+4
C060_9	60	158	-25	0%	<1	-25	22	5465	10000	0+5
C070_9	70	215	-27	0%	<1	-26	1	9855	10000	4+5
C080_9	80	306	-29	0%	<1	F.E.	-	-	-	T.L.
C090_9	90	407	-29	0%	1.0	F.E.	-	-	-	T.L.
C100_9	100	508	-30	0%	2.0	F.E.	-	-	-	T.L.
C110_9	110	615	-32	0%	5.1	F.E.	-	-	-	T.L.
C120_9	120	729	-32	0%	45	F.E.	-	-	-	T.L.
C125_9	125	787	-34	0%	55	F.E.	-	-	-	T.L.
C250_9	250	3141	-43	40%	T.L.	F.E.*	-	-	-	T.L.

F.E.
Failed Embed

T.L.
Time Limit (1 hour)

If you fail twice…

If you fail twice…

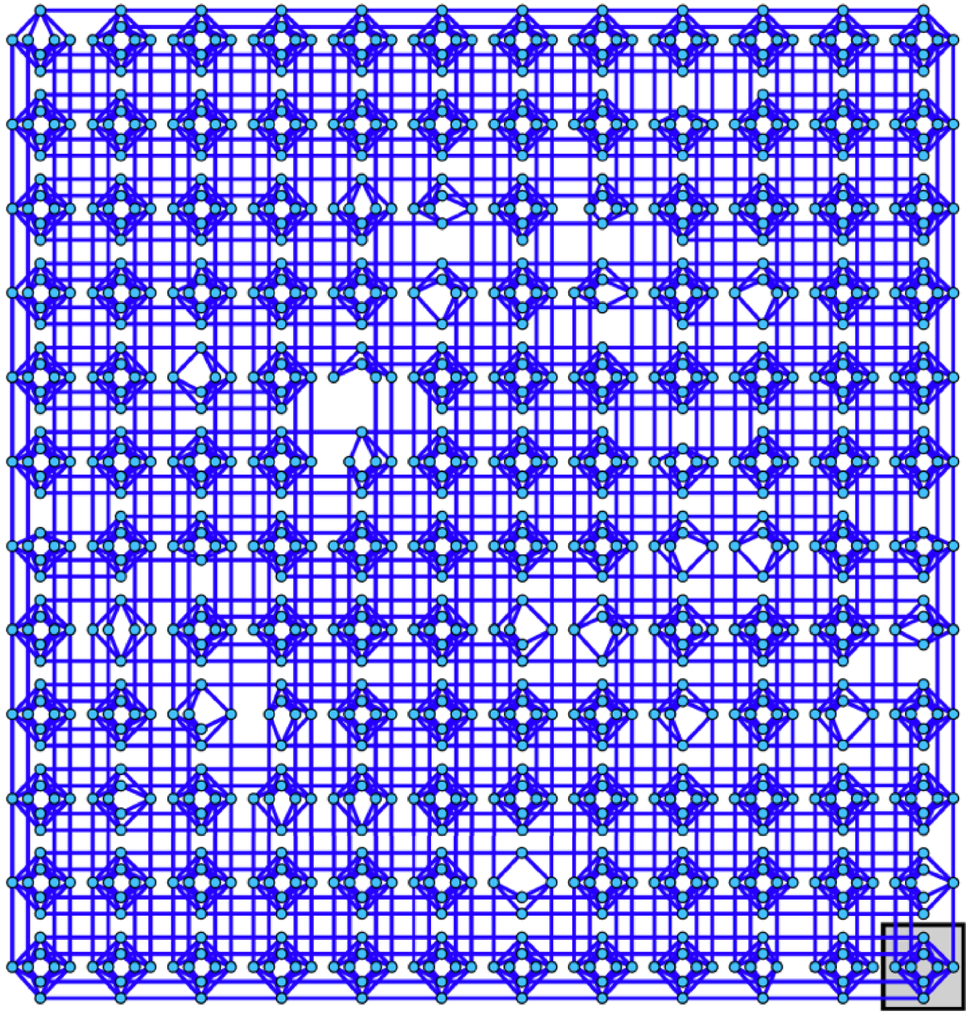
Try, Try, Again!

