Prioritizing the Purchase of Spare Parts Using an Approximate Reasoning Model

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Key Words: Spare Parts, Approximate Reasoning, Fuzzy Sets, Possibility Theory

SUMMARY & CONCLUSIONS

The complexity of a spare parts prioritization model should be consonant with the amount and quality of data available to populate it. When production processes are new and the reliability database is sparse and represents primarily expert knowledge, an approximate reasoning (AR) based model is appropriate. AR models are designed to emulate the inferential processes used by experts in making judgments. We have designed and tested such a model for the planned component production process for nuclear weapons at Los Alamos National Laboratory.

The model successfully represents the experts' knowledge concerning the frequency and consequences of a part failure. The use of linguistic variables provides an adaptable format for eliciting this knowledge and a consistent basis for valuing the effect on production of different parts. Ranking the parts for inclusion in a spare parts inventory is a straightforward transformation of the AR output. The basis for this ranking is directly traceable to the elicitation results. AR-based models are well-suited to prioritization problems with these characteristics.

1. INTRODUCTION

The industrial practice of maintaining an inventory of spare parts for equipment to minimize lost production time is commonplace. Conceptually, the management of such an inventory is also straightforward: one wishes to maximize production while minimizing the costs associated with the spare parts inventory. In practice, such models can be quite complex, depending on the production process, the nature of the equipment and associated failures, and the consequences of lost production. For example, one might expect to find a model based on a discrete object simulation for the process, individual reliability models for equipment, and sophisticated representations of recovery timing and part acquisition times—all of which have been benchmarked using historical data for the production system. In this paper, we consider a complex system for which the critical data needed to build a conventional prioritization model are lacking.

The process studied here is the planned production at Los Alamos of components for United States nuclear weapons. The essential features associated with the design of a spare parts program for this process can be summarized as follows.

- The production line is new and under development.
- Some of the equipment is old and not directly replaceable.
- Quantitative reliability data are sparse.
- The times to diagnose the failure, schedule the repair, obtain the needed parts, and effect the repair are known, at best, in a very uncertain and approximate way.

Under these circumstances, it is difficult to populate a detailed parts prioritization model. We examine whether an AR-based model with data requirements that are commensurate with the sparse and uncertain qualitative data available provides the basis for a logically consistent parts prioritization program*.

Our approach to decision analysis problems with such characteristics is based on the theory of AR (Refs. 2 and 3). AR models are designed to emulate the inference process used by an expert in making a decision. Our work at Los Alamos has been directed at extensions of AR to handle complex systems for which the underlying physical phenomena are incompletely understood. Knowledge of such systems normally exists with a group of experts—no single expert has a complete understanding of the system. We use deductive logic models called process trees to represent this group knowledge (Refs. 4 and 5). Process trees are capable of representing many possible realizations of a physical process in a single logical equation. We use "possible" here in the context of possibility theory (Ref. 6). Possibility theory provides a direct link between process trees and AR decision models and facilitates hybrid representations of uncertainty that make it possible to consistently represent an expert's confidence in his knowledge. Decision-making is a process too and therefore is amenable to representation with a process tree. We refer to this combination of process trees to represent physical and decision processes, AR decision models, and the use of hybrid uncertainty measures as logic-evolved decision models (LED) (Ref. 7).

In this paper, we show how AR and the extensions to it incorporated in LED can be applied to spare parts prioritization. We present a decision process tree for this problem and show how it can be realized in an AR model. The elements and functioning of the AR model are discussed, and a hybrid representation of uncertainty—fundamental to the decision process—is examined. The inputs and uncertainty measures are primarily expert knowledge, and the techniques

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*In a previous paper, we considered the problem of making reliability assessments for weapons when the available data are comparably sparse (Ref. 1).

used to elicit this information are described. We present representative results obtained using the AR model and discuss the observed model performance to date.

2. DECISION PROCESS TREE

The starting point for the prioritization model is the construction of a decision process tree. The tree is a logical representation of the fundamental decision process. A simplified and condensed version of the process tree used for this problem is shown in Figure 1. The tree is constructed deductively. That is, one begins with a general description of the issue for which a decision model is required, here "Prioritization Strategy" (Node A in the figure) and then develops in turn the various items that logically constitute such a strategy.

The deductive aspect of the tree development is quickly apparent. The overall process is decomposed as a strategy for evaluating each component, and an aggregation operation to make judgments among the complete set of components. The aggregation operation will employ a cost strategy and a set of aggregation constraints. It was decided early in the project that the aggregation portion of the analysis would be deferred. Instead, the initial prioritization would be a simple rank ordering of the components based on a single metric, the output of the AR model.

The natural context in which to evaluate individual components is in terms of risk. For this problem, the possible description of "Consequence of a Fault Event" (Node B in Fig. 1) could be quite detailed, taking into account factors such as the magnitude of the shortfall and the sensitivity of the overall process to it. However, it was clear that the information needed to make such distinctions useful did not yet exist. This meant that the primary emphasis would be on the likelihood of production shortfalls and that the effect would have to be treated in a simple, approximate manner. Logically, "Shortfall Likelihood" (Node C) is determined by the "Production Disruption Rate" and the "Restoration Time Distribution." That is, how often do components fail, how badly do they disrupt production, and how long does it take to recover? Each of these is developed in the tree in turn. For example, "Restoration Time Distribution" is composed of "Warning Time," "Diagnosis Time," "Set Up Time," "Repair Time," and "Restoration Time." In the actual tree, each of these is developed in additional detail.

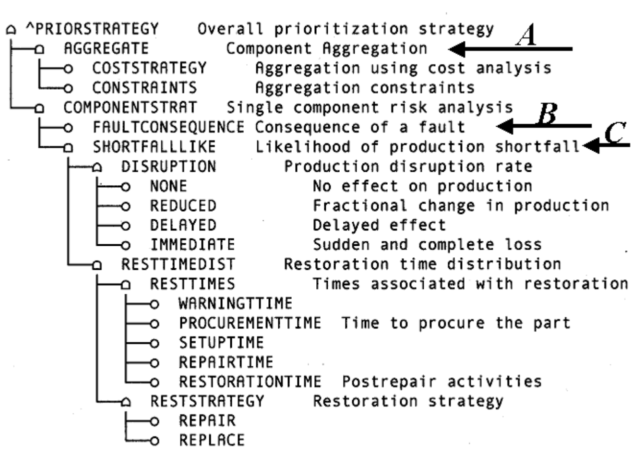

Figure 1. Process Tree for Prioritization Strategy.

3. INFERENTIAL MODEL

The process tree provides the basis for the construction of the inferential model that is used to make judgments about the components to be prioritized. There are two basic aspects of the construction process: developing the inferential structure and defining the elements associated with the structure that make it possible to arrive at a conclusion.

The top-level structure of the inferential model used for parts prioritization is shown in Fig. 2. It corresponds to the logical structure deduced in the process tree. There is a module that deals with the timing and duration of important activities and a module that is concerned with conditional production reduction likelihood. Also shown are two simple nodes (A and B in Fig. 2). Node A represents the operation of inferring the production loss likelihood, L_P, from the fault frequency, f, and the conditional likelihood of production loss, L_C, and Node B chains this result with the time delay attributable to procurement, Δt_P, to infer production impact[†]. Production impact is related to "Risk for Each Component" in the process tree.

3.1 *Operation of an AR Model*

The reader may be unfamiliar with the operation of an AR model. In this section, we briefly describe the operators employed in AR. There are three basic operations to be carried out to perform the sequence of evaluations shown in Fig. 2: (1) *conversion of the input* information on the parts of interest into the proper form, (2) *solution of the forward-chaining inferential chain*, and (3) *translation of the output into metrics* to bin and order the parts for inclusion in the inventory. These steps correspond to the circled numbers in Fig. 3.

