# Application of Extreme Value Statistics for Structural Health Monitoring


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# Four Goals for Structural Health Monitoring

- Step 1: Damage Identification
- Step 2: Damage Localization
- Step 3: Damage Quantification
- Step 4: Damage Prognosis

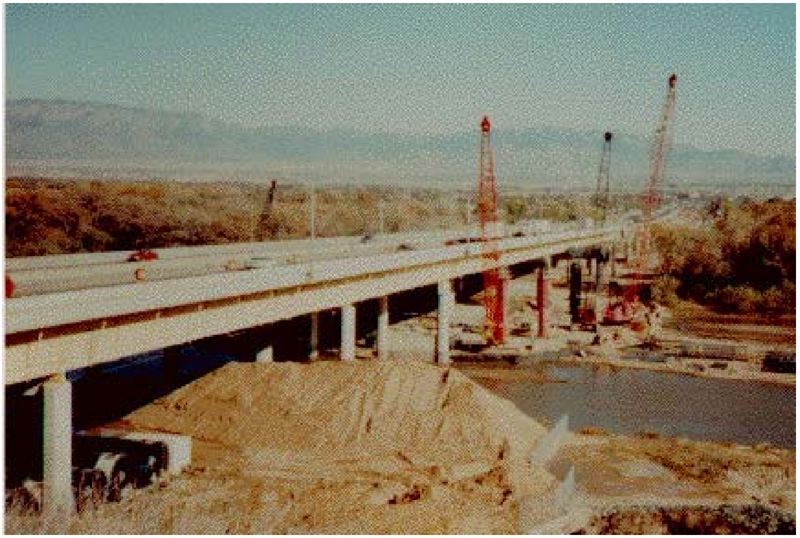
# Structural Health Monitoring

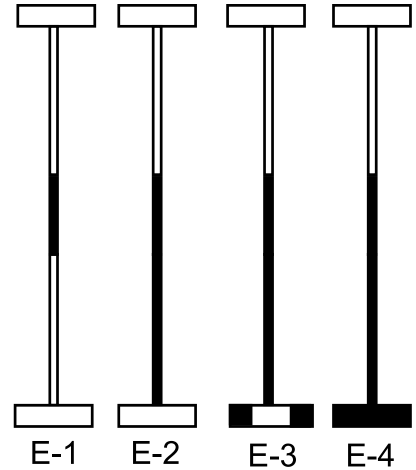
**Is this bridge damaged? ➡ Perform pattern comparison.**

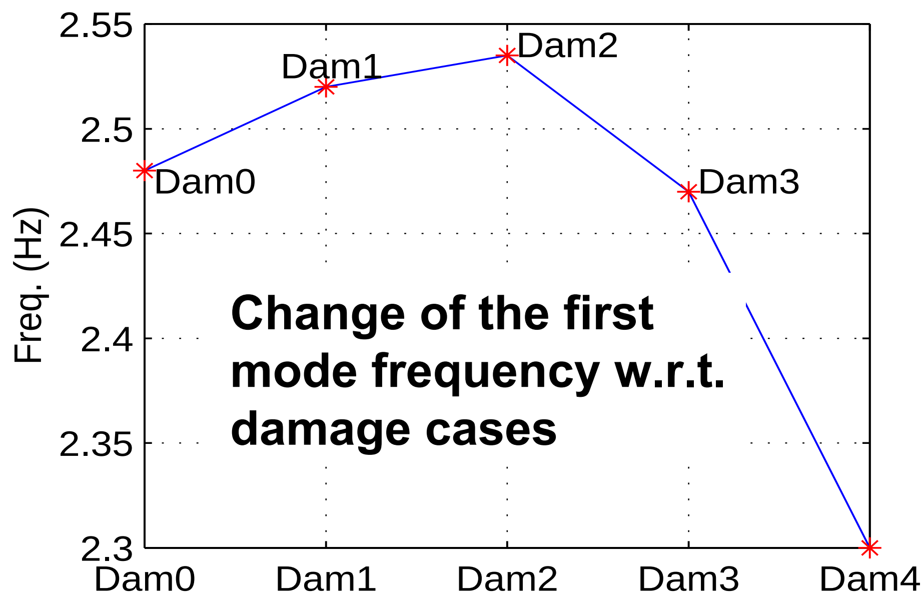
# Five Steps for Structural Health Monitoring

- Step 1: Operational Evaluation
- Step 2: Data Acquisition
- Step 3: Data Normalization
- Step 4: Feature Extraction
- Step 5: Statistical Inference

