Late-Time Quadratic Growth in Single-Mode Rayleigh-Taylor Instability

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The growth of the single-mode Rayleigh-Taylor instability has been investigated using Direct Numerical Simulations (DNS) [1]. The results show that, at long times and sufficiently high Reynolds numbers, the bubble acceleration becomes stationary, indicating mean quadratic growth. This is contrary to the general belief that single-mode RTI reaches constant bubble velocity at long times. A new stage, chaotic development, was found at sufficiently high Re values. During this stage, the instability experiences seemingly random acceleration and deceleration phases, due to complex vortical motions, with strong dependence on the initial perturbation shape. Nevertheless, the mean acceleration of the bubble front becomes constant, with little influence from the initial shape of the interface. As Re is lowered to small values, the later instability stages are subsequently no longer reached. Therefore, the results suggest a minimum Reynolds number and development time necessary to achieve all stages of single-mode RTI development, requirements that were not satisfied in the previous studies.

Rayleigh-Taylor instability (RTI) is an interfacial instability that occurs when a high-density fluid is accelerated or supported against gravity by a low-density fluid. This instability is of fundamental importance in a multitude of applications ranging from fluidized beds, oceans and atmosphere, to Inertial Confinement Fusion (ICF) and supernovae [2]. Thus, the loss of target performance in ICF is associated with the development of RTI. Better knowledge about the growth of the instability and its dependence on initial conditions can help more accurately predict important practical problems. Using DNS, we have systematically studied the development and dependence on initial conditions of single-mode RTI [1]. Besides its own interest, single-mode RTI has also been used as a building block for the study of multimode RTI development. Despite its apparent simplicity, single-mode RTI is still not well understood and continues to be the focus of research in experimental (with both 2D and 3D perturbations), numerical, and theoretical studies.

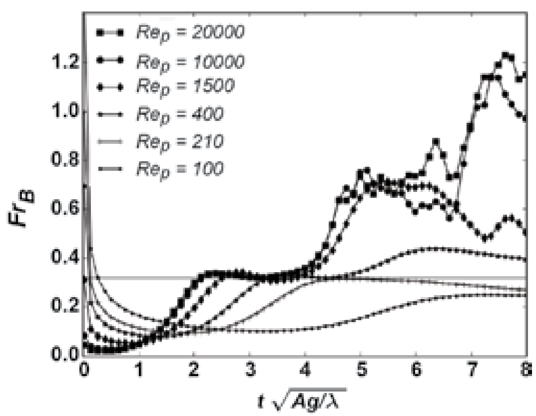

Fig. 1. The effect of the Reynolds number on the normalized bubble front velocity, Fr_b.

All simulations have been performed with the CFDNS code [3]. We have carried out extensive resolution studies to ensure that the solution is converged and the flow symmetries are preserved. The simulation results also show excellent agreement with the Linear Stability Analysis (LST), the analytical prediction of Goncharov [4], and the experimental results of Waddell et al. [5].

The development of single-mode RTI is usually divided into a number of stages, depending on which physical effect dominates the instability growth. We found that this development and the transition (or lack thereof) to the subsequent stages is strongly influenced by a perturbation Reynolds number, defined by $\mathrm{Re}_p = \lambda\sqrt{A/(A+1)\lambda g}$. The full range of development stages can be obtained at high enough values of the Reynolds number. Until recently, it was believed that the bubble front approaches and maintains a constant "terminal velocity" at late times (i.e., linear bubble height growth, $H_B \sim t$), based on the assumption that the flow remains potential near the tip of the bubble. Our results show that the flow does indeed reach such a regime (Fig. 1); however, while the vorticity is zero in this region due to symmetry conditions, the induced velocities due to vortical motions inside the bubble become strong enough to render such potential flow solutions inadequate at long times. Thus, at high enough Reynolds numbers, the Kelvin-Helmholtz instability generates vortices at the interface as the two fluids move in opposite directions. These vortices are first generated near the initial interface, then break into vortex pairs (in the 2D case) or rings (in the 3D case), that self-propagate towards the tips of the bubbles and spikes. The induced velocities generated by the vortex pairs or rings add bursts of acceleration to the instability growth, until these reach the tips of the bubbles and spikes, where they break into smaller structures. However, vortexes are continuously being generated at the interface and the process continues. The resulting flow exhibits complex vortical interactions (Fig. 2). Although the flow should preserve the initial reflectional symmetries (i.e., the axes of the bubbles and spikes), the motions in between these symmetry lines become chaotic and the bubble acceleration becomes stationary, with non-zero mean. The instantaneous evolution of the layer shows strong sensitivity to the initial conditions; however, the mean acceleration has little influence from the initial perturbation shape. The vortical interactions, neglected in previous

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studies, transform the constant velocity growth into mean quadratic growth at high enough Reynolds numbers (Fig. 3).