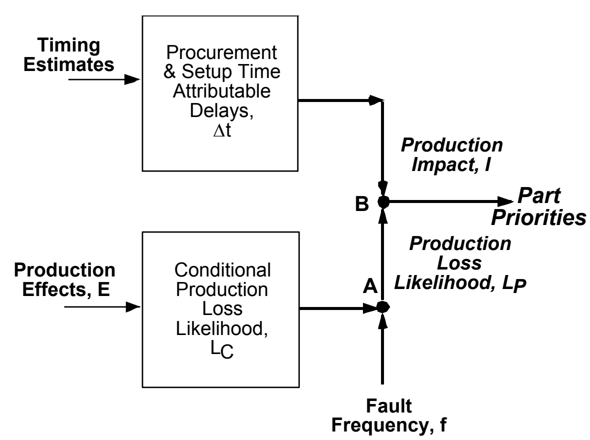

Figure 2. Top Level of the Inferential Structure for Prioritization.

[†] This chaining of inferences is characteristic of AR models.

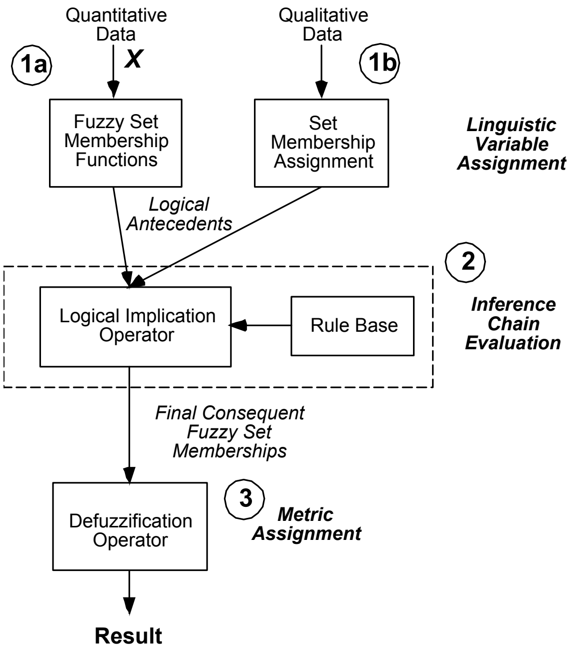

Figure 3. Overview of Operation of an AR Model.

Step 1: *Conversion of the input.* The data to be considered can be either quantitative (Circle 1a in Fig. 3) or qualitative (Circle 1b); both will be converted into the same internal form. The data inputs for the timing module for example are quantitative. We will see shortly how these inputs are first operated on algebraically to obtain a time delay associated with procurement. A quantitative input is represented by the symbol X in Fig. 3. AR models treat variables as linguistics—natural language expressions.

We denote the corresponding linguistic variable for X as Lin(X). We chose to represent the time delay with the following descriptors: negligible, small, medium, large. These descriptors are referred to as the universe of discourse, U = {Negligible, Small, Medium, Large}. The elements in a universe of discourse are treated as fuzzy sets (Refs. 8 and 9). A linguistic variable has associated with it a degree of membership vector, D, that defines to what extent it belongs to each set. For example, if Δt_P = 1000 days, then it is surely large, and $D(\Delta t_P)$ = [0, 0, 0, 1], denoting that in this case Δt_P has only membership in {Large}. The assignment to D given an X is done using fuzzy set membership functions. The membership functions for Δt_P are shown in Fig. 4. For example, a delay time of 5 days corresponds to $D(\Delta t_P)$ = [0,.5,.5,0]. The D-vector is the internal representation of the linguistic variable "Procurement Time Delay."

Some of the inputs may be better expressed qualitatively to begin with; they already are in linguistic variable form. The input to the conditional likelihood module in Fig. 2 is "Effect on Production," E. It represents the result of a failure—the effect, given the failure. We use a universe of discourse for E of U(E) = {None, Reduced, Delayed, Immediate}, where "immediate" is understood to mean instantaneous and complete. The assignment of D(E) is done by interpreting the expert elicitation. For example, if the expert judges that the part failure could result in either immediate or a delayed effect, with immediate more likely, then this would be encoded as D(E) = [0, 0, .25, .75]. This can be interpreted as a "degree of belief." We will consider the possible interpretations of D-vectors and their relationship to uncertainty in more detail below.

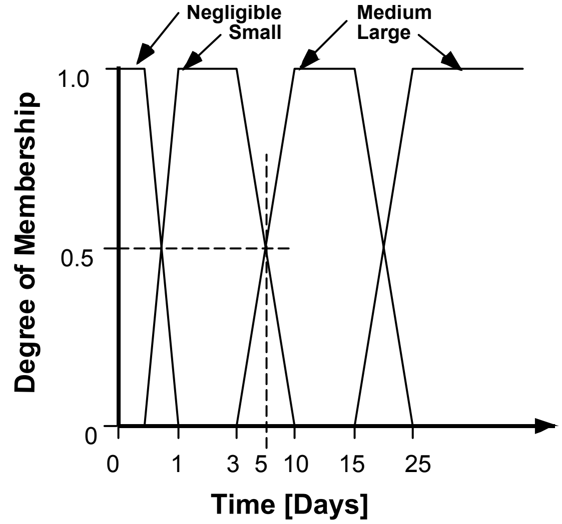

Figure 4. Membership Functions for Delay Time, Δt_P.

Step 2: *Solution of the inference chain*. The fundamental logical operation in AR is implication, which is performed at each inference node in the model. A node has multiple inputs—the antecedents, A—and a single output—the consequent, C—all with their corresponding universes of discourse. For simplicity, consider the case where there are two elements in A, {α, β}. The inference drawn is of the form: "α is α_I and β is β_j, *AND IF* α_I and β_j *THEN* C_k, *THEREFORE* C_k", where i, j, and k refer to particular elements in the respective universes of discourse. This statement is a tautology, hence the justification for the use of the word "*THEREFORE*." The assignment of k given i and j is done with a rule base. This rule base describes how a combination of antecedent fuzzy sets implies a particular set in U(C). To make this concrete, consider Node A in Fig. 2. The universes of discourse for the antecedents are $U(L_C)$ = {Very Unlikely, Unlikely, Likely, Nearly Certain} and U(f) = {Rare, Seldom, Anticipated, Routine}. The consequent is "Production Loss Likelihood", L_P with $U(L_P) = U(L_C)$. The reader will recognize that this inference looks like the product of a conditional probability and a frequency. The rule base for this node is given in Table 1. The shaded entry is the rule: *If f is Seldom and L_C is Likely then L_P is Unlikely*.

Recall that both antecedents are represented internally as D-vectors, and in general, these vectors will have more than one non-zero entry. This means that multiple rules, and therefore assignments of L_P, are occurring simultaneously. A consistent resolution of this situation is obtained using the max-min rule (Ref. 8). This can be expressed as

$$\mu_{\Re} = \underset{\forall (n,m)\to\Re}{Max}\left(Min\left(\kappa_n, \sigma_m\right)\right) \ . \tag{1}$$

Table 1. Rule Base for Likelihood of Production Loss, L_P

Fault Frequency, f	Conditional Production Loss Likelihood, L_C			
	Very Unlikely	*Unlikely*	*Likely*	*Nearly Certain*
Rare	Very Unlikely	Very Unlikely	Very Unlikely	Very Unlikely
Seldom	Very Unlikely	Very Unlikely	Unlikely	Unlikely
Anticipated	Very Unlikely	Unlikely	Likely	Likely
Routine	Very Unlikely	Unlikely	Likely	Nearly Certain

In this formula, κ_n and σ_m are elements n and m of fuzzy input membership vectors κ and σ, and $\Re$ is a particular element output by the rule. For example, assume that D(f) = [0, 0, .25, .75]—the failure frequency has no membership in the sets "Rare" or "Seldom" and non-zero membership in "Anticipated" and "Routine"[‡] and $D(L_C)$ = [0, .3, .9, 0], corresponding to non-zero membership in "Unlikely" and "Likely." Applying the max-min operator to these D-vectors yields $D(L_P)$ = [0, .3, .75, 0]. A natural language interpretation of this vector is that L_P is between unlikely and likely and closer to being likely.