# Environmental Variation



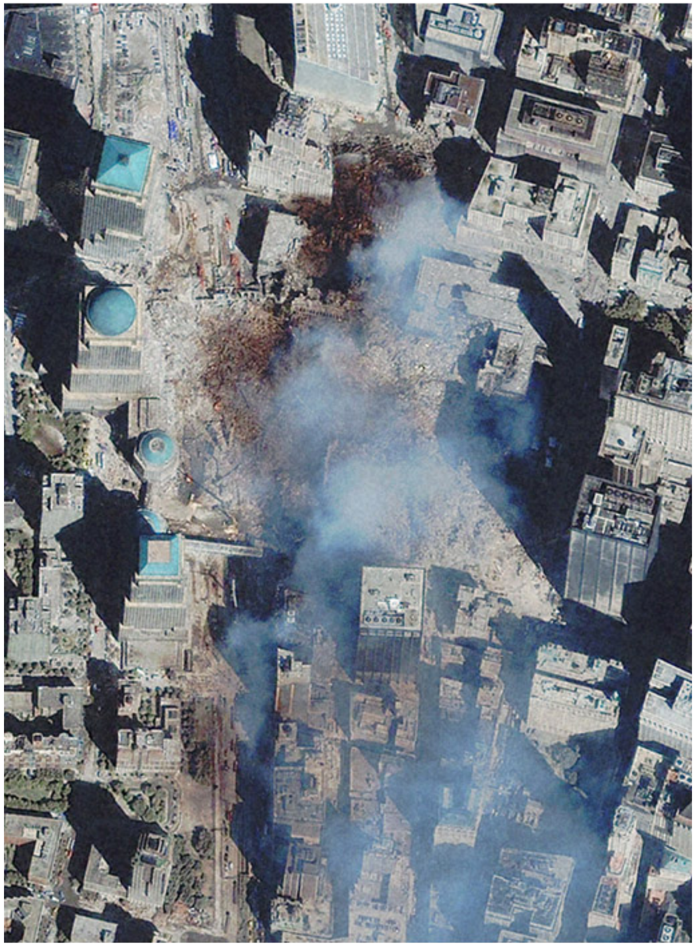


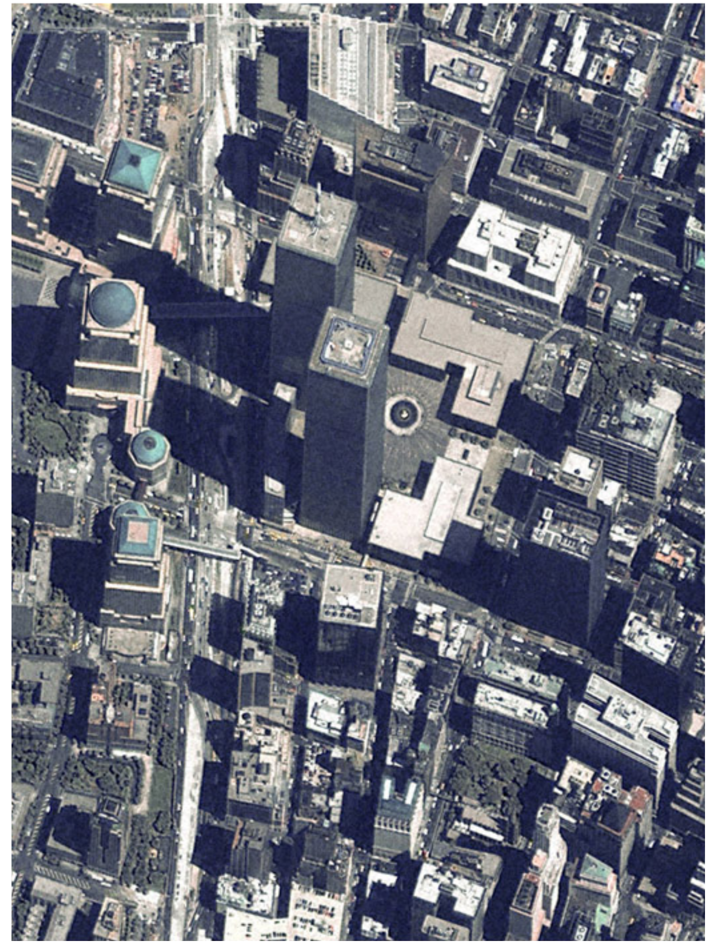
# Example of Debris in Expansion Joint of the Alamosa Canyon Bridge

This debris can effect the boundary conditions of the structure and its response to environmental changes

