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MEASUREMENTS WITH A RECUPERATIVE SUPERFLUID STIRLING REFRIGERATOR

A. Watanabe, G. W. Swift, and J. G. Brisson[1]

Condensed Matter and Thermal Physics Group
Los Alamos National Laboratory, Los Alamos NM 87545

ABSTRACT

A superfluid Stirling refrigerator cooled to 168 mK using a 4.9% ^{3}He-^{4}He mixture and exhausting its waste heat at 383 mK. Cooling power versus temperature and speed is presented for 4.9%, 17%, and 36% mixtures. At the highest concentration, a dissipation mechanism of unknown origin is observed.

INTRODUCTION

The superfluid Stirling refrigerator (SSR) uses a ^{3}He-^{4}He liquid mixture[2] as a working fluid in a Stirling cycle. It operates at temperatures below 2 K where the ^{4}He component of the working fluid is superfluid. The ^{3}He component of the working fluid, to first approximation, behaves thermodynamically as an ideal gas in the inert background of superfluid ^{4}He. Using pistons equipped with a superleak bypass, it is possible to compress and expand the ^{3}He solute "gas" alone. The SSR is a Stirling machine equipped with these "superleaked" pistons to take advantage of the properties of the ^{3}He solute to cool below 1 K. The principle was first demonstrated by Kotsubo and Swift[3,4] in 1990; a more practical refrigerator was studied by Brisson and Swift[5–8] beginning in 1992.

The refrigerator used in the present work, shown in Fig. 1, is a modified version of that discussed by Brisson and Swift.[5,6] The major modification is the addition of a ^{3}He evaporation refrigerator that cools the hot platform on which the hot pistons are mounted. With this arrangement, the hot platform temperature can be maintained at between 0.3 K and 1 K. This refrigerator is actually two SSR's operating 180° out of phase with each other, which allows the use of a counterflow heat exchanger to act as a regenerator for each of the SSR's. Our counterflow regenerator,[8] or recuperator, consists of CuNi tubes silver soldered in a hexagonally close packed array with alternating rows corresponding to each "half" of the SSR.

The pistons are made with welded bellows which have wavy segments that nest into one another to minimize the dead volume. The hot platform pistons are rigidly connected together and driven sinusoidally by a rigid push rod from a room temperature drive. The cold platform pistons are similarly driven. Linear position sensors are mounted on a pumped ^{4}He platform maintained at 1.2 K to monitor the positions of the pistons. The hot piston's stroke ranged from 2.41 mm to 4.85 mm, corresponding