Step 3: T*ranslation of the output into metrics.* The result of a set of forward-chained inferences is a single consequent and one D-vector. In our model, this is "Production Impact", I with U(I)={Negligible, Small, Medium, Large}. To use I as our metric for ranking parts for inclusion in a spare parts inventory, it must be converted to a single numerical value that reflects the meaning of the D-vector. This process is called defuzzification. We have discussed defuzzification in the context of an AR decision model at length in a previous paper (Ref. 10) and restrict ourselves here to a short summary.

The first step in defuzzification is to define membership functions, $M_I(I)$, i = 1, n, similar to those in Fig. 4, for the fuzzy sets in U(I), These membership functions should be chosen so that there is an approximate relationship between the intervals on which they are defined and the linguistics. We then find the fuzzy union of the sets defined by

$$C = \bigcup_i D_i M_i \qquad ,i = 1,n \tag{2}$$

Fuzzy union is a max operation and the result is the outer envelope of the membership functions weighted by the numerical values in D(I). If we view this union as a possibility distribution, then it is natural by the analogy with probability to compute an expected value, λ as

$$\lambda(I) = \int x\, C(x)\, dx \,/ \int C(x)\, dx \ . \tag{3}$$

This is referred to as centroid defuzzification and provides a numerical value for ranking purposes. The membership functions can be adjusted so that the centroid values approximately reflect the set linguistics scale. It is also possible to convert the centroid into a non-fuzzy linguistic by selecting the set for which the centroid has the maximum degree of membership denoted as Lin(λ). We then use λ(I) to order and Lin(λ(I)) to bin the I vectors for each spare part. Note that because several time durations are represented in the model as random variables, it follows that λ(I) and Lin(λ(I)) are random variables as well. We will discuss this aspect of the model further in Sec. 5. With these preliminaries, we are now ready to take a closer look at the details of the model of Fig. 2.

3.2 *Attributable Delay Time Module*

The outputs of the timing module are measures of the delays attributable to procurement and non-procurement sources. The relationship of the timing factors is shown in Fig. 5. The initial datum is the detection time. If failure is not immediate, then there is a warning period, at the end of which failure occurs. We present results showing the effect of warning time. It is assumed that procurement will not begin until diagnosis of the problem is complete. In contrast, we assume that setup will begin immediately upon detection. This is conservative relative to the effect of a part's failure. It also reflects the reality that preparations for repairs in a nuclear facility can be very complex and time-consuming and that the process owners are well aware of this fact. The situation shown in Fig. 5 corresponds to the case where procurement is the critical path item, $\Delta t_P > 0$; setup is completed before the new part is available. Thus, the repair process is held up waiting for the part. Note that if a spare were available, then the procurement phase would be complete at the conclusion of the diagnosis period. The durations are treated as uniformly distributed random variables. The endpoints of these

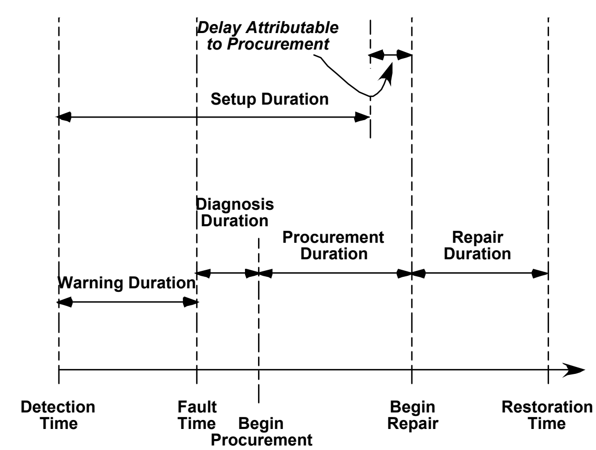

Figure 5. Timing Relations Associated with Production Interruption.

[‡]There is no requirement that the set memberships sum to 1.0

distributions correspond to lower and upper bound estimates obtained from the subject matter experts (SMEs).[§]

3.3 *Conditional Production Loss Likelihood Module*

This part of the AR model was revised several times because of difficulties encountered in eliciting the required information. The following approach was found to be a reasonable compromise between the desire for a detailed, elegant model and the realities of interacting with SMEs who are unfamiliar with the concepts of AR and probability and are extremely busy trying to bring production on line.[**] We were concerned with eliciting L_C directly from the experts because it would be difficult to document their reasoning and it was clear that delayed or partial losses would be poorly represented. We chose instead to elicit information on the effect of the production change (E) resulting from a component failure. We used multiple rule bases similar to Table I to evaluate the relative importance of delayed or reduced production. The D-vector from this chain is combined with the "none" and "immediate" effects in a four antecedent rule base to obtain L_C. $D(L_C)$ is the output from this module and is the antecedent for Node A of Fig. 2 as discussed above.

3.4 *Determination of Effective Production Impact*

At this point, both of the antecedents to Node A in Fig. 2 have been calculated. The consequent here is the effective production impact associated with failure of the part, I. The inference is performed using the rule base in Table II and the max-min operator. Reference to Table 2 shows that I is of the functional form I(magnitude of consequence, frequency). Therefore, the linguistics used to represent production impact apply equally well to risk and provide the link back to the decision process tree.

4. FUZZY SETS AND UNCERTAINTY IN AR[††]

We have made several references above to "degree of belief" in reference to D-vectors. To interpret a D-vector properly, it is necessary to discuss briefly the relationship between fuzzy sets, possibility, and sources of uncertainty. Fuzzy sets are most often used to represent *ambiguity*. Ambiguity arises when it is not possible to assign an object to a single set. For example, is a room "Cold," "Comfortable," "Hot," or somewhere in between? Memberships in fuzzy sets vary smoothly, so we can easily represent situations where the temperature is perceived to be "in between". Another useful perspective exists. One can interpret a degree of membership as a measure of the possibility that an object belongs to a set. That is, the assignment of set memberships induces a possibility distribution. When linguistic variables are used to represent concepts such as "conditional probability" or "likelihood," it is this latter interpretation that is operative. That is, the set memberships are a measure of *outcome uncertainty*. Possibility is an imprecise outcome uncertainty measure. A precise measure is probability.[‡‡] The choice of which measure to use will depend on the available data. When sufficient data exist to construct an acceptable probabilistic model or where the expert description of outcome uncertainty is best represented as a probability distribution, then it is preferable. For most cases involving large amounts of epistemic (state of knowledge) uncertainty, we have found possibility measures to be more compatible with expert knowledge and judgment. Both possibility and probability are used in the prioritization model. AR models are able to aggregate the various aspects of uncertainty using rule bases. We refer to such aggregations as hybrid uncertainty.