# Statistical Pattern Recognition Paradigm for SHM

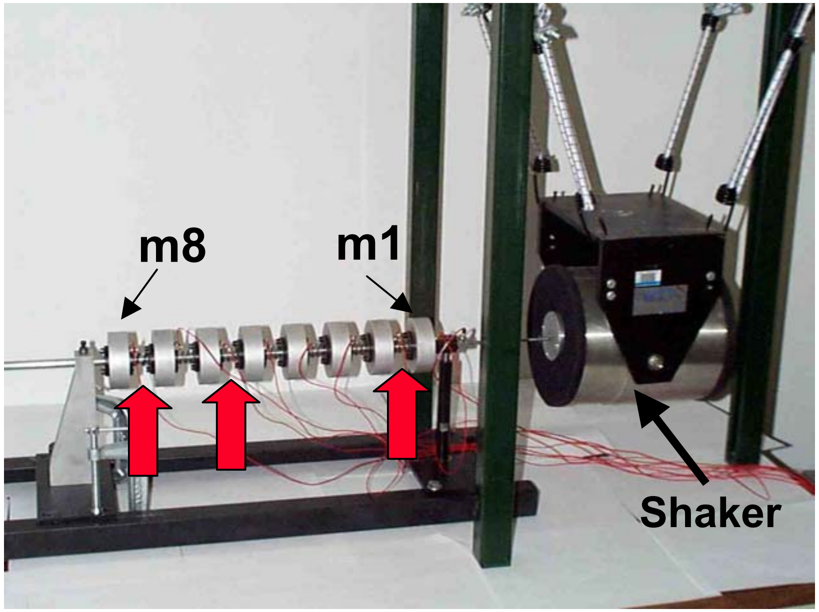


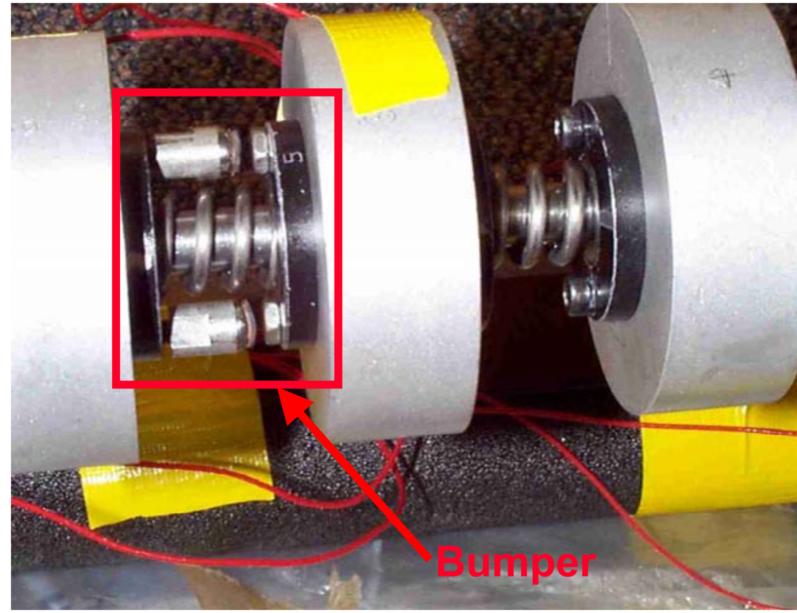
After Sep. 11, 2001



Before Sep. 11, 2001

# Data Normalization Example [Sohn et al. 2002]



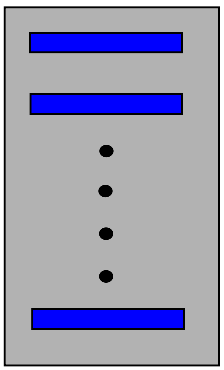

**List of time series employed in this study**

| Case | Description | Input level | Data # per input | Total data # |
|---|---|---|---|---|
| 0 | No bumper | 3, 4, 5, 6, 7 Volts | 15 sets | 75 sets |
| 1 | Bumper between m1-m2 | 3, 4, 5, 6, 7 Volts | 5 sets | 25 sets |
| 2 | Bumper between m5-m6 | 3, 4, 5, 6, 7 Volts | 5 sets | 25 sets |
| 3 | Bumper between m7-m8 | 4, 5, 6, 7 Volts | 5 sets | 20 sets |