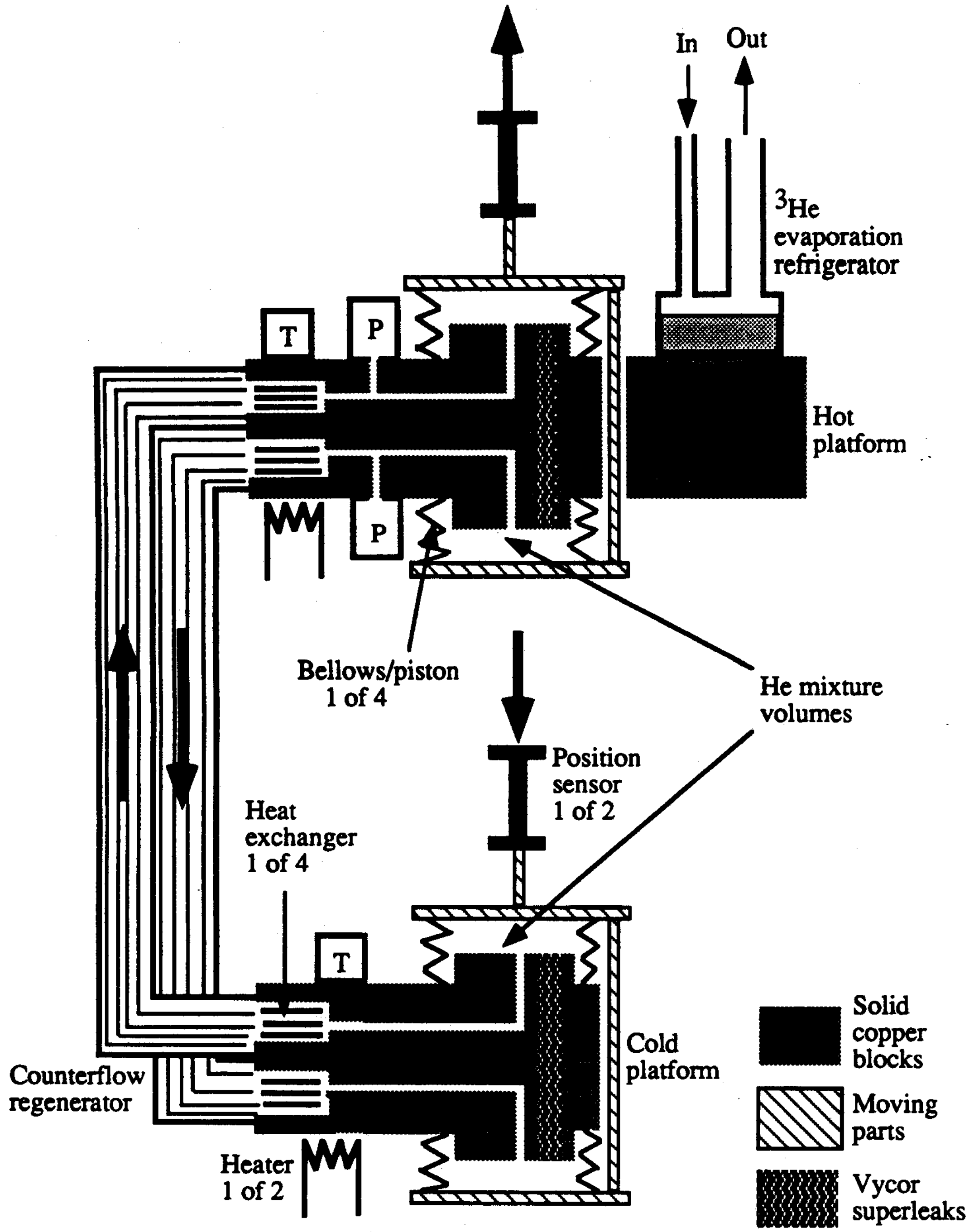

Fig. 1. The SSR. Pressure gauges are delineated with a "P", thermometers with a "T". The pumped ^{4}He platform, which is located above the hot platform, is not shown. The arrows in the regenerator indicate the direction of fluid flow corresponding to the direction of piston motion shown.

to a swept volume of 0.763 cm^3 to 1.53 cm^3. The cold piston's stroke ranged from 2.34 mm to 4.93 mm, corresponding to a swept volume of 0.737 cm^3 to 1.56 cm^3. The superleaks shown in Fig. 1 are cylinders of Vycor glass. These allow the superfluid ^{4}He to freely flow between the halves of the refrigerator during its operation. Flexible diaphragm pressure gauges are connected to each half of the SSR at the hot platform. After loading the SSR with ^{3}He-^{4}He mixture of desired concentration, the two fill lines into each of the SSR halves are sealed with mechanically actuated low temperature valves mounted on the pumped ^{4}He platform. If we did not use these valves, operation of the SSR would cause the mixture to oscillate up and down the fill capillaries and put a significant heat load on the refrigerator. Heaters and calibrated germanium thermometers are mounted on the outside of the hot and cold platforms.