3rd time's a charm?

Key Idea: Best-Case Scenario for D-Wave

- Design the problem on the QPU chip
  - No embedding needed!
  - No chains needed!
- {1,-1} coefficients, no accuracy issues!

Case 1 - All Negative



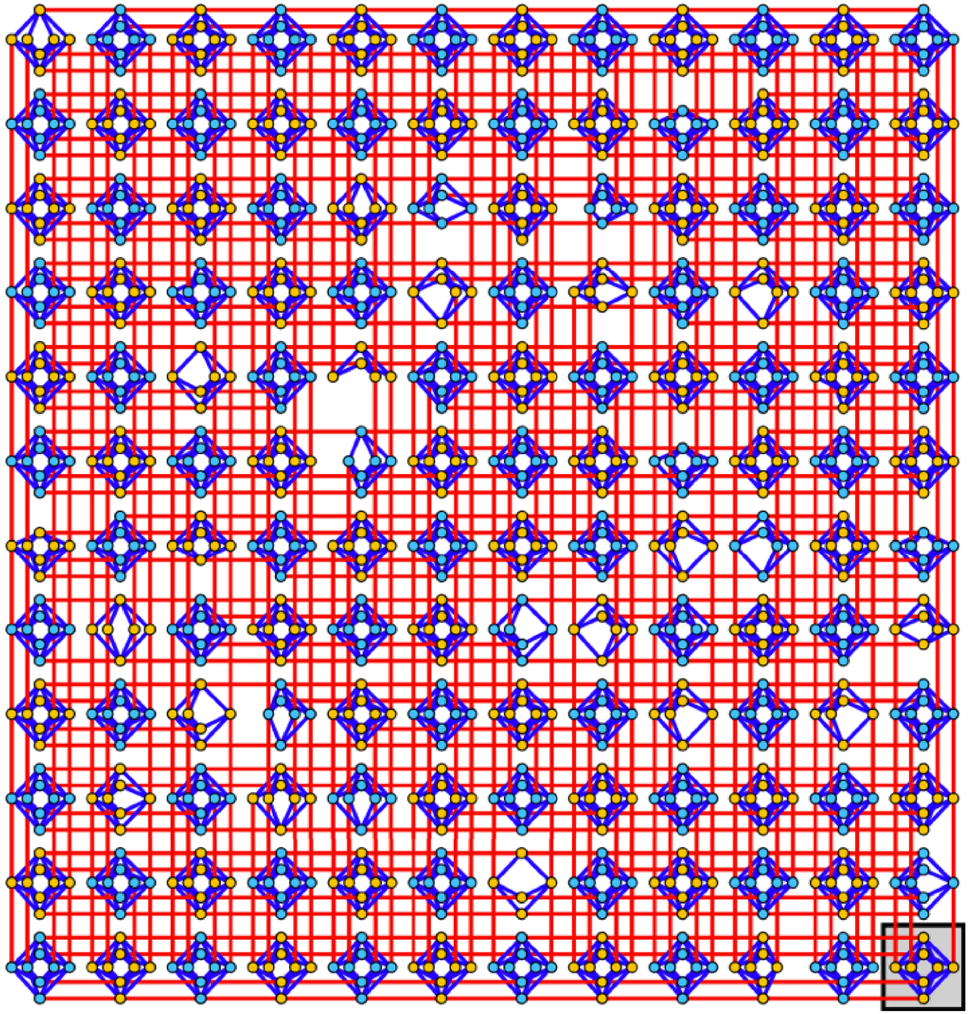
D:wave

-4168, 9982/10000, 4 sec.

GUROBI OPTIMIZATION

-4168, 0% gap, <1 sec.