# Outline of Damage Diagnosis using AR-ARX, Auto-Associative Network and Hypothesis Test

## 1. COLLECT BASELINE DATA

$x(t)$

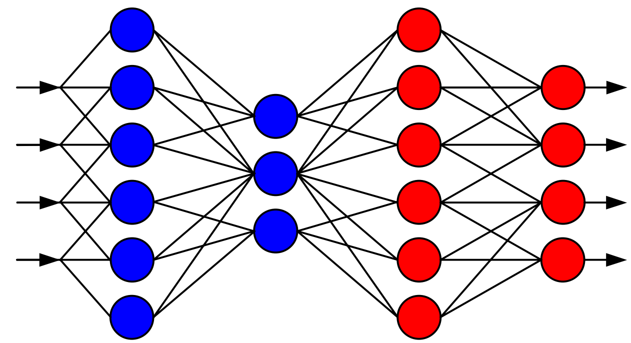


Time series from various operation & environmental conditions

## 2. FEATURE EXTRACTION

$$\varepsilon_x(t) = x(t) - \sum_{i=1}^{a} \alpha_i \, x(t-i) - \sum_{j=1}^{b} \beta_i \, e_x(t-j)$$

## 3. DATA NORMALIZATION



## 4. FEATURE EXTRACTION

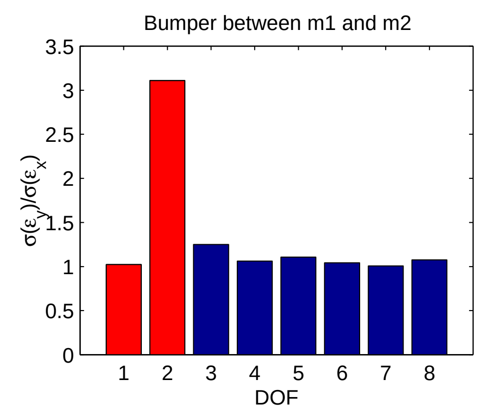
$$\varepsilon_y(t) = x(t) - \sum_{i=1}^{a} \hat{\alpha}_i \; x(t-i) - \sum_{j=1}^{b} \hat{\beta}_i \; e_x(t-j)$$

## 5. STATISTICAL INFERENCE

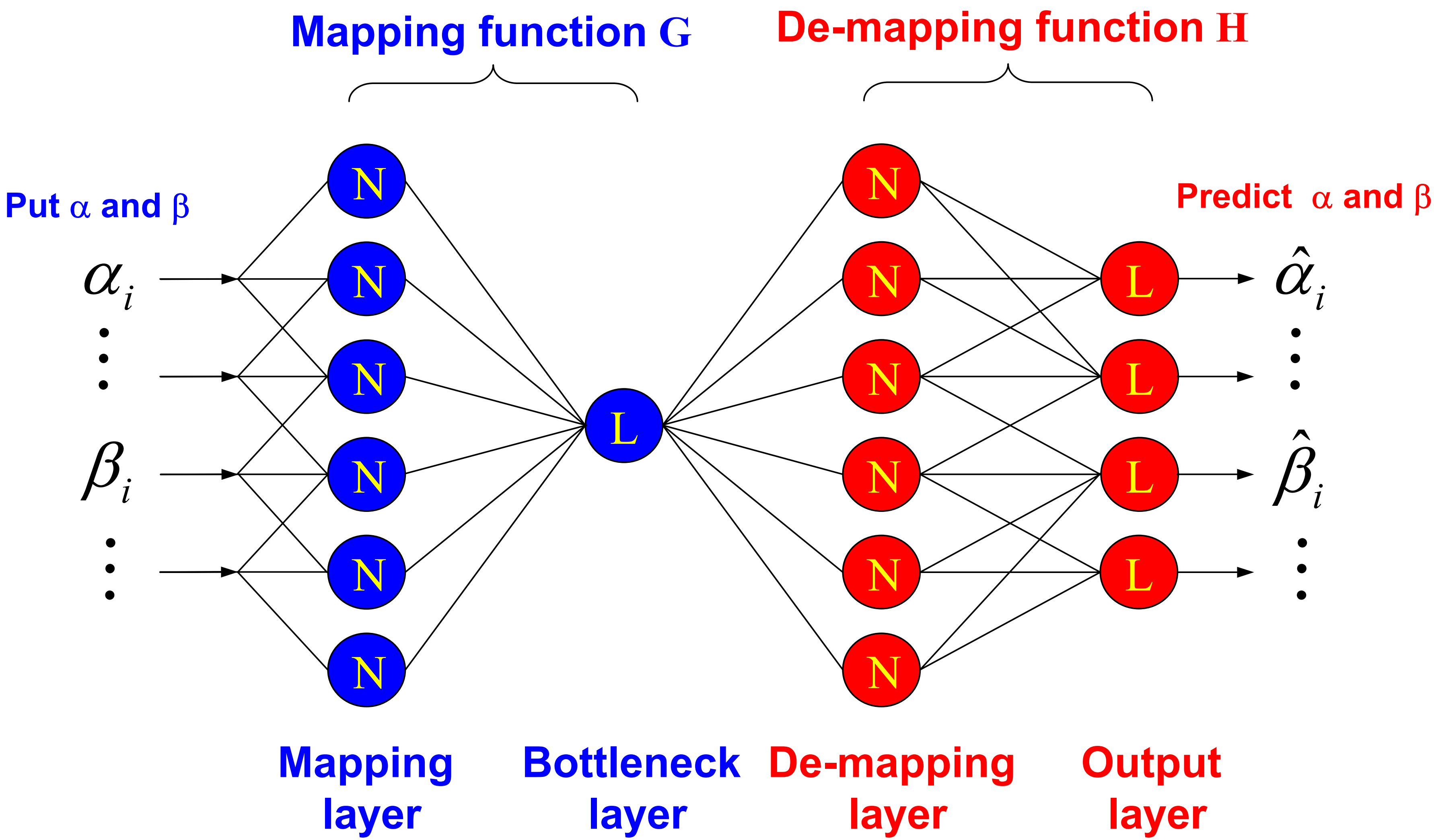
$$H_o : \sigma(\varepsilon_y) \leq \sigma(\varepsilon_x)$$