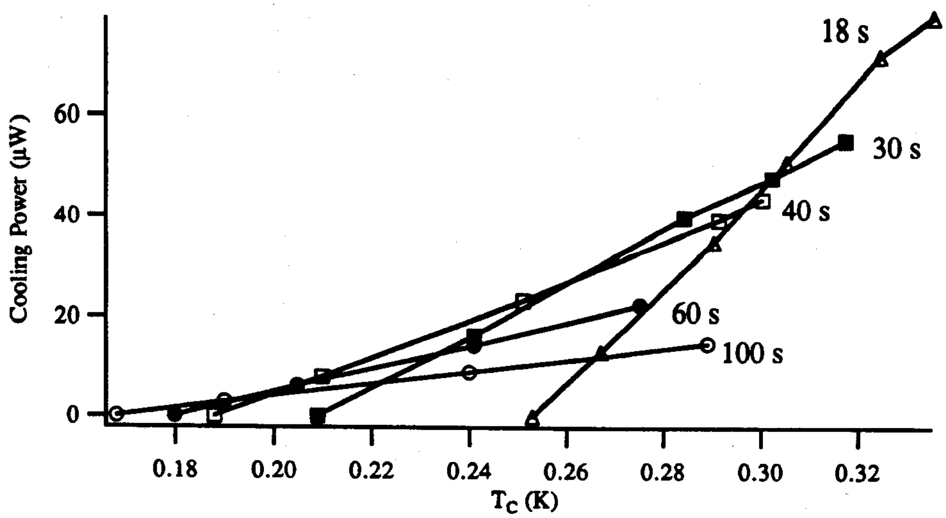

Fig. 2. The SSR's cooling power vs the cold platform temperature for a 4.9% mixture. Here and in the following figures, each set is labeled with the SSR's period, and the lines are only guides to the eye. The hot and cold piston swept volumes were 1.53 cm^3 and 1.56 cm^3, respectively. The hot platform temperature was approximately 0.450 K, 0.417 K, 0.407 K, 0.400 K, and 0.387 K for the period of 18, 30, 40, 60, and 100 seconds, respectively.

RESULTS AND DISCUSSION

The SSR's cooling power shown in Figs. 2 through 5 was measured as a function of the mixture's average concentration, hot platform temperature T_H, cold platform temperature T_C, and the operating speed. T_C was actively controlled by a feedback controller that delivered current to the heater mounted on the cold platform, and the heat delivered averaged over five to ten periods yielded the cooling power. With the exception of the 4.9% mixture, T_H was actively controlled by a combination of the ^{3}He evaporation refrigerator and a feedback controller delivering current to the heater mounted on the hot platform. For the 4.9% mixture, T_H was kept as cold as the ^{3}He evaporation refrigerator allowed. The SSR's minimum period, the time necessary to complete one cycle, was limited by the ^{3}He pot's capability in all of the runs. Running the SSR too fast put a large heat load on the hot platform and resulted in the ^{3}He pot quickly running dry. Concentrations given in this paper are molar per cent of ^{3}He.

Figure 2 shows the SSR's cooling power at various speeds as a function of cold platform temperature for a 4.9% mixture. The SSR reached a minimum temperature of 168 mK with a period of 100 s and hot platform temperature of 383 mK. Figure 2 also shows that there is an optimum operating speed at each temperature to maximize the cooling power. The upper envelope of the lines defines this SSR-^{3}He refrigerator system's maximum cooling power as a function of temperature.

It is of interest to model the mixture's concentration as a function of position in the refrigerator corresponding to the experimental conditions measured above. Because of the temperature difference, the concentration in the cold platform pistons is higher than the "nominal" concentration, and that in the hot platform pistons is lower. For low concentrations of ^{3}He, the mixture should obey[5]

$$P_3(\rho, T) + P_f = \text{constant} \tag{1}$$

throughout the refrigerator, where P_3 is the osmotic pressure of ^{3}He, ρ is the mixture's ^{3}He density, T is the temperature, and P_f is the fountain pressure. Approximate values of $P_3(\rho, T)$ can be obtained using the ideal Fermi gas equation of state.[9] It was assumed that the hot and cold piston volumes are 3.2 cm^3. The regenerator, which has a volume of 1.0 cm^3, was modeled as six discrete volumes with a linear temperature profile. The number of ^{3}He particles in the refrigerator is constant, which means