Case 2 - Checker Board



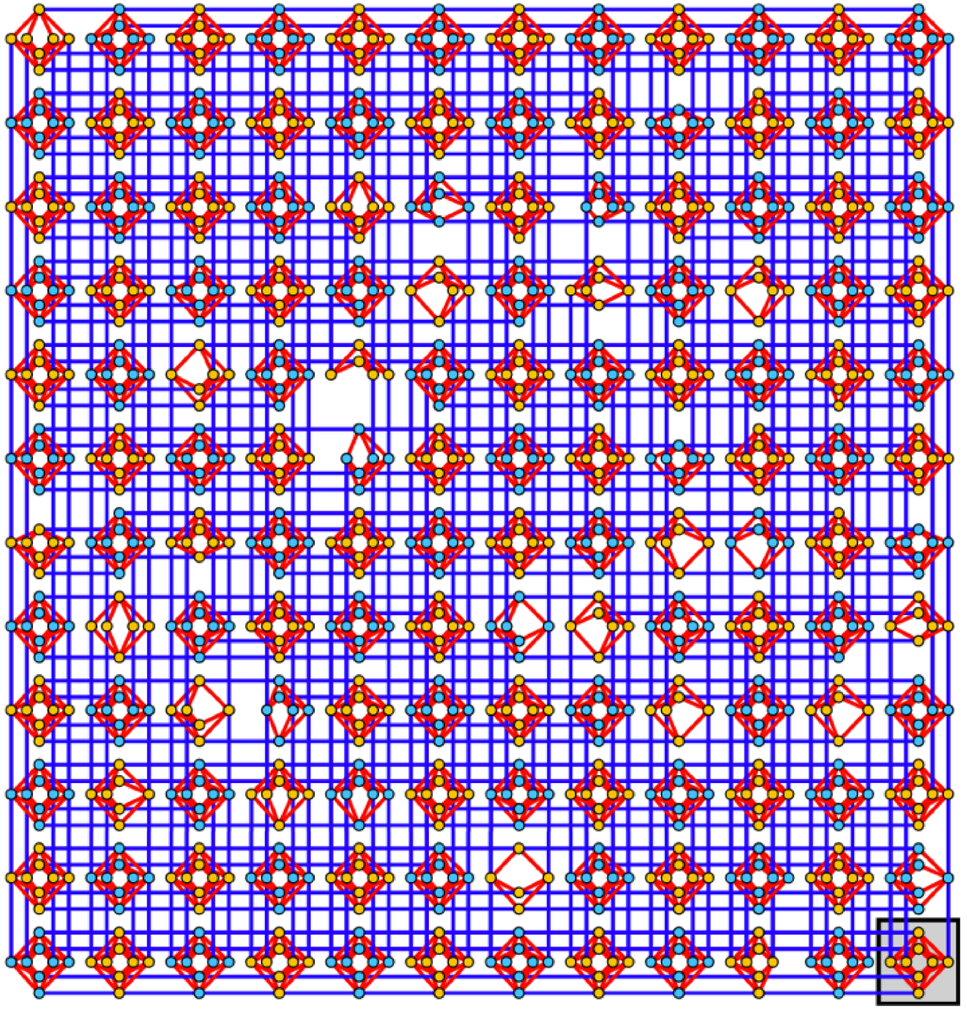
D:wave

-4168, 9980/10000, 4 sec.

GUROBI OPTIMIZATION

-4168, 0% gap, <1 sec.

