

# Statistical Mechanics of the Geometric Control of Flow Topology in 2D Turbulence

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We apply the maximum entropy principle of statistical mechanics to 2D turbulence in a new fashion to predict the effect of geometry on flow topology. We consider two prototypical regimes of turbulence that lead to frequently observed self-organized coherent structures. Our theory predicts bistable behavior that exhibits hysteresis and large abrupt changes in flow topology in one regime; the other regime is predicted to exhibit monostable behavior with a continuous change of flow topology. The predictions are confirmed in fully nonlinear numerical simulations of the 2D Navier-Stokes equation. Finally, implications of this work are briefly considered.

Large-scale flows in the atmosphere and ocean are characterized by stable density stratification, rotation, and small vertical-to-horizontal aspect ratios. These characteristics render turbulence at large scales quasi-2D, leading to a cascade of energy to large scales—a feature that is in contrast to the cascade of energy to small scales in commonly occurring, smaller-scale, 3D turbulence. A consequence of the inverse cascade is the self-organization of turbulence to form long-lived (large-scale) coherent structures such as the Great Red Spot and other vortices and jets in the Jovian atmosphere, the subtropical jet-stream and other vortical structures that are associated with storm systems in the earth’s atmosphere, and mesoscale eddies and alternating jets in the world’s oceans. Mechanisms that explain the formation of such coherent structures are in part amenable to statistical mechanical considerations [1,2].

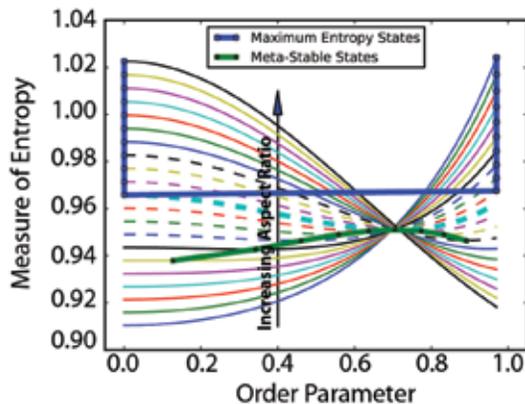


Fig. 1. Maximum entropy-based prediction for jet regime: The different smooth lines (without symbols) show the variation of a measure of entropy (y-axis) with order parameter (x-axis) at different aspect ratios. In this regime, the blue line connecting the filled blue circles (maximum entropy states) jumps abruptly as the aspect ratio is varied. The presence of meta-stable states over an intermediate range of aspect ratios indicates bistable behavior and hysteresis.

In this work [3], in the idealized setting of 2D turbulence, we consider the effect of domain geometry on flow topology. In particular, we consider regimes of turbulence that are conducive to the formation of two prototypical self-organized structures—jets or vortices—and apply the maximum entropy principle of statistical mechanics to produce a theory of flow topology in the two cases as the domain aspect ratio is varied.

Figures 1 and 2 summarize the results of the theory. In these figures, a measure of entropy is plotted as a function of the order parameter at a given value of aspect ratio; this is repeated for various values of the aspect ratio (different smooth lines). The filled blue circles indicate the “maximum entropy state” for each value of the aspect ratio, whereas the stars indicate meta-stable states if present at that value of aspect ratio. The blue line connecting the filled blue circles is the locus of

maximum-entropy states as the aspect ratio is varied. In Fig. 1, we consider the regime of turbulence that leads to the jet state; in Fig. 2, the regime of turbulence leads to vortex dipoles. In the jet-regime (Fig. 1) the locus of maximum entropy states exhibits a discontinuous jump as a function of aspect ratio, whereas in the vortex-regime (Fig. 2) there are no discontinuous jumps—the maximum entropy states continually deform as a function of the aspect ratio. Furthermore, in the jet-regime, there are meta-stable states for a range of aspect ratios (green stars), allowing for bistable behavior and hysteresis over this range of aspect ratios.

Figures 3 and 4 show the numerical verification of the theoretical predictions of Figs. 1 and 2 in a fully nonlinear setting. In Figs. 3 and 4, a measure of the order parameter is plotted as a function of the aspect-ratio parameter, as obtained in fully nonlinear simulations of the 2D Navier-Stokes equations. Also indicated in the colored panels are the long-time, self-organized, quasi-stationary flow patterns that result in the two different regimes and how they deform as a function of the aspect ratio. In the jet-regime of Fig. 3, the discontinuous jump from flow in the x-direction to flow in the y-direction is evident, as is also the regime of bistability over a range of aspect ratios surrounding 1, all in accordance with the theoretical predictions of Fig. 1. On the other hand, the fully nonlinear simulations of the 2D Navier-Stokes equations in the vortex-regime confirm that, in this regime, the flow topology changes continuously as a function of the aspect ratio, again in accordance with the theoretical predictions of Fig. 2.