$$\sum \rho_i(T) V_i = \rho_{avg} V_{tot}, \tag{2}$$

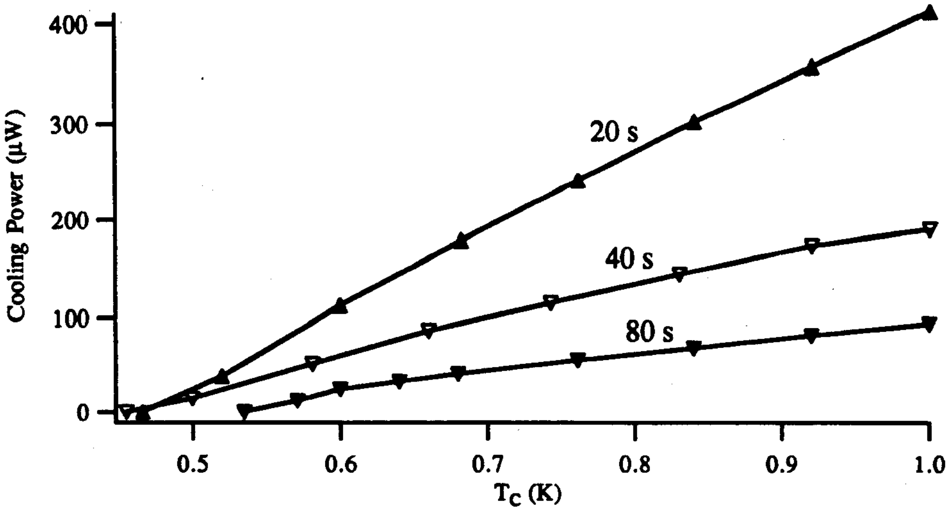

Fig. 3. The SSR's cooling power vs the cold platform temperature for a 17% mixture and T_H = 1 K. The hot and cold piston swept volumes were 0.763 cm^3 and 0.737 cm^3, respectively.

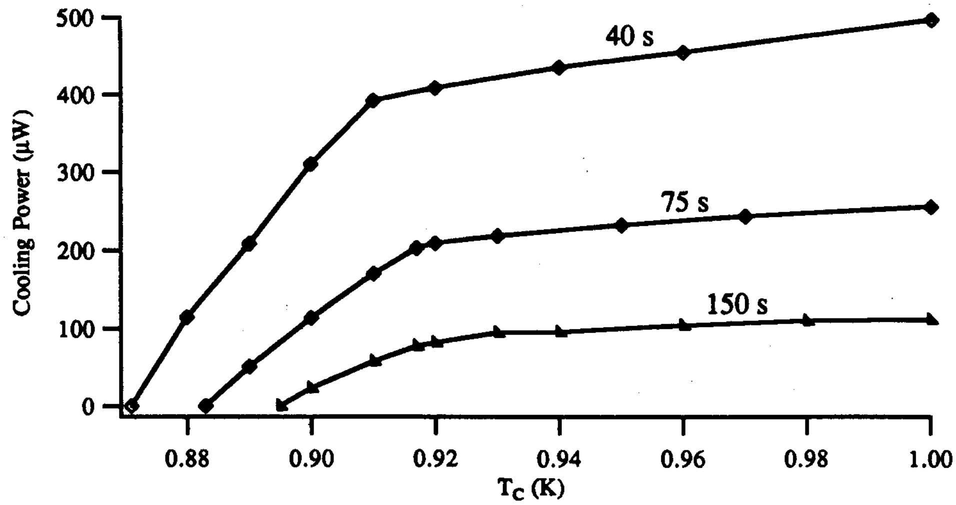

Fig. 4. The SSR's cooling power vs the cold platform temperature for a 36% mixture and T_H = 1 K. The hot and cold piston swept volumes were 0.767 cm^3 and 0.737 cm^3, respectively.

