Collective Decoherence of the Superpositional Entangled States in Quantum Shor Algorithm

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The quantum Shor algorithm solves the problem of factorization of large integers. A quantum computer is needed in order to implement this algorithm. The most important problem preventing an experimental realization of the quantum computer is decoherence, i.e., the destructive influence of environment on quantum states in the quantum computer register. We considered the collective decoherence for the quantum Shor algorithm when all spins interact with the same environment [1]. This model describes, for example, the decoherence caused by low-frequency phonons in a solid-state spin quantum computer.

Different approaches to suppress this collective decoherence have been proposed. One approach is based on logical quantum bits (qubits). Each logical qubit is represented by a group of physical qubits in such a way that the states of the logical qubits are free from decoherence (decoherence-free subspaces). It has been shown that universal quantum computation is possible within decoherence-free subspaces. In another (more abstract) approach, the quantum information is represented in terms of conserved quantities of the quantum system, which are not affected by noise (noiseless subsystems). Both decoherence-preventing schemes require additional computational resources and complexity. Our results are important for preserving quantum entanglement without the application of complicated decoherence-preventing strategies.

Each quantum bit in our quantum computer is represented by a particle with spin 1/2. The environment is represented by a system of oscillators with continuously distributed frequencies. We analyzed the interaction between the environment and the z-components of all spins. The process of decoherence in the quantum computer is caused by entanglement between the thermal environment and the quantum computer.

In the system under consideration, different states of the quantum superposition are affected differently by the environment. We have shown that the probability p of very fast decoherence rate (superdecoherence) for the quantum Shor algorithm is extremely small. In particular, $p \sim \exp[-(\Delta I_z)^2]$, where $|\Delta I_z|$ is the sum of the differences $|b_i - c_i|$. Here b_i and c_i are the ith bits ($b_i = 0,1$; $c_i = 0,1$; $I = 1,2,\ldots L$) of the numbers b and c which form a superposition of two states in the quantum register. As Fig.1 demonstrates, the probability P of the appearance of large $|\Delta I_z|$ in the quantum Shor algorithm is small.

In order to estimate the decoherence time, we assumed that the size of the sample is 1 cm, the speed of the elastic waves is 500 m/s, and the temperature is 1 mK. Then, using our results we estimated the decoherence time to be of the order of 0.04 s. This long decoherence time allows one to implement thousands of quantum logic operations before the quantum state is destroyed by the environment.

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[1] G.P. Berman, et al., *Phys. Rev. A* **71**, 032346 (2005).

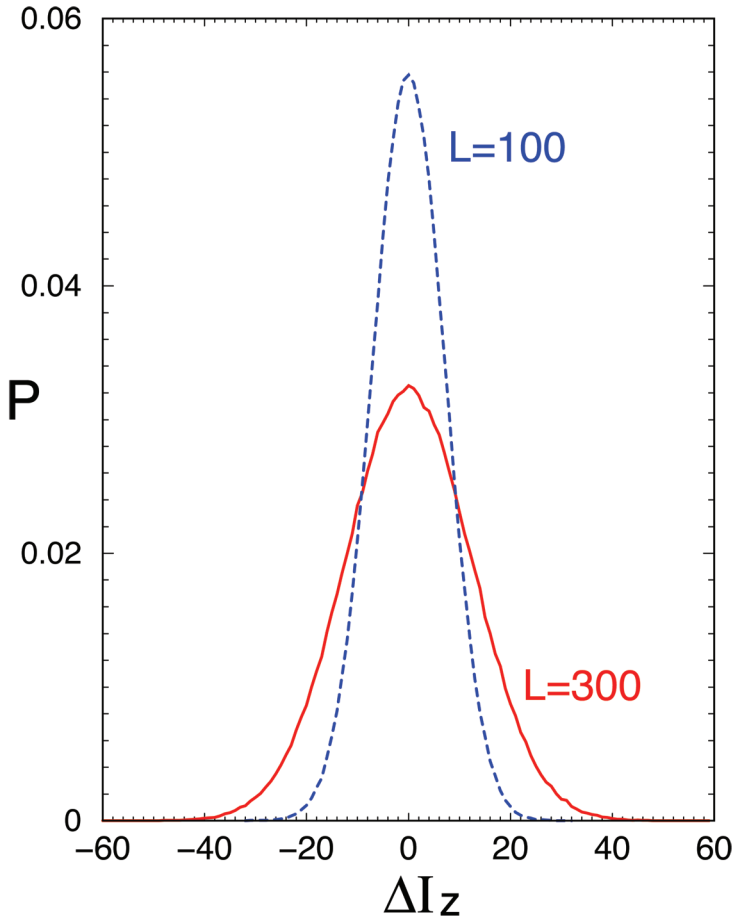

Fig. 1.
The figure shows the probability P of the appearance of large $|\Delta I_z|$ in the quantum Shor algorithm is small.