Direct Search Methods with Equality Constraints

David W. Dreisigmeyer, HPC-4

Direct search methods are derivative-free algorithms for function minimization. They are popular optimization methods for nonsmooth, noisy, and discontinuous. Additionally, a user may not have access to or trust the available derivative information. One major attraction of derivative-free algorithms is the ease with which they can be coded. A typical optimization problem for a direct search method is (P1):

$$\begin{aligned} &\textit{minimize}: && f(x) \\ &\textit{subject to}: && g(x) \leq 0. \end{aligned}$$

We'll take the inequality constraints in (P1) as defining some full dimensional region Ω. The way that direct search methods typically precede is to lie out some mesh around the current iterate in Ω. The function $f(x)$ is then evaluated at the points on the current mesh.

A major difficulty with direct search methods is their ability to handle problems of the form (P2):

$$\begin{aligned} &\textit{minimize}: && f(x) \\ &\textit{subject to}: && g(x) \leq 0 \\ & && h(x) = 0. \end{aligned}$$

The reason is that the equality constraints define a subspace M of Ω that is vanishingly small. The probability of a mesh point actually lying on M and satisfying the equality constraints is 0. The way around this is to treat M as a Riemannian manifold, at least locally [1]. Then, if we start with a point on M we can efficiently stay on M as the direct search method proceeds.

What is actually done is to restate (P2) as a problem in the tangent space of some point on M. Each vector ω in the tangent plane will correspond to a unique point y on M. We assign to ω the function values $f(y)$ and $g(y)$. That is, we pullback the objective function and inequality constraints from M to the tangent plane. But the tangent plane is just a copy of some Euclidean space, so we can now employ standard direct search methods in the tangent plane. The situation is illustrated in Fig. 1.

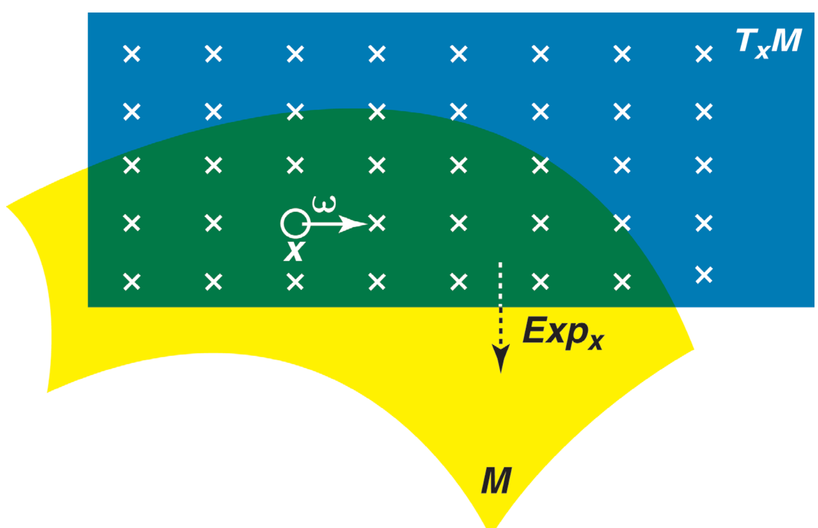

Fig. 1.
General setup for performing a direct search over a manifold. The Exp_x function is a mapping from the tangent space T_xM to the manifold M.

Our technique is especially useful if M is a Lie group or some other well-understood manifold [2]. In this case, closed form solutions are available for the mapping from ω to y. However, even when M is defined as a level set, finding the mapping from the tangent space to M has proven feasible [1, 3].

[1] D. W. Dreisigmeyer, "Equality Constraints, Riemannian Manifolds and Direct Search Methods," submitted to *SIAM J. Optimiz. (SIOPT).*
[2] D. W. Dreisigmeyer, "Direct Search Algorithms over Riemannian Manifolds," submitted to *SIAM J. Optimiz. (SIOPT).*
[3] D. W. Dreisigmeyer, "Direct Search Methods over Lipschitz Manifolds," submitted to *SIAM J. Optimiz. (SIOPT).*

For more information, contact David Dreisigmeyer at dreisigm@lanl.gov or visit Data-Driven Modeling and Analysis (DDMA) Team Web site, http://ddma.lanl.gov/

Funding Acknowledgements
This research was supported by the NNSA tri-Lab Advanced Simulation and Computing Program.