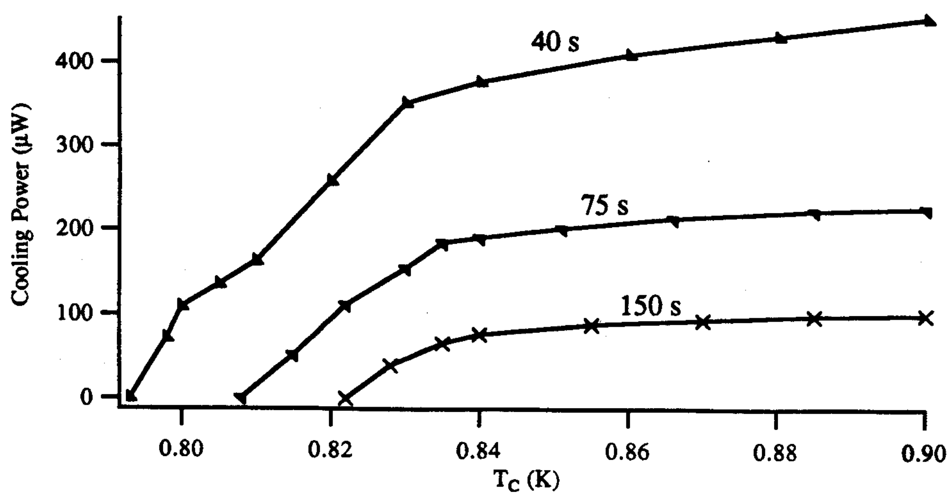

Fig. 5. The SSR's cooling power vs the cold platform temperature for a 36% mixture and T_H = 0.9 K. The hot and cold piston swept volumes were 0.767 cm^3 and 0.737 cm^3, respectively.

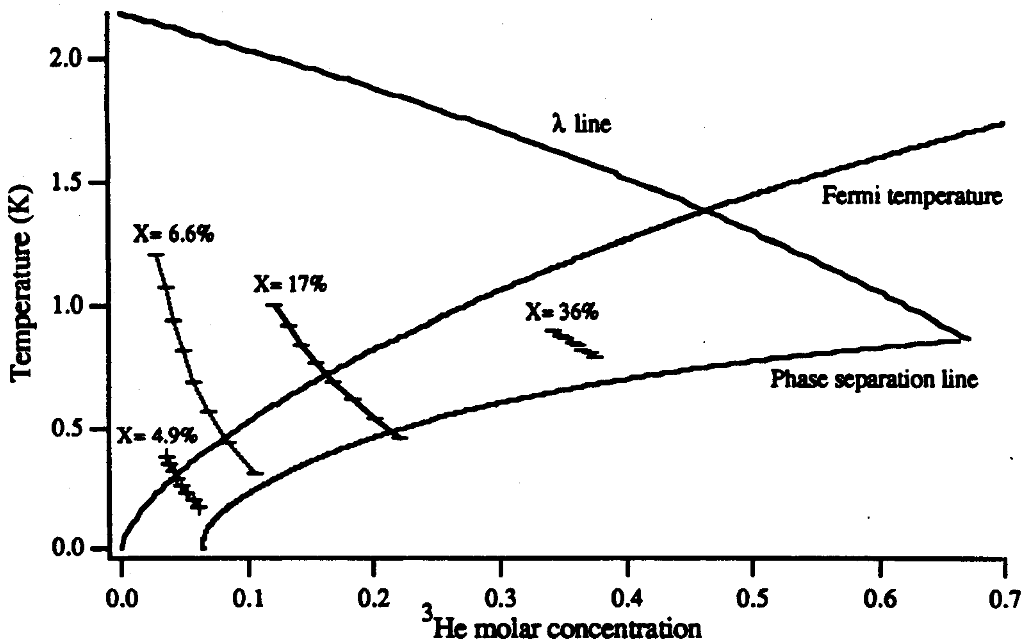

Fig. 6. The mixture's state as a function of position in the refrigerator for four different average concentrations, superimposed on a phase diagram. Each curve is labeled with the mixture's average concentration. Three of the curves are for operating conditions of this paper; the 6.6% curve is for the conditions of Ref. 6. The ^{3}He Fermi temperature as a function of concentration is also shown. In each curve, the top point is the state of the mixture in the hot piston, the bottom point is the state of the mixture in the cold piston, and the six intermediate points represent the mixture states in the regenerator.