Case 3 - Frustrated Checker Board



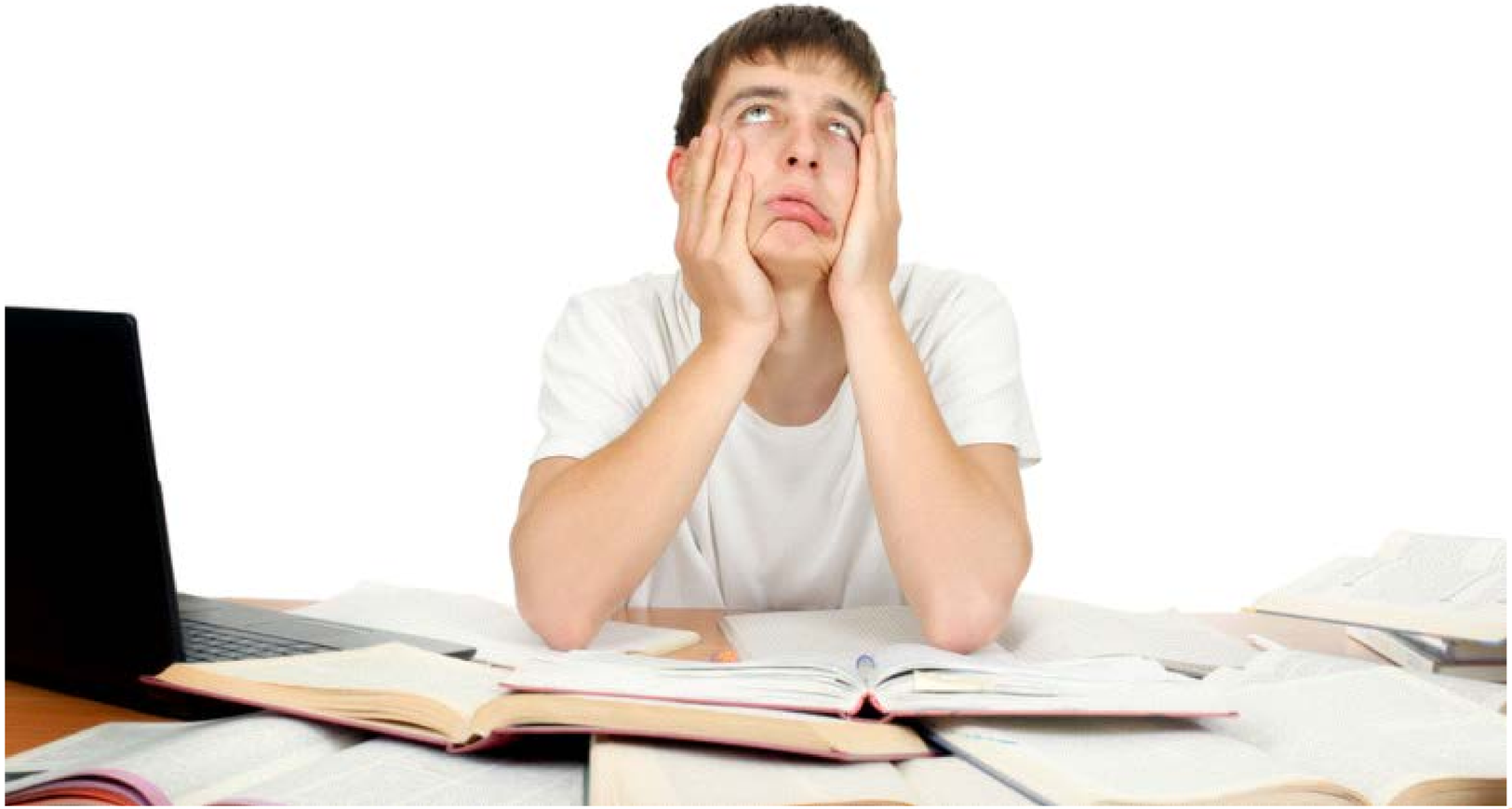
D:wave

-3070, 4269/10000, 4 sec.

GUROBI OPTIMIZATION

-3070, 0% gap, <1 sec.