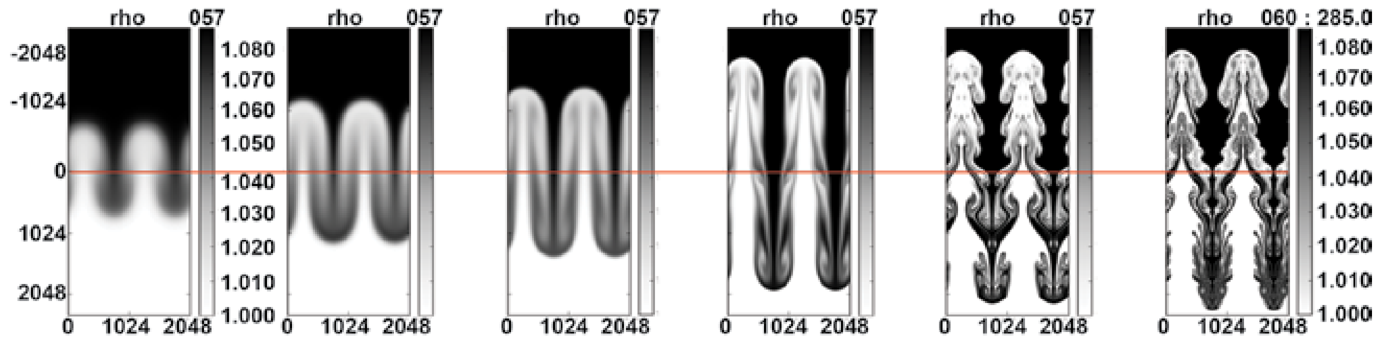
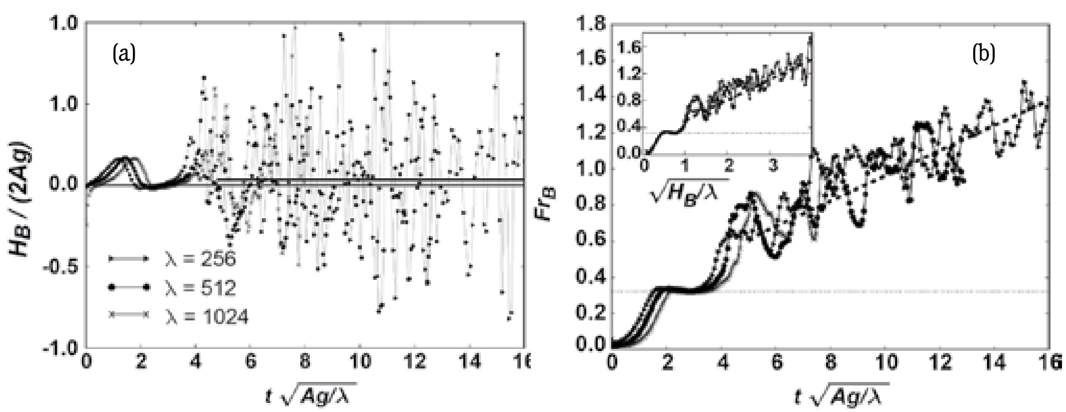


Fig. 2. Density contours at late time for different Reynolds number simulations. From left to right, Re_p= (a) 100, (b) 210, (c) 400, (d) 1,500, (e) 10,000 and (f) 20,000.

These results show that there is no fundamental difference between the single-mode and multimode RTI growth, as previously believed. In addition, the results reconcile the apparent contradiction between the previously believed constant velocity single-mode growth and recent results showing fast quadratic growth (with α values larger than those routinely obtained in numerical simulations) for initial perturbations with a pronounced peak at $k = 1$ [6]. Therefore, single-mode RTI results could be used to understand the growth of laterally confined RTI (when the $k = 1$ mode dominates the spectrum).

The perturbation Reynolds number, Re_p, has very important implications on the Large Eddy Simulations (LES) of single mode RTI. In LES, with or without explicit subgrid scale (SGS) modeling, an "effective" perturbation Reynolds number can be defined using the SGS (or numerical) viscosity. LES of single-mode RTI with coarse mesh may suffer from a low Re_p effect, due to an insufficient range of scales allowed by the mesh. Such an effect can be clearly seen in the Implicit Large Eddy Simulations (ILES) results of Ramaprabhu et al. [7] (see their Figs. 11 and 12), Glimm et al. [8] (their Fig. 1), Francois et al. [9], and Ramaprabhu et al. [10]. In those studies, the changing of meshes [7,8] or numerical methods [9] gave different growth rates, due to the implicit change in the effective perturbation Reynolds number. In coarse mesh (or lower effective Re_p) simulations, the later instability stages were not observed. Even when the instability reached a growth stage beyond the constant velocity regime (e.g., [10]) the flow returned to bubble front velocities smaller than the potential flow result, presumably due to insufficiently high Reynolds numbers. In addition, when small-scale vortical motions were generated and seemingly influenced the growth, there is a clear interference between the physical and numerical vorticity production mechanisms, indicated by the breaking of the symmetries that should be preserved by the flow. Due to the sensitivity of the instability growth to the vortical motions, this raises significant questions on the relevance of ILES techniques to the single-mode RTI.

Fig. 3. (a) Bubble front acceleration, and (b) bubble front velocity evolution at Re_p>10,000 showing mean quadratic growth.



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