$$H_1 : \sigma(\varepsilon_y) \geq \sigma(\varepsilon_x)$$

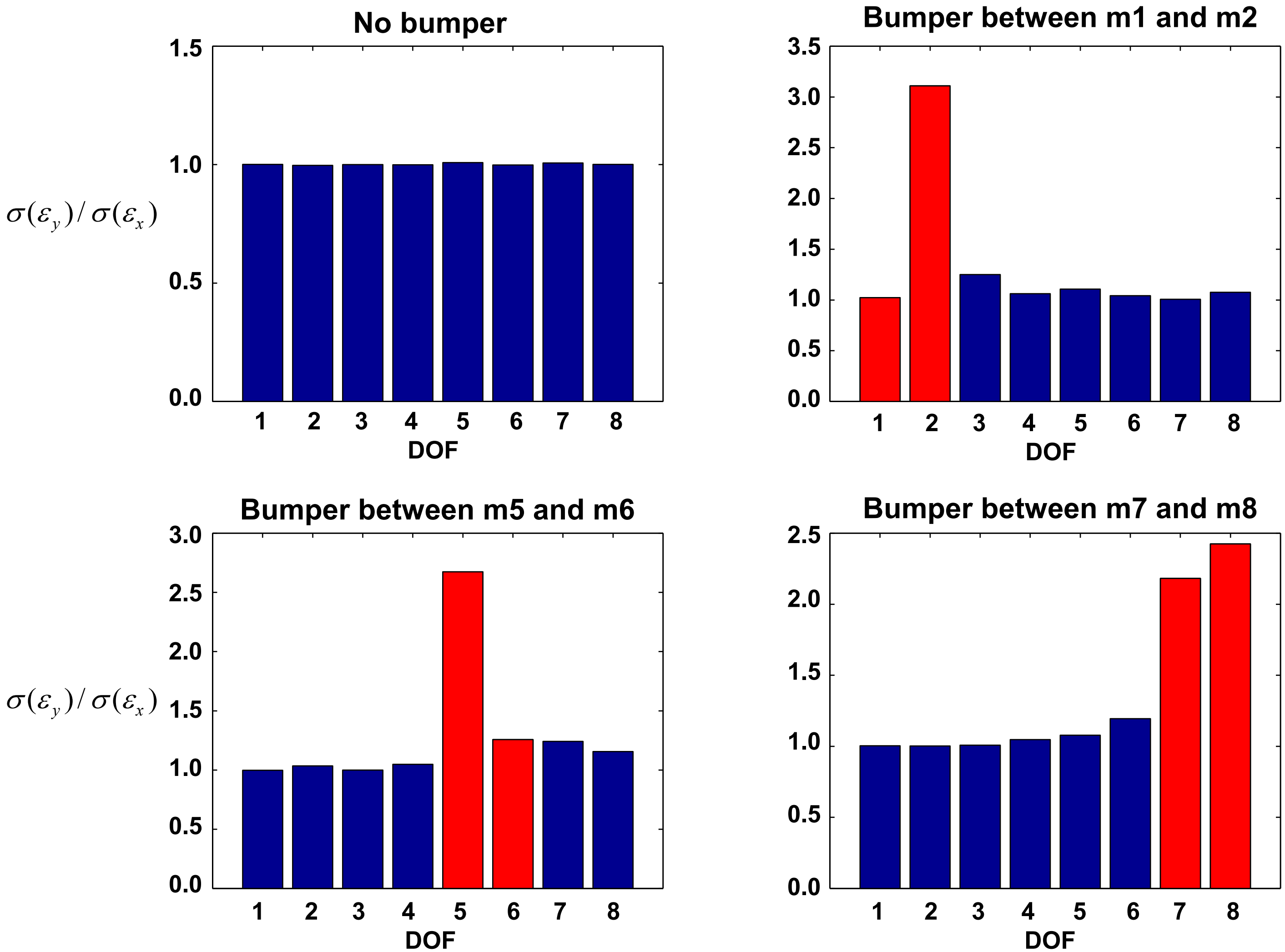
## 6. DECISION MAKING



# Auto-Associative Neural Network for Data