D:wave VS GUROBI OPTIMIZATION



D:wave VS GUROBI OPTIMIZATION

D:wave VS GUROBI OPTIMIZATION

Marc

(T-4)

Andrey

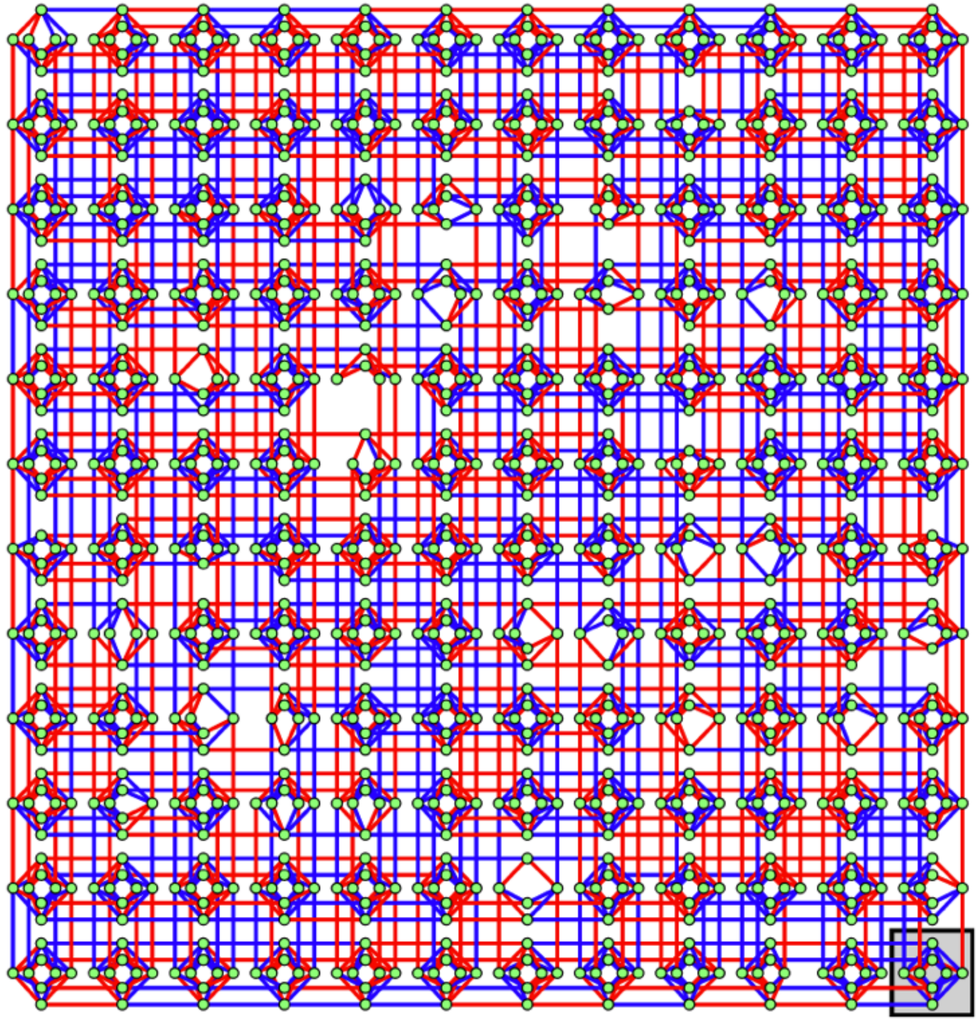
(CNLS/T-4)

