Numerical Schemes for Hydrodynamics Based on Multidimensional Riemann Solvers

William W. Dai, HPC-4; Paul R. Woodward, B. Kevin Edgar, University of Minnesota

Numerical schemes based on Riemann solvers have been widely used in applications of hydrodynamics. Typically these schemes are using 1-D Riemann solvers at interfaces between numerical cells. In Lagrangian or arbitrary Lagrangian Eulerian (ALE) calculations, it is always desired to obtain the flow velocities at grid points. Truly multidimensional high-order shock-capture schemes could play an important role in Advanced Simulation and Computing (ASC) code projects. Approximate multidimensional Riemann solvers, which solve Riemann problems at grid points instead of interfaces between cells, naturally serve the purpose.

Multidimensional Riemann problems have not been mathematically solved yet. Therefore, developing numerical schemes based on multidimensional Riemann solvers is still a challenge. But the hope is that all the 1-D Riemann solvers actually used in numerical schemes are approximate, but not exact. We expect approximate multidimensional Riemann solvers to play an important role in future numerical simulations for hydrodynamics.

In this paper, we briefly outline a numerical scheme based on an approximate multidimensional Riemann solver. In the scheme, we use an approximate solver for 2-D and 3-D Riemann problems to calculate time-averaged values of flow variables at each grid point. Time-averaged fluxes at interfaces between numerical cells are calculated from the time-averaged values of the variables. The scheme is truly multidimensional, is second-order accurate in both space and time, and satisfies conservation laws for mass, momentum, and total energy exactly.

Numerical Schemes

The set of 2-D Euler equations is

$$\rho \frac{dU}{dt} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y}. \qquad (1)$$

Here, $U \equiv (v, u_x, u_y, e)^T$, superscript T stands for transpose, ρ is mass density, $v \equiv 1/\rho$, u_x and u_y are the components of flow velocity, e is the specific total energy, and F_x and F_y are fluxes in the x- and y- directions respectively. A 2-D Riemann problem is Eq.(1) with a set of constant states for each region surrounding a point; for example, four constant states in the four quadrants in a structured mesh. At each time step within a simulation, what are known are mass, momentum, and energy in each numerical cell. The initial condition surrounding each grid point naturally corresponds to a 2-D Riemann problem.

If Eq.(1) is integrated over a cell and one time step, $0 < t < \Delta t$, the following equation will be obtained.

$$\langle U \rangle = \langle U \rangle_0 + \frac{\Delta t}{\Delta m} \{ \overline{\oint F_x dy} + \overline{\oint F_y dx} \}. \qquad (2)$$

Here, $\langle U \rangle$ is a space-averaged value of U over the cell at $t = \Delta t$, $\langle U \rangle_0$ is its initial value, $\oint$ is the integral counterclockwise along the perimeter of the cell, and the bar over the integrals stands for the time average during the time step. In our scheme, the time-averaged integral is approximately calculated through the time-averaged values obtained from an approximate multidimensional Riemann solver. To get the second-order accuracy of the scheme, the states surrounding a

grid point will not be the states of the cells, but are the states on domains of dependence.

Results and Discussions
The scheme has been tested in an image processing program, ALE, calculations (Fig. 1). Currently, artificial viscosity has not been introduced into the scheme, but some form of artificial viscosity may be necessary for real equation of state when very strong shocks are present. In the future, we plan to extend the scheme for unstructured meshes.

Figure 1 shows the pressure in an ALE calculation at $t = 0.0002$ for a 2-D Riemann problem. The initial pressures on the four quadrants are 10^6, 1.0, 10^6, 10 respectively. The initial velocity is zero, and initial density is unity everywhere.

For more information contact William W. Dai at dai@lanl.gov.

Funding Acknowledgements
This research was supported by the NNSA tri-Lab Advanced Simulation and Computing Program.

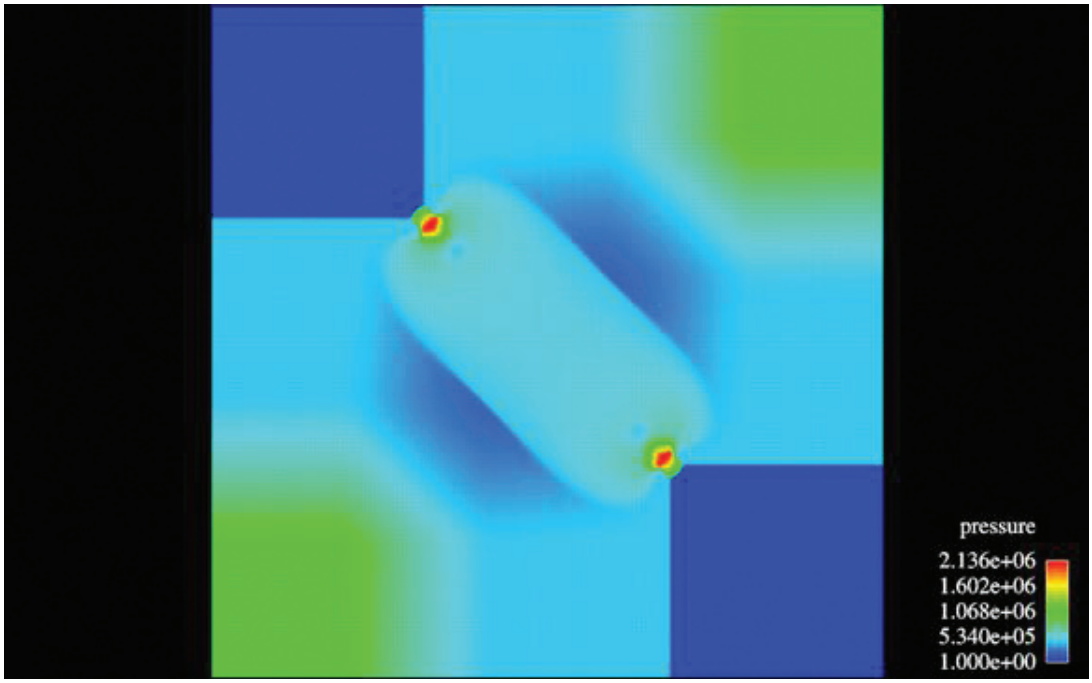

Fig. 1.
ALE calculation at **t** *= 0.0002 for a 2-D Riemann problem. The initial pressures on the four quadrants are 10^1, 1.0, 10^6, 10 respectively. The initial velocity is zero, and initial density is unity everywhere.*