5. ELICITATION PROCESS

The spare part prioritization project is a small part of the much larger effort to manufacture weapon components. This meant that a complete flow sheet on the planned process was available and that SMEs for each aspect of the process had already been identified. The value of an AR model is strongly dependent on the design of the elicitation program (Ref. 12). We developed a standard elicitation form in co-operation with several SMEs who were willing to participate in a pilot program. Following testing of the finalized elicitation program, the major elicitation began.

The elicitation began by asking each previously identified SME what manufacturing responsibilities he/she had. These were tied to the process flow sheet. For each identified subprocess, the expert was asked to generate an equipment list. The failure modes were identified for each item of equipment. For each failure mode, the expert explained whether the equipment was replaced or repaired. If repair was the preferred option, then the parts needed were identified. These parts are candidates for inclusion in the spare parts inventory and are the objects to be rank-ordered. The SMEs were given copies of the elicitation package before the elicitation session. The package includes a complete glossary describing each of the variables and explanations of the set descriptors used to represent them.

6. PRIORITIZATION RESULTS

To date we have analyzed the data from 5 SMEs covering approximately 40 parts. Figure 6 shows the median production impact centroids for these parts for the base case (without warning) and with warning. A number of observations can be made. For about half the parts examined, either the SME responded that failure of the part would have no effect on production because redundant equipment is available or the expected delays are attributable to set up, not procurement. Such parts have small values of $\lambda(I)$, corresponding to $\mathrm{Lin}(\lambda(I)) \rightarrow$ "Negligible" and do not need to be included in a spare parts inventory. For several parts (Part Ids 18–20 in Fig. 6), the effect of warning time was

[§] We have found the use of a uniform distribution to represent an expert's estimate for the range of a numerical quantity such as time duration to be generally consistent with how experts estimate such quantities.

[**] Most of the SMEs are technicians who are very familiar with part of the process and the associated equipment.

[††] For a more detailed discussions of these issues see References 6 and *Fuzziness and Probability* by S. Thomas (Ref.11)

[‡‡] Probability satisfies the law of the excluded middle. Possibility and fuzzy sets do not and are therefore described as imprecise.

Table 2. Rule Base for Production Impact, I

Attributable Time Delay, Δt_D	Production Loss Likelihood, L_P			
	Very Unlikely	*Unlikely*	*Likely*	*Nearly Certain*
Negligible	Negligible	Negligible	Negligible	Negligible
Small	Negligible	Negligible	Moderate	Moderate
Medium	Negligible	Small	Large	Large
Large	Very Unlikely	Small	Large	Very Large

observable; the warning time duration was comparable to the procurement duration. Parts with large λ(I) are ones that stop production immediately upon failure, fail fairly frequently and have long procurement times that exceed set up requirements. Even when warning time is available for these parts, the duration is too short to affect the production impact. These are good candidates for the spare parts inventory.

As noted earlier, the timing duration data from the experts are represented as uniform random variables. In some cases the ranges given for the procurement and warning times can vary significantly. In such cases it is important to consider the probability distribution function for I. Typically we present the binned and ranked results at the median and ninetieth percentiles.

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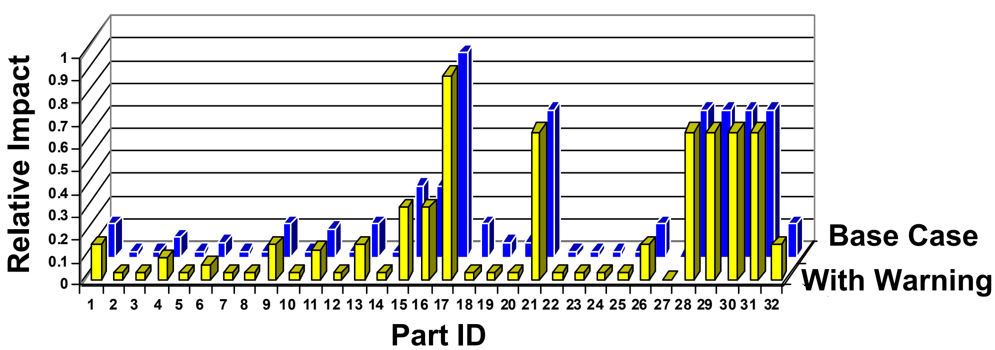

Figure 6. Median Prioritization Results.

Prioritizing the Purchase of Spare Parts Using an Approximate Reasoning Model

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Reliability and Maintenance Symposium

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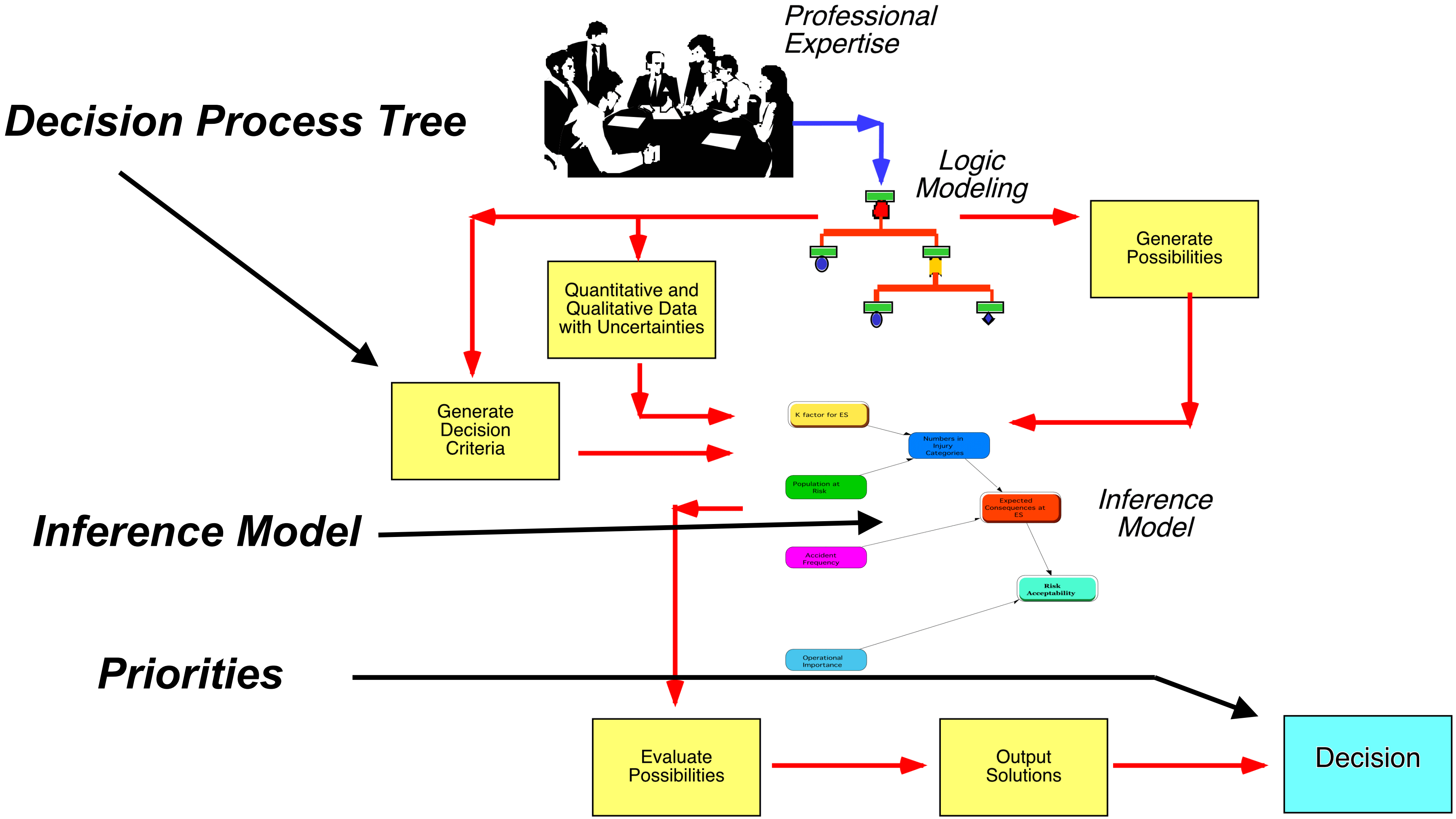
Objective: ***Design a spare parts prioritization model for a production process***