Sidhant

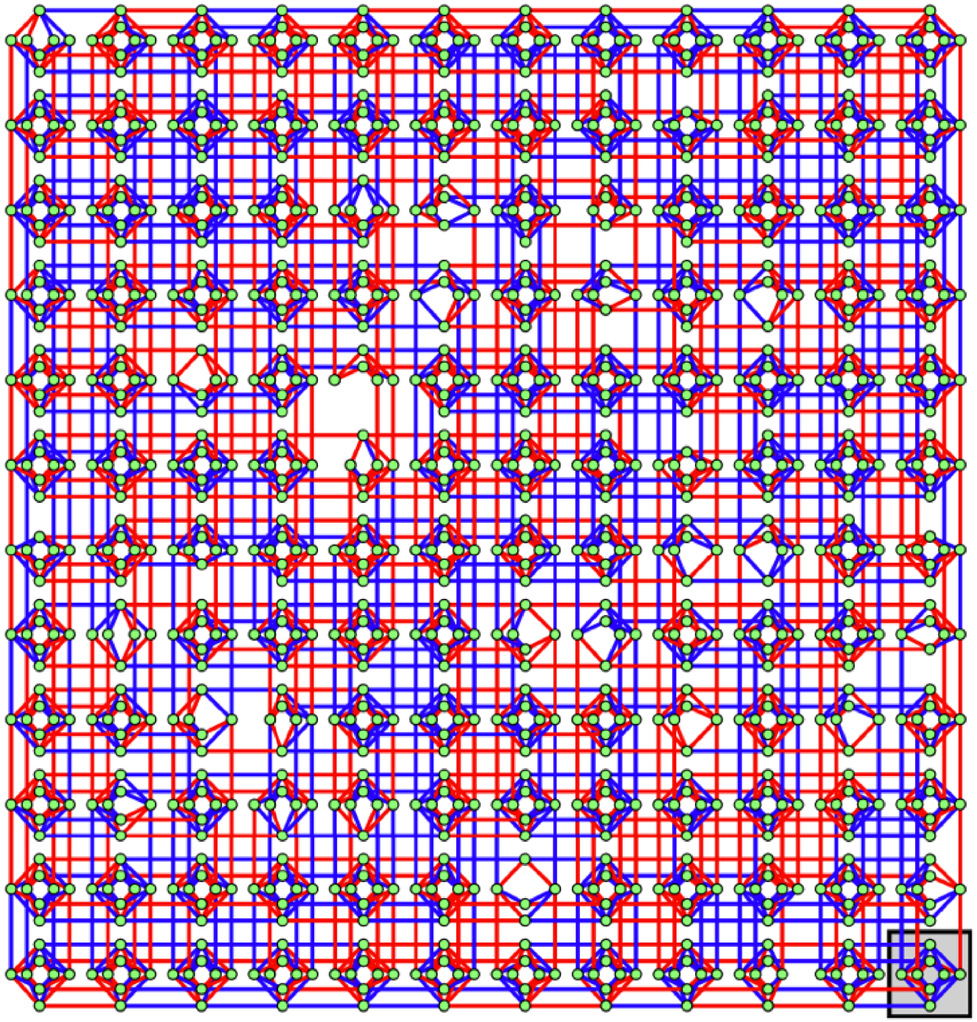
(T-5)

BATHTUB ROW

Case 4 - RAN1



Case 4 - RAN1



D:wave

!

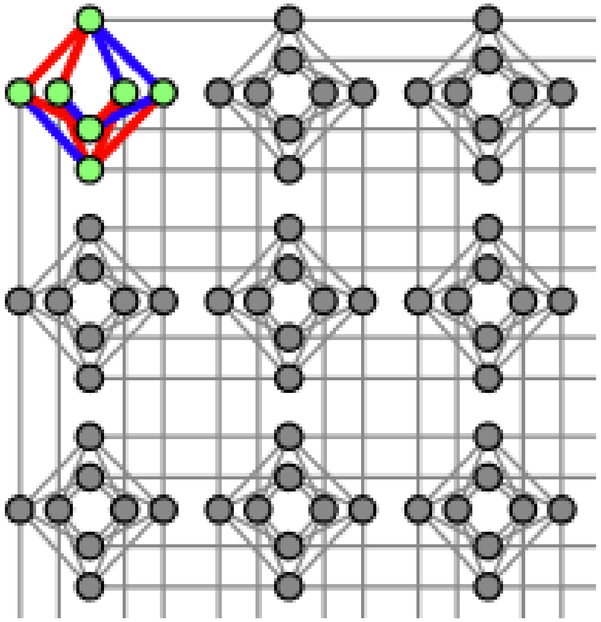
-1894, 67/10000, 5 sec.