where $\rho_i(T)$ and V_i are the ^{3}He density and volume at various points in the refrigerator, ρ_{avg} is the average ^{3}He density in the refrigerator, and V_{tot} is the total volume of the refrigerator. With Eq. (2) as a constraint, we can solve iteratively for the mixture's concentration as a function of position in the refrigerator, given the hot and cold platform temperatures and average concentration. Figure 6 shows the results of the calculation for four sets of operating conditions, superimposed on a ^{3}He-^{4}He phase diagram. The top point in each curve is the state of the mixture in the hot piston, the bottom point is the state of the mixture in the cold piston, and the six intermediate points represent the mixture states in the regenerator.

We can make several observations using Fig. 6. First, for all the conditions modeled the state of the ^{3}He in the cold piston is in the Fermi regime. In addition, when the SSR was loaded with an average concentration of 36%, the entire refrigerator was operated below the Fermi temperature. Our model for the 17% mixture in the SSR suggests that there is phase separation in the cold piston. Unfortunately, we have no direct experimental evidence for phase separation in the 17% data.

The experimental cooling power results for the 36% mixture, Figs. 4 and 5, show a "cutoff" temperature where the cooling power drops off sharply as a function of temperature. Clearly, this is not due to phase separation of the mixture, as a ^{3}He-^{4}He mixture does not phase separate at 0.92 K regardless of the concentration.[10] Yet the sharpness of the transition suggests that it is caused by an abrupt transition in fluid properties. This transition may be caused by a critical velocity. The ^{3}He component may be exceeding a critical velocity in the 0.8 mm diameter capillary connecting the piston to the heat exchanger in the cold platform. As the cold platform gets colder, the concentration in the cold piston increases according to Eq. (1), and the net effect may be to decrease the critical velocity. For a saturated mixture below 100 mK, Zeegers et al.[11] empirically found a critical velocity of

$$v_c = (K/d)\ln(d/d_0) \tag{3}$$

where K=0.05 cm^2/s, d is the diameter of the capillary, and d_0=15 μm. Although both the temperature and the concentration used to find Eq. (3) are significantly lower than

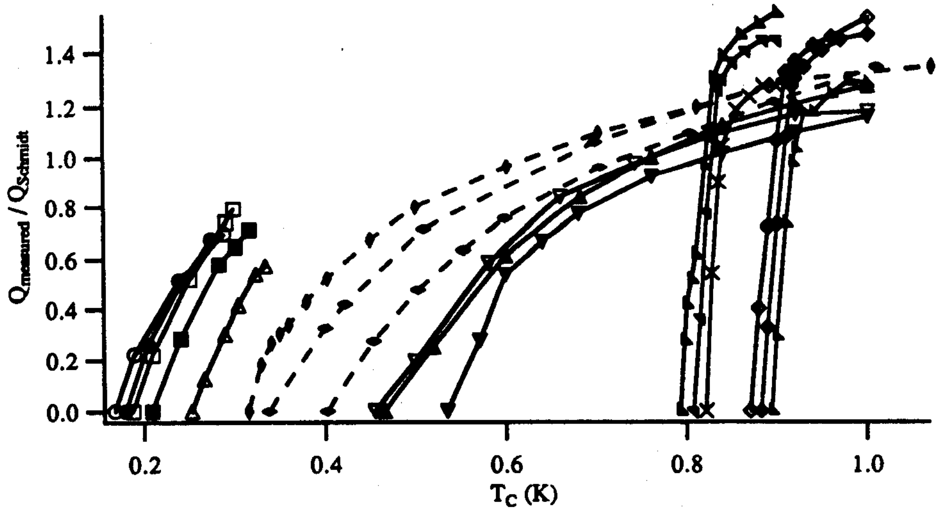

Fig. 7. The measured SSR cooling power normalized by the cooling power calculated from Schmidt analysis vs cold platform temperature. The symbols correspond to those shown in Figs. 2 through 5. The dashed lines represent the normalized cooling power of data of Ref. 6.

those used in our refrigerator, we will use this expression because we are unaware of any other published data for the critical velocity of superfluid ^{3}He-^{4}He mixtures. For a 0.8 mm diameter capillary, this expression yields 2.5 cm/s. In our experiment, the maximum ^{3}He velocity in the capillary during the cycle ranged from around 2.3 cm/s for the period of 150 s to around 8.5 cm/s for the period of 40 s, which is of the same order of magnitude as that given by Eq. (3).