In future work, we will build upon the above results to develop an explanation of the low-frequency regime transitions that have been observed in the non-equilibrium setting of this problem [4,5]. Consequently, insights developed here should be useful in developing a better understanding of the phenomenon of low-frequency regime

transitions, which are a pervasive feature of the weather and climate systems. Familiar occurrences of this phenomenon—wherein extreme and abrupt qualitative changes occur, seemingly randomly, after very long periods of apparent stability—include blocking in the extra-tropical winter atmosphere, the bimodality of the Kuroshio extension system, the Dansgaard-Oeschger events, and the glacial-interglacial transitions.

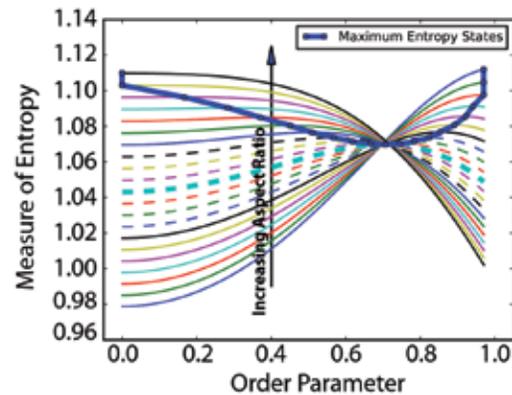


Fig. 2. Predictions for vortex regime: Flow topology evolves continuously as a function of aspect ratio. Consequently, there are neither abrupt transitions nor hysteretic behavior.

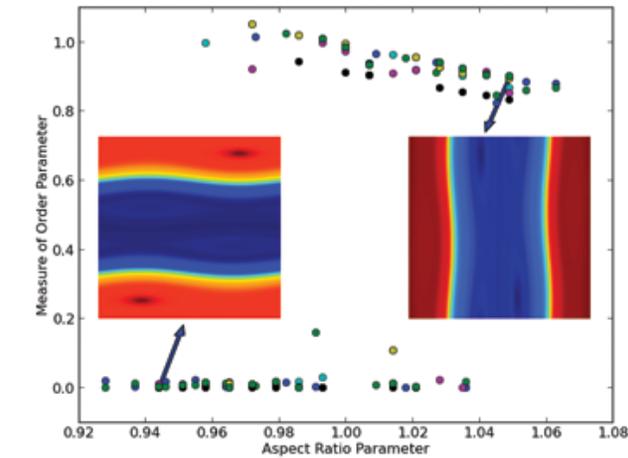


Fig. 3. Order parameter as a function of aspect ratio as obtained in fully nonlinear simulations of the 2D Navier-Stokes equations. Jet regime, abrupt transitions, bistable regimes, and hysteretic behavior are observed in accordance with the theoretical predictions of Fig. 1. Flow configurations on the two branches are shown in color insets.

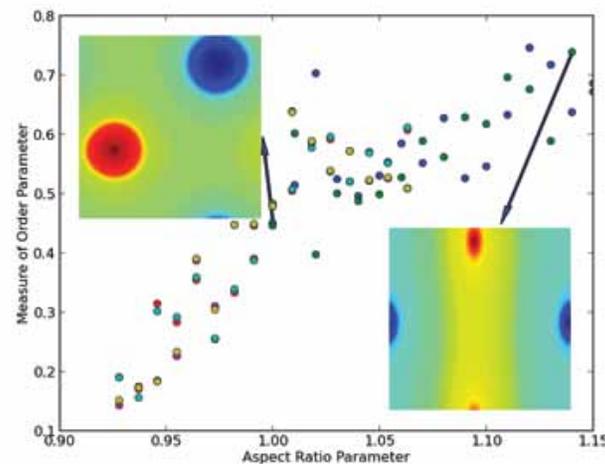


Fig. 4. Results of nonlinear simulations in the vortex regime. Continuous transformation of flow topology with geometry is observed in accordance with theoretical predictions of Fig. 2.

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