Process Characteristics:

- **the production line is new and under development**
- **some of the equipment is old and not directly replaceable**
- **quantitative reliability data is sparse**
- **the times to diagnose the failure, schedule the repair, obtain the needed parts and effect the repair are known, at best in a very uncertain and approximate way.**

These factors make it difficult to populate a conventional model

Approach: ***Use approximate reasoning to represent the data and design an inferential model compatible with the problem characteristics.***



Decision Process Tree

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^PRIORSTRATEGY          Overall prioritization strategy   A
  AGGREGATE             Component Aggregation
    COSTSTRATEGY        Aggregation using cost analysis
    CONSTRAINTS         Aggregation constraints
  COMPONENTSTRAT        Single component risk analysis   B
    FAULTCONSEQUENCE    Consequence of a fault   C
    SHORTFALLLIKE       Likelihood of production shortfall
      DISRUPTION          Production disruption rate
        NONE                No effect on production
        REDUCED             Fractional change in production
        DELAYED             Delayed effect
        IMMEDIATE           Sudden and complete loss
      RESTTIMEDIST        Restoration time distribution
        RESTTIMES           Times associated with restoration
          WARNINGTTIME
          PROCUREMENTTIME     Time to procure the part
          SETUPTIME
          REPAIRTIME
          RESTORATIONTIME     Postrepair activities
        RESTSTRATEGY        Restoration strategy
          REPAIR
          REPLACE
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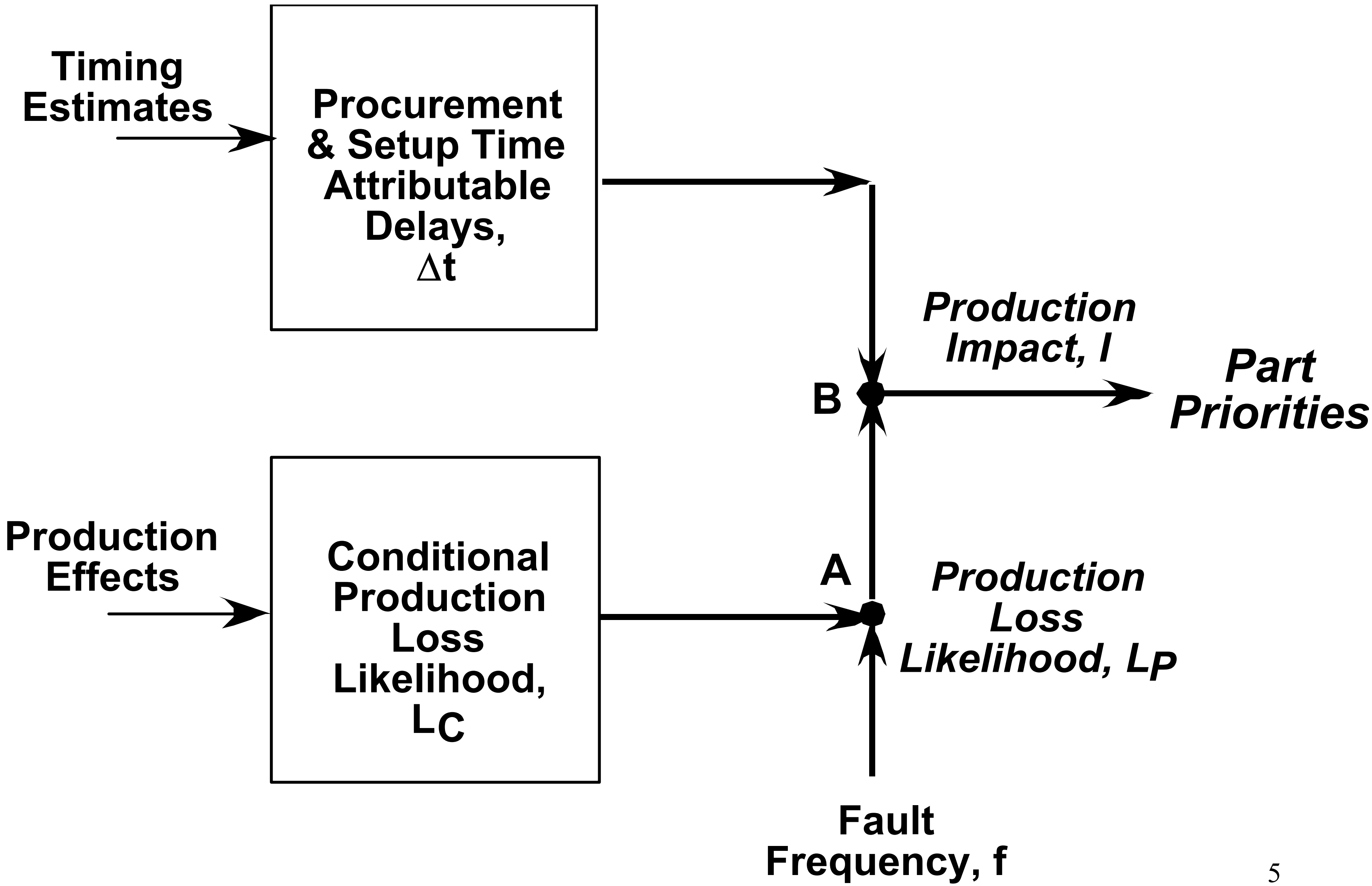
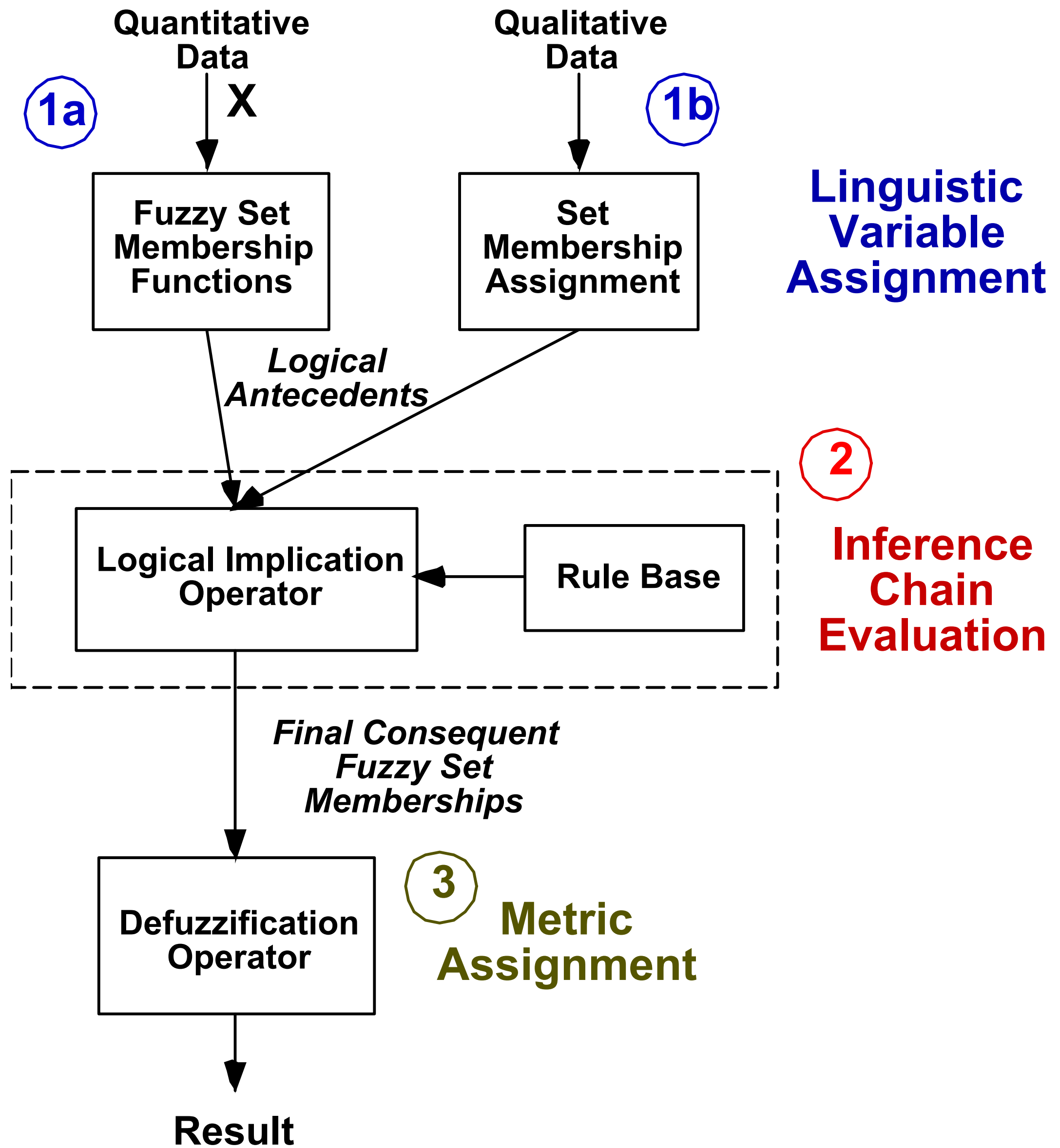
Inferential Structure