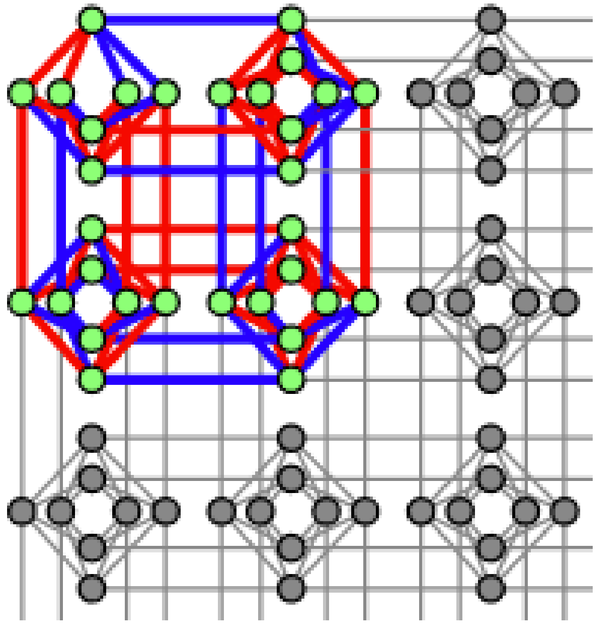
GUROBI OPTIMIZATION

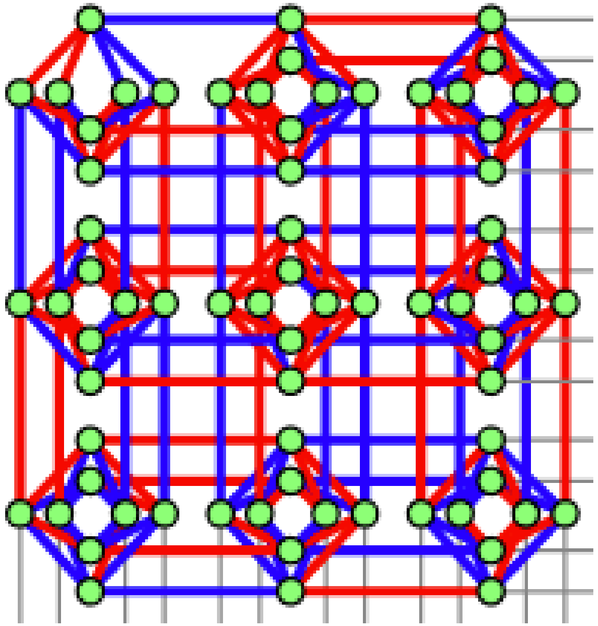
-1838, 9.9% gap, T.L. (1h)

