# Network Uncertainty on Electric Power Grids

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Electric grid operators decide how much power each of their generators should produce based on an assumed network state. The state can change as safety devices trigger or lines go down, but these events may not be observed directly so estimates are needed from a limited number of flow measurements at generators and transformers. Demands at internal nodes are usually not measured but can be assigned probability distributions. We estimate the present state of a network and its uncertainty by posterior probabilities that each state in a bank of alternatives could have produced the observed measurements. Randomly drawn demands are repeatedly run through the bank of network simulators to produce statistical distributions of power flows. Flow measurements at any point in time can be used to calculate state probabilities across the bank of models. Ambiguity about the current state suggests optimizing power generation with explicit hedging against unfavorable possibilities.
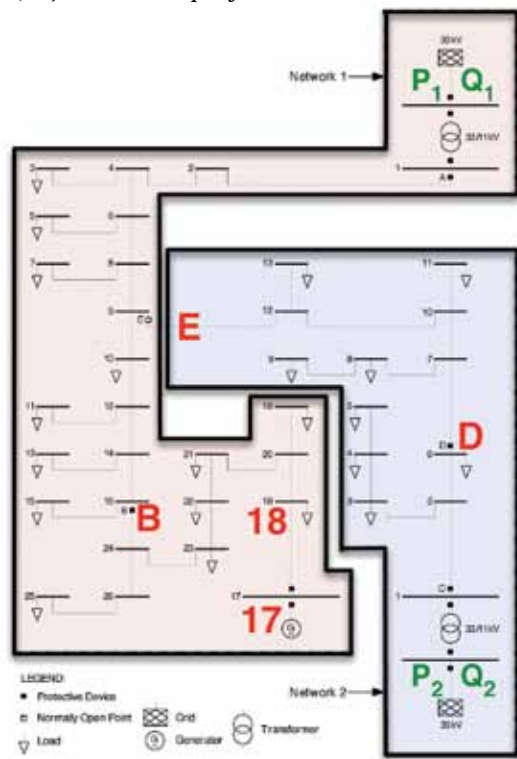
Infrastructure networks have a prominent role in national and global security because we increasingly depend on them to deliver commodities such as information and energy. However, they are difficult to monitor because of their complexity. We are developing methods for inference about unobserved parts of a network (links, nodes, flows, attributes) using science-based simulations to match measurements on observable components. Our focus is on electric power.

Modern power grid operations (optimizing generation, monitoring exchanges, modifying safety devices, restoring damage, and determining criticality) depend heavily on understanding the current state of the grid. Operators typically have very limited real-time observations of voltages, flows, consumption, production, and even topology, so state estimation is a major topic of interest in the power engineering community [1-3].

Traditional state estimation methods [1,3] are based on relatively simple statistical analyses such as least-squares fitting within a greedy topology search. These methods produce a single estimate of the network configuration that matches available measurements within reason. A more recent approach [2] assumes that a bank of models contains all of the important configurations and attempts to directly estimate a single most likely model. Best-fitting state estimates generally work well, but occasional failures have produced disastrous consequences. For example,

root causes of the 2003 northeast blackout included problems with state estimators combined with operator error [4,5]. If uncertainties were integrated into operations, control algorithms could hedge against unfavorable states that are less likely but still plausible.

We are developing statistical methods to identify the current topology of a power network based on partial measurements. The method that follows uses importance sampling to estimate probabilities for each member of a model bank and thereby constrain the set of plausible grid configurations and appropriately represent uncertainty in the current state.

The small network of Fig. 1 from [2] has 39 buses (nodes) within two subnets that are connected to a larger transmission grid at the top and bottom of the figure. Several devices (denoted by squares) can trigger automatically and disconnect parts of the network or link the two subnets together. As in [2], the model bank consists of a set of network configurations representing contingencies that need to be guarded against. Red labels in Fig. 1 indicate the five major contingencies.

The operator measures only the real and reactive power flowing between the transmission grid and each subnet (Ps and Qs in the figure). Information from these measurements should be used to decide how much power to generate at node 17, trading-off the cost to buy power from the grid and the risk of overloading lines in the contingencies where protective devices have altered the nominal distribution topology.

In addition to the few direct measurements of power, the operator can use probability distributions of historical loads for each internal node and a physics-based alternating current (AC) solver that computes
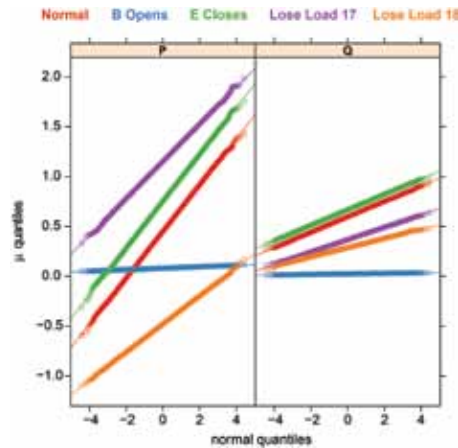

Fig.1. A small test network with measurements P1, Q1, P2, Q2 at two nodes and five contingencies (B, E, D, 17, 18) that alter the topological state.