Except for this unusual behavior at 36% concentration, cooling power exhibits remarkable linearity in T_C. This was also true for measurements[6] for a 6.6% average mixture concentration. This linearity can be understood as follows. Assuming equal clearance volumes and swept volumes in the hot and the cold pistons and with 90° phasing between the pistons, the isothermal Schmidt analysis[12] of the Stirling cycle predicts a cooling power in the cold platform of

$$\dot{Q}_{Schmidt} = f\frac{\pi}{2}\left(\frac{V_{sw}}{V_0}\right)^2 MRT_H\left(\frac{\alpha}{1+\alpha}\right)^2 \Big/ \left(1-\frac{V_r}{V_0}\frac{\alpha\ln\alpha}{1-\alpha^2}\right)^2 \tag{4}$$

to lowest order in swept volume, where f is the frequency, V_{sw} is the piston swept volume, V_0 is the average volume of each piston, M is the mass of ^{3}He in the refrigerator, R is the gas constant per mass of ^{3}He, $\alpha = T_C/T_H$, and V_r is the volume of the regenerator. We have multiplied the standard Schmidt result by twice the operating frequency because our refrigerator comprises two Stirling refrigerators operating in parallel. For a small regenerator volume, the temperature dependent part of Eq. (4) is very nearly linear in T_C for our operating range of $T_C/T_H > 0.3$. We can write

$$\dot{Q}_{measured} = \dot{Q}_{Schmidt} - \dot{Q}_{losses} \tag{5}$$

where $\dot{Q}_{losses}$ accounts for thermal loads on the cold platform due to thermal conduction from the hot platform, imperfect thermal contact of fluid in the regenerator, and other smaller sources. Thermal conduction and regenerator losses are roughly proportional[8] to $T_H - T_C$. Since both $\dot{Q}_{Schmidt}$ and $\dot{Q}_{losses}$ are approximately linear in T_C, Eq. (5) predicts that $\dot{Q}_{measured}$ will also be approximately linear in T_C.

The measured cooling powers normalized by those given by Schmidt analysis are plotted in Fig. 7. The normalized cooling powers extrapolate to between 1.1 and 1.6 near $T_H = T_C$, where the regenerator losses are predicted to be negligible. Without the regenerator losses, the normalized cooling power should be 1 when $T_H = T_C$. The deviation from this value can be attributed[7], in part, to the deviation of the working fluid's thermodynamic behavior from that of an ideal Boltzmann gas. The deviation is enhanced further by the non-isothermal compression and expansion in the real refrigerator. The latter effect can be estimated by considering the thermal penetration depth δ_κ in the ^3He-^4He mixture, given by $\delta_\kappa^2 = \kappa/\pi f \rho c_p$, where κ is the thermal conductivity of the mixture and ρ and c_p are the density and the heat capacity of ^3He. Assuming $c_p = 5R/2$ and using published figures[13] for κ, δ_κ varies between 1.3 mm and 4.2 mm for our operating conditions at 1.0 K. Since the fluid is as much as 6 mm away from isothermal solid surfaces, part of the fluid behaves more adiabatically than isothermally. This increases the pressure amplitude above that predicted by the Schmidt model, leading to a cooling power enhancement[7] of the order of 30%. Figure 7 nevertheless indicates that the cooling power is roughly proportional to the concentration of the working fluid.

In summary, the SSR has cooled to 168 mK with the hot platform being held at 383 mK, demonstrating the ability to cool to temperatures well below that attainable with a continuous ^3He evaporation refrigerator. The SSR has also been operated with mixtures of up to 36% concentration with proportionately higher cooling powers.

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