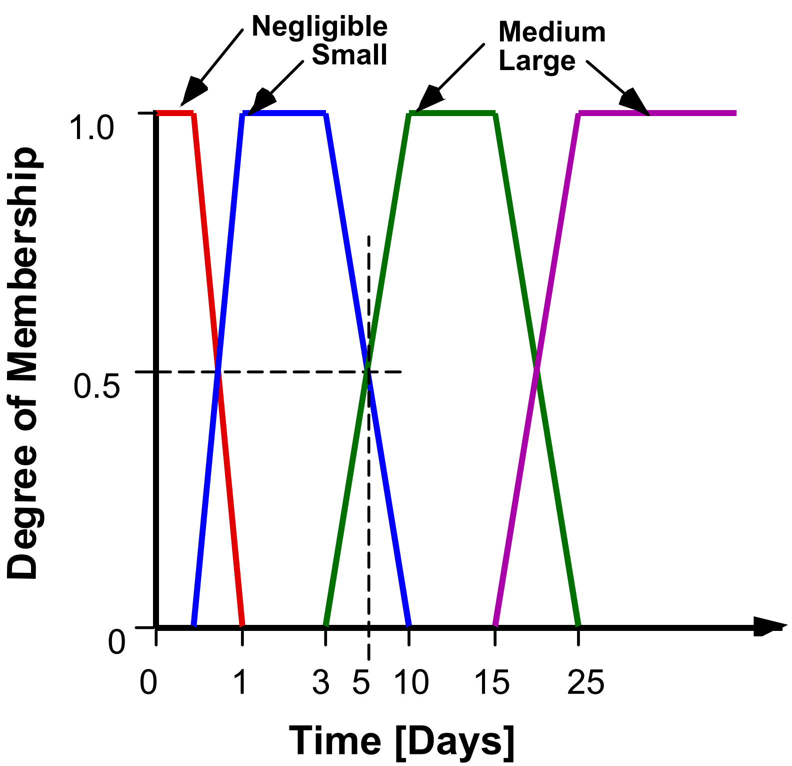
Operation of an AR Model



Step 1: Assignment of Linguistic Variables

Quantitative Input

Delay Time:
[Negligible, Small. Medium, Large}



D(ΔT = 5) = [0, .5, .5, 0]

Qualitative Input

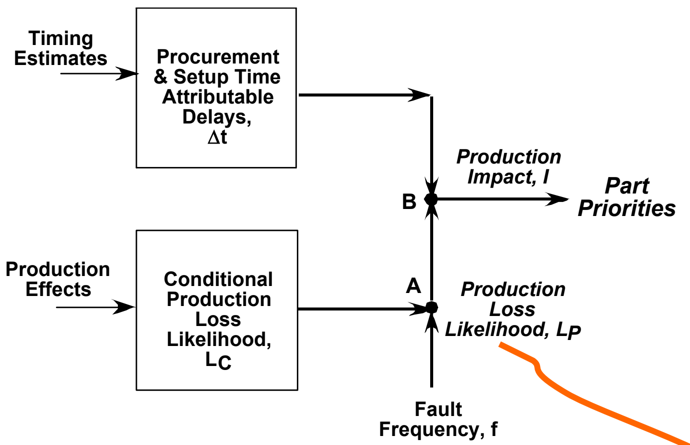
Effect on Production:
{None, Reduced. Delayed, Immediate}

Expert Elicitation

"immediate or delayed with immediate more likely"

D(E) = [0, 0, .25, .75]

Step 2: Solution of the Inference Chain



$$f \wedge L_C \Rightarrow L_P$$

	Conditional Production Loss Likelihood			
	Very Unlikely	*Unlikely*	*Likely*	*Nearly Certain*
Rare	Very Unlikely	Very Unlikely	Very Unlikely	Very Unlikely
Seldom	Very Unlikely	Very Unlikely	Unlikely	Unlikely
Anticipated	Very Unlikely	Unlikely	Likely	Likely
Routine	Very Unlikely	Unlikely	Likely	Nearly Certain

Fault Frequency
{Rare, Seldom, Anticipated, Routine}

[0, 0, .25, .75]

Production Loss Likelihood
{Very Unlikely, Unlikely, Likely, Nearly Certain}

Antecedents ***Consequent***

[0, 0, .3, .75]