Fig.2. Normal probability plots of real (P) and reactive (Q) flows corresponding to P1 and Q1 in Fig. 1.

Fig. 3. Classification regions for five contingencies using just measurements P1 and Q1. Color encodes the maximum contingency probability with one being red. Regions of orange, yellow, and green indicate ambiguity in the topological state of the network.
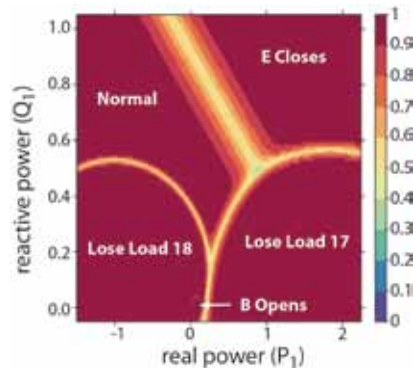
power flowing over lines, provided the interconnection topology and all loads and generation amounts are known.

This setup is small but exemplary of typical power grid state estimation in that only a small fraction of relevant quantities are measured directly while many others can only be given probability distributions. The physical constraints of electrical power flow provide further information relevant to the decisions that need to be made.

Given a measurement (P1, Q1, P2, Q2), we use standard importance sampling to efficiently calculate state probabilities. The method requires pre-computing, for each enumerated contingency, the mean and covariance of flows at measurement nodes using loads chosen from historical probability distributions. The AC power solver converts the random loads into random flows for each of the contingencies in the model bank. The resulting flows should be transformed to produce approximate Gaussian quantities for ease of implementing the importance sampler. Figure 2 shows normal probability plots of real (P) and reactive (Q) flows corresponding to P1 and Q1 in Fig. 1 when loads at internal nodes have standard deviations that are 20% of historical average values. Linearity in the plots shows that the flows are Gaussian with means and covariances that depend on the network topology. Measured flows are modeled with 1% standard deviation from the actual values. This completes the off-line preparation for state estimation by importance sampling.

When a measurement (P1, Q1, P2, Q2) is obtained, the importance sampler draws many candidates for the corresponding actual flows, using a standard deviation of 2% around the measured values to cover the full range of possible measurement errors. Estimates of the probabilities for each contingency in the model

bank are obtained through standard importance sampling formulae [6] that use weighted averaging to combine the candidate flows with the Gaussian distributions of Fig. 2.

Importance sampling is fast and accurate for this problem and can easily produce updates of the state probabilities at one-second intervals. Figure 3 shows classification regions for five contingencies using just measurements P1 and Q1. Color encodes the maximum contingency probability with one being red. Regions of orange, yellow, and green indicate ambiguity in the topological state of the network. When measurements fall in the ambiguous regions, decisions such as optimal generation should hedge against state uncertainty. In these cases, the best single estimate of the state is not an adequate description of inherent ambiguities in the partially observed network.

We have applied importance sampling to model bank state estimation for the IEEE Reliability Test System [7], a network of 72 buses and 125 lines, with measurements on five lines that connect three subnets. The model bank consisted of 125 contingencies, one for each of the lines going down. In this case, the measurements generally leave large ambiguities about the network state. The research opportunity is to determine whether a few additional measurement points could resolve much of the uncertainty.

Future research objectives are to optimize power generation under state uncertainty and to incorporate probabilistic risk of overloading lines due to states that are plausible but unfavorable for efficient power distribution. This objective requires quantifying uncertainty of all unobserved flows on the grid. We are exploring the use of Gaussian process emulators to support these inferences with a manageable number of runs of the AC power flow solver.

[1] Clements, K.A. and P.W. Davis, *IEEE Trans Power Syst* **3**, 1748 (1988).

[2] Singh, R. et al., *IEEE Trans Power Syst* **25**, 1329 (2010).

[3] Wu, F.F and W.E. Liu, *IEEE Trans Power Syst* **4**, 176 (1989).

[4] Anderson, G. et al., *IEEE Trans Power Syst* **20**, 1922 (2005).

[5] U.S. Canada Power System Outage Task Force, "Final Report on the August 14, 2003 Blackout in the United States and Canada," technical report, DOE (2004).

[6] Ripley, B.D., *Stochastic Simulation*, Wiley, New York (1987)

[7] Reliability Test System Task Force of the Application of Probability Methods Subcommittee, *IEEE Trans Power Syst* **14**, 1010 (1999).