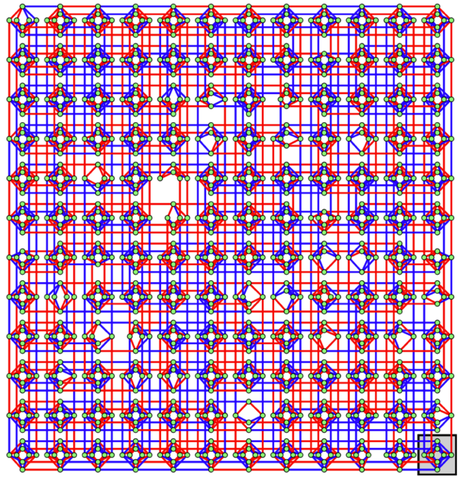
The RAN1 Family

$\mathcal{C}_1$ $\mathcal{C}_2$ $\mathcal{C}_3$ $\ldots$ $\mathcal{C}_{12}$









The RAN1 Family

Case	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{E}\rvert$	gurobi Best Sol.	gurobi Opt. Gap	gurobi Time	dwave Best Sol.	dwave Best	dwave Inf.	dwave Samples	dwave Time
C01	7	12	-8	0%	<1	-8	9984	0	10000	0+3
C02	31	75	-43	0%	<1	-43	9969	0	10000	0+3
C03	71	187	-121	0%	<1	-121	8553	0	10000	0+3
C04	127	345	-213	0%	9	-213	5937	0	10000	0+4
C05	193	525	-325	0%	157	-325	5684	0	10000	0+4
C06	275	746	-474	0%	523	-474	7	0	10000	0+4
C07	377	1036	-634	4.7%	T.L.	-634	454	0	10000	0+4
C08	487	1337	-835	4.1%	T.L.	-837	3	0	10000	0+6
C09	613	1680	-1014	7.9%	T.L.	-1038	216	0	10000	0+5
C10	758	2086	-1268	7.9%	T.L.	-1288	4	0	10000	0+5
C11	921	2554	-1502	11.5%	T.L.	-1550	14	0	10000	0+5
C12	1100	3068	-1838	9.9%	T.L.	-1894	67	0	10000	0+5

T.L.

Time Limit (1 hour)

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T.L. Time Limit (1 hour)

BIG DEAL!

The RAN1 Family

			gurobi			dwave				
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BIG DEAL!

Looking to the Future

- I have drunk D-Wave **Kool-Aid**
- RAN1 convinced me that the **DW2X** has huge potential

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- RAN1 convinced me that the **DW2X** has huge potential
- I believe, in the next 5 years, QPU's will be very **disruptive** to optimization research
- **Key Issue:** QPU potential is currently unaccessible on classic benchmarks

Unlocking D-Wave's Potential

- Need **multi-disciplinary** R&D
  - Functional form of the QPU Behavior (**physics**)
  - Significant progress on Embedding Algorithms (**computer science**)
  - Incorporating a QPU into established Optimization Algorithms (**optimization research**)

Making a Smooth Transition

GUROBI OPTIMIZATION ? → D:wave

SAPI

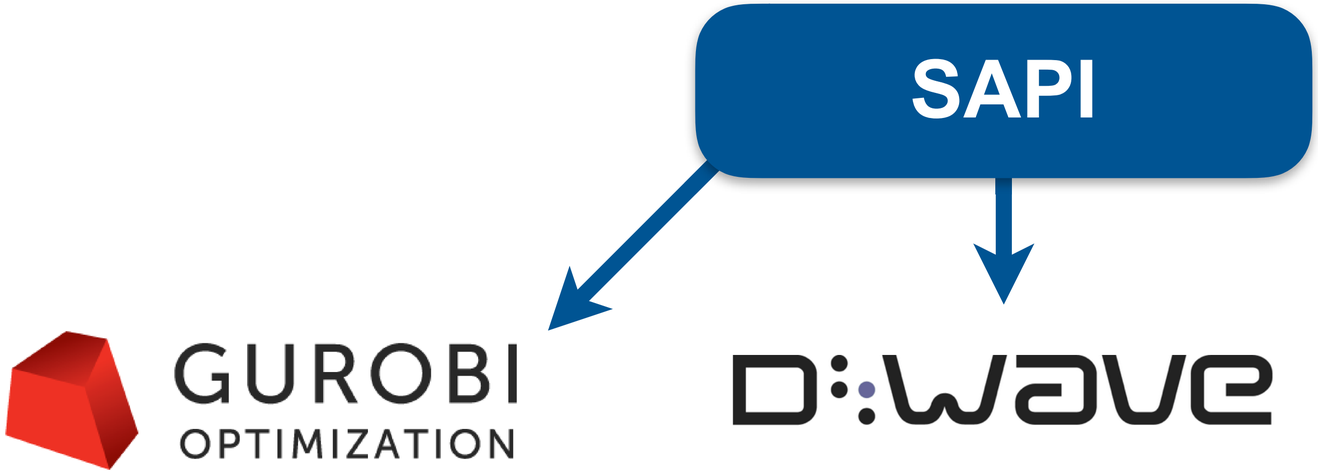
↓

D:wave

Making a Smooth Transition

GUROBI OPTIMIZATION ? ⇒ D:wave

"G-Wave" - QPU Simulator



Q?

Special Thanks

Misha
(T-4)