Normalization



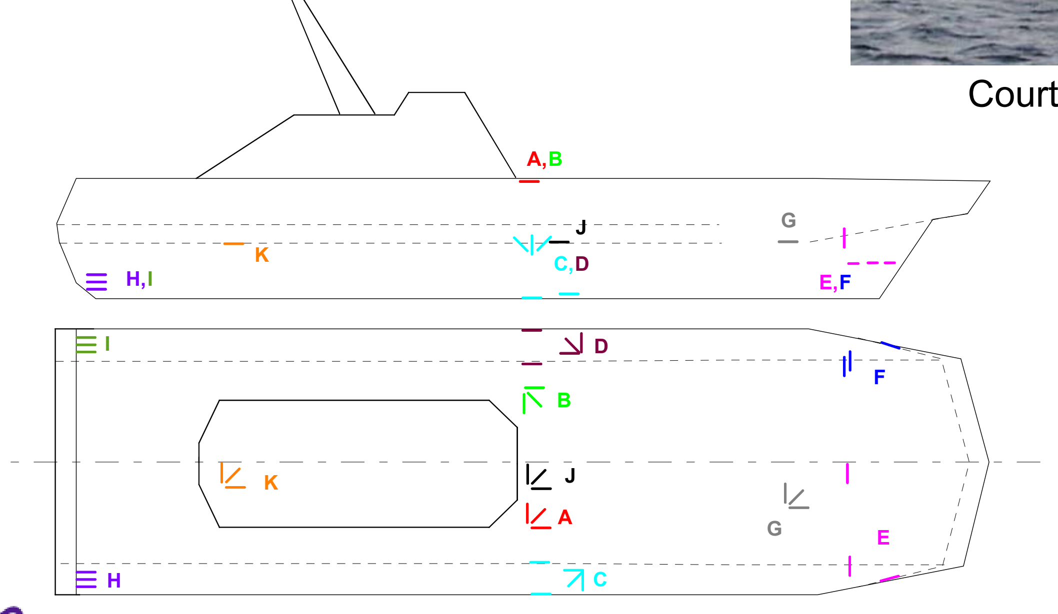
# Damage Localization



# Real World Application [Sohn et al. 2001]

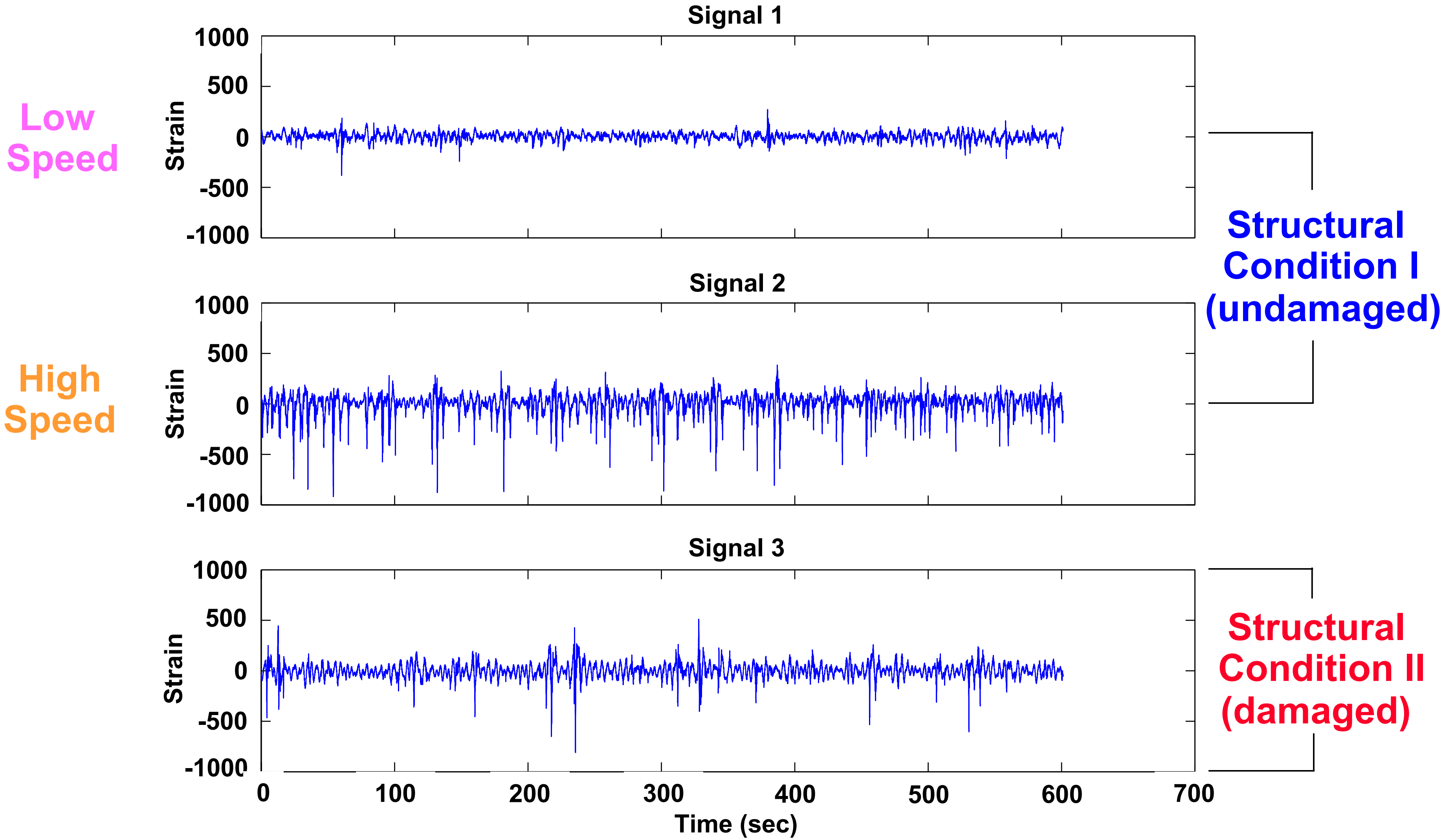