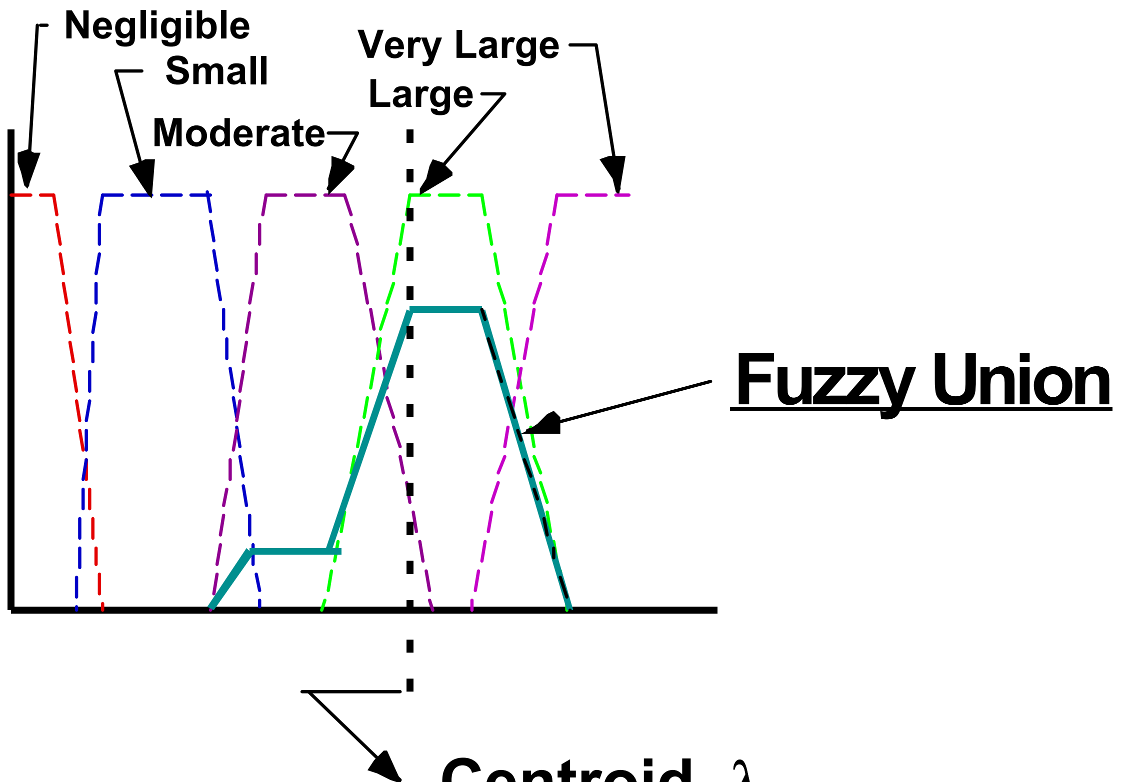
[0, 0, .3, .9]

Conditional Production Loss Likelihood
{Very Unlikely, Unlikely, Likely, Nearly Certain}

Step 3: Translation of the Output into Metrics

Production Impact: {Negligible, Small, Moderate. Large, Very Large}

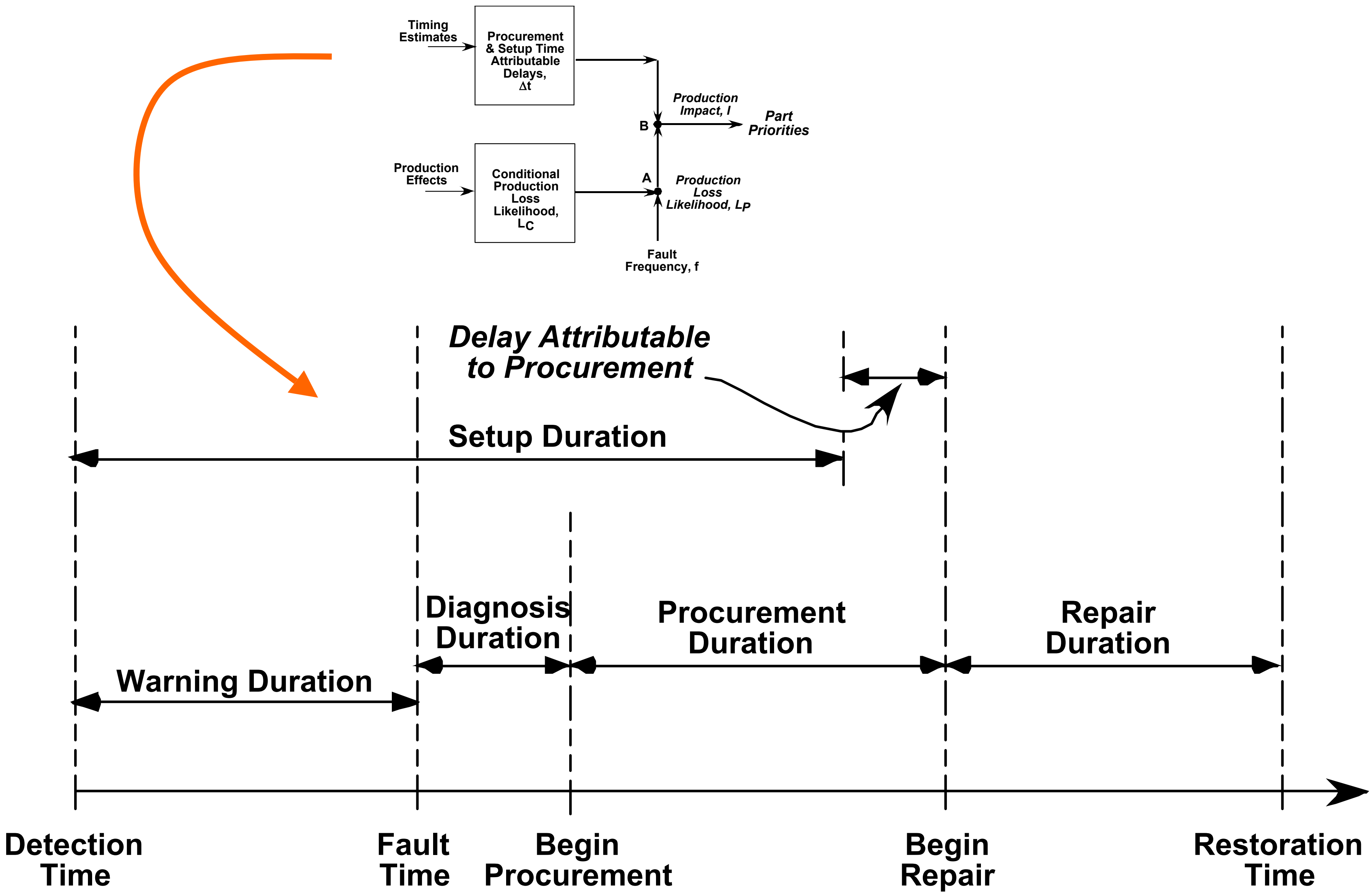
D(I) = [0, 0, .2, .75, 0]



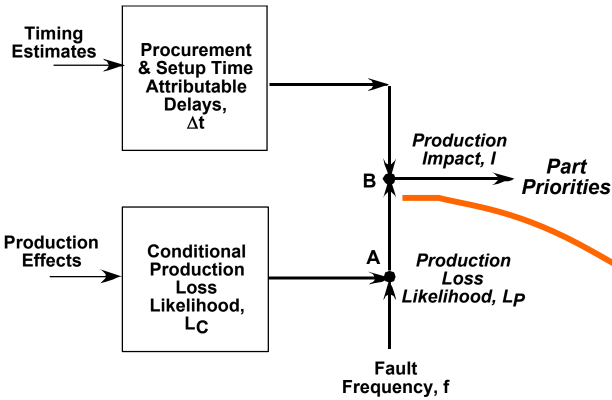
Centroid, λ

Linguistic, **Lin(λ) = "Large"**

Time Delays and Linguistic Variables



Inferring Production Impact



$$\Delta T_P \wedge L_P \rightarrow I$$

	Production Loss Likelihood			
	Very Unlikely	*Unlikely*	*Likely*	*Nearly Certain*
Negligible	Negligible	Negligible	Negligible	Negligible
Small	Negligible	Negligible	Moderate	Moderate
Medium	Negligible	Small	Large	Large
Large	Very Unlikely	Small	Large	Very Large

Procurement Time Delay
{None, Small, Medium, Large}

[0, .2, .8, 0]