## (a) Surface-Effect Fast Patrol Boat

Courtesy of Naval Research Laboratory

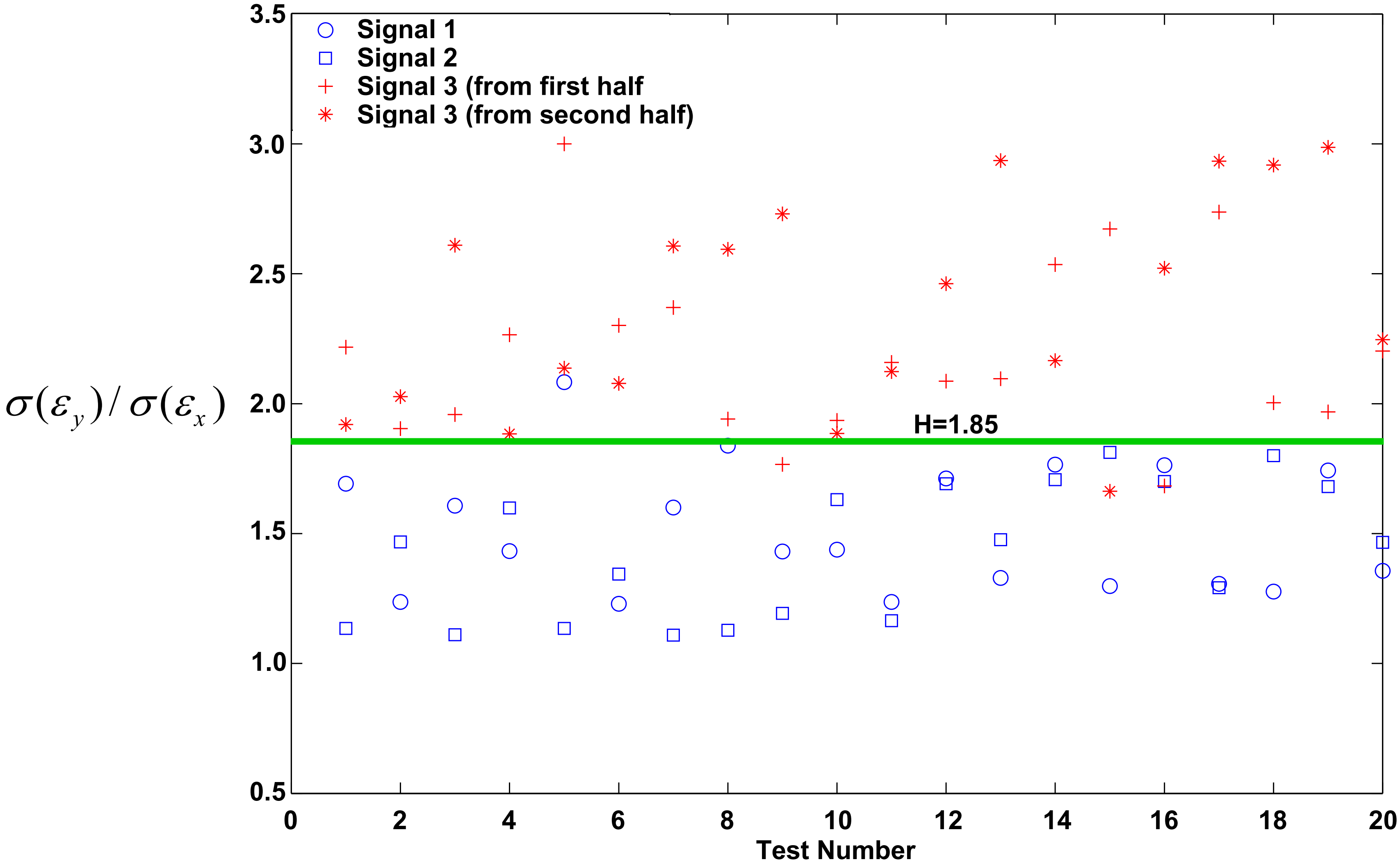



## (b) Fiber optic strain gauges

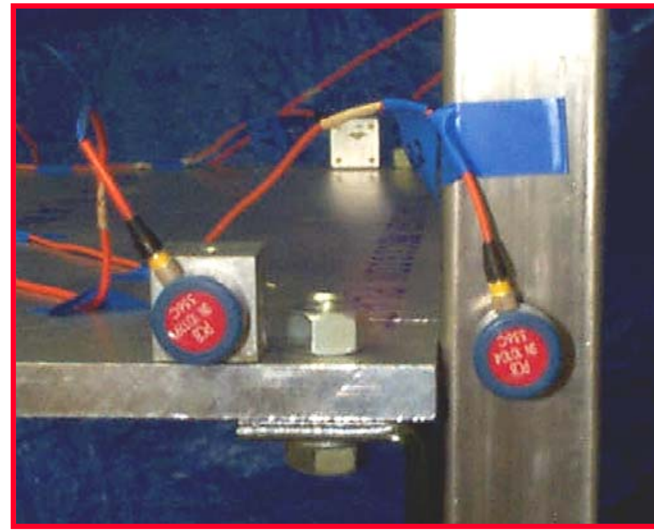
# Raw Dynamic Strain Time Series Data

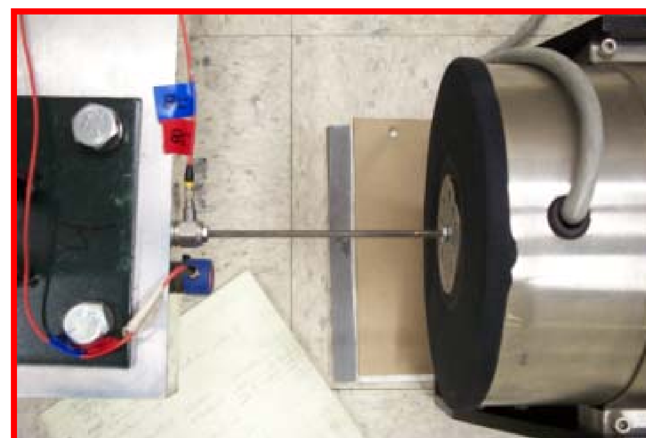


# Damage Classification using $\sigma(\varepsilon_y)/\sigma(\varepsilon_x)$