Production Impact
{Negligible, Small, Moderate, Large, Very Large}

Antecedents ***Consequent***

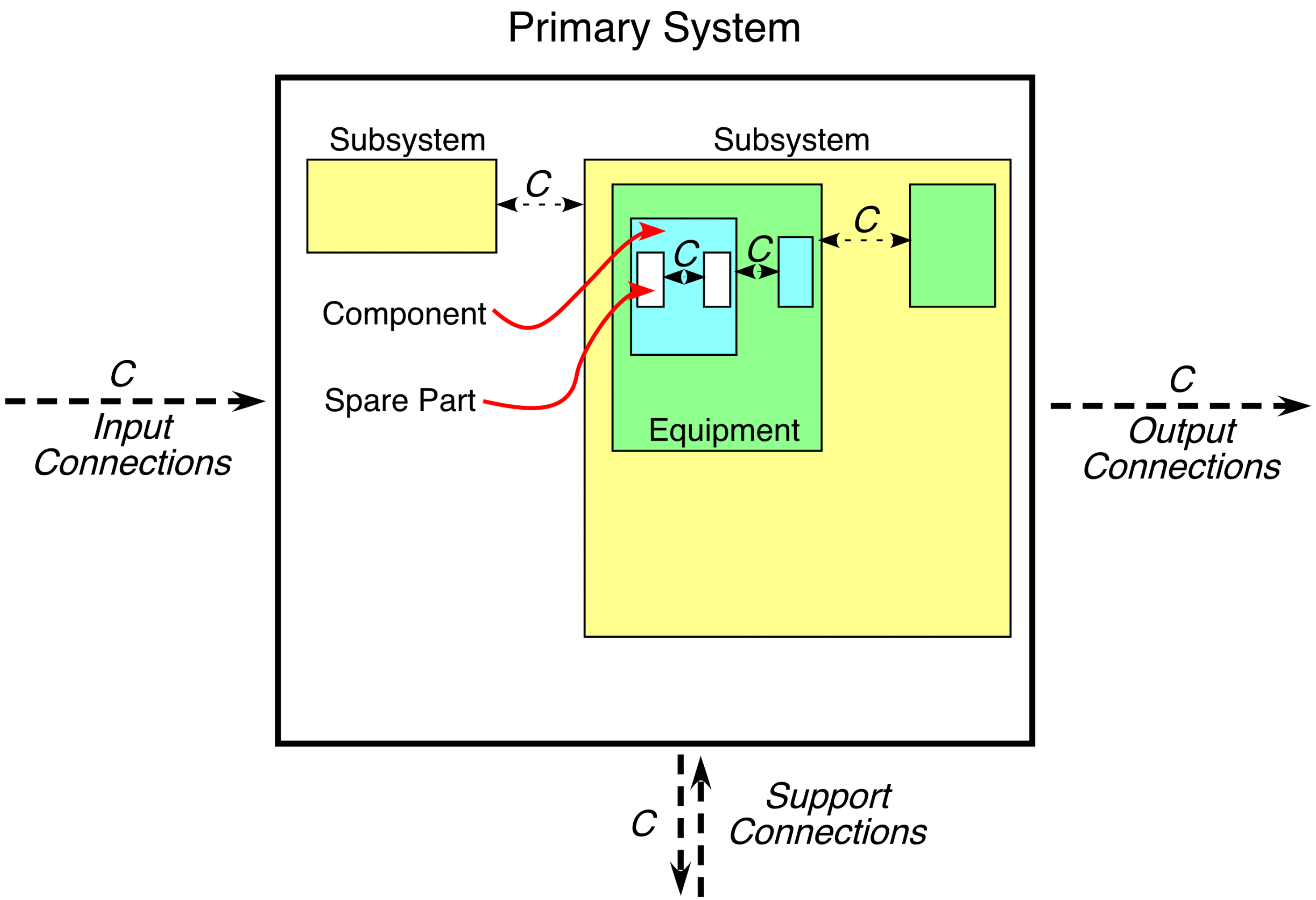
[0, 0, .2, .75, 0]

[0, 0, .3, .75]

Production Loss Likelihood
{Very Unlikely, Unlikely, Likely, Nearly Certain}

Production impact is moderate to large

Expert Elicitation – Part Identification



Expert Elicitation – Failure Characterization

Failure Frequency Linguistics

(*Rare*: Never expected to occur during project lifetime
***Seldom*: Surprised if occurs during project lifetime**
***Anticipated*: Occurrence once every couple of years**
Routine:* Occurs one or more times per year*)

Elicited Set Memberships [0,1]

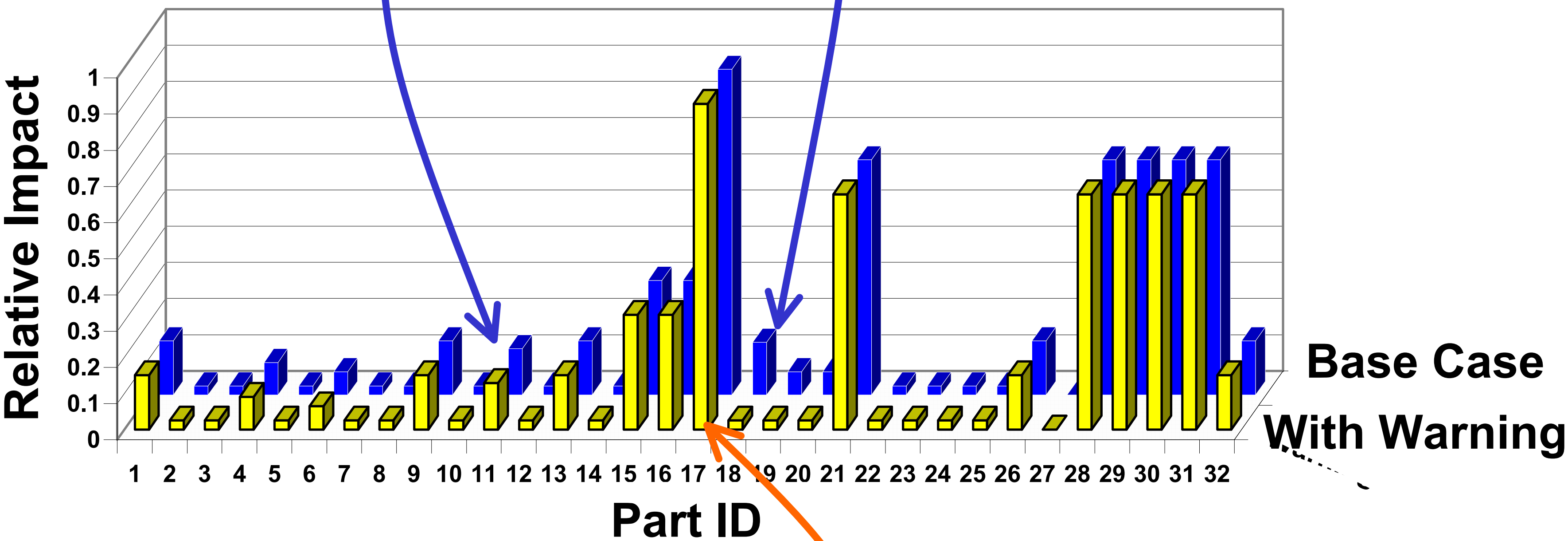
Rare	*Seldom*	*Anticipated*	*Routine*

Production Impact Results for Initial Study

Warning Importance:

No Effect

Some Effect



Production Loss Likelihood: Likely

Failure frequency: Anticipated

Procurement Time Delay: Large

Conclusions

- ***AR is a good tool for prioritizing when knowledge is approximate and uncertain.***
- ***The process tree logic model helps to create comprehensive and reviewable inferential models***
- ***An AR ranking process is consistent and traceable – ensures that parts are purchased based upon a commonly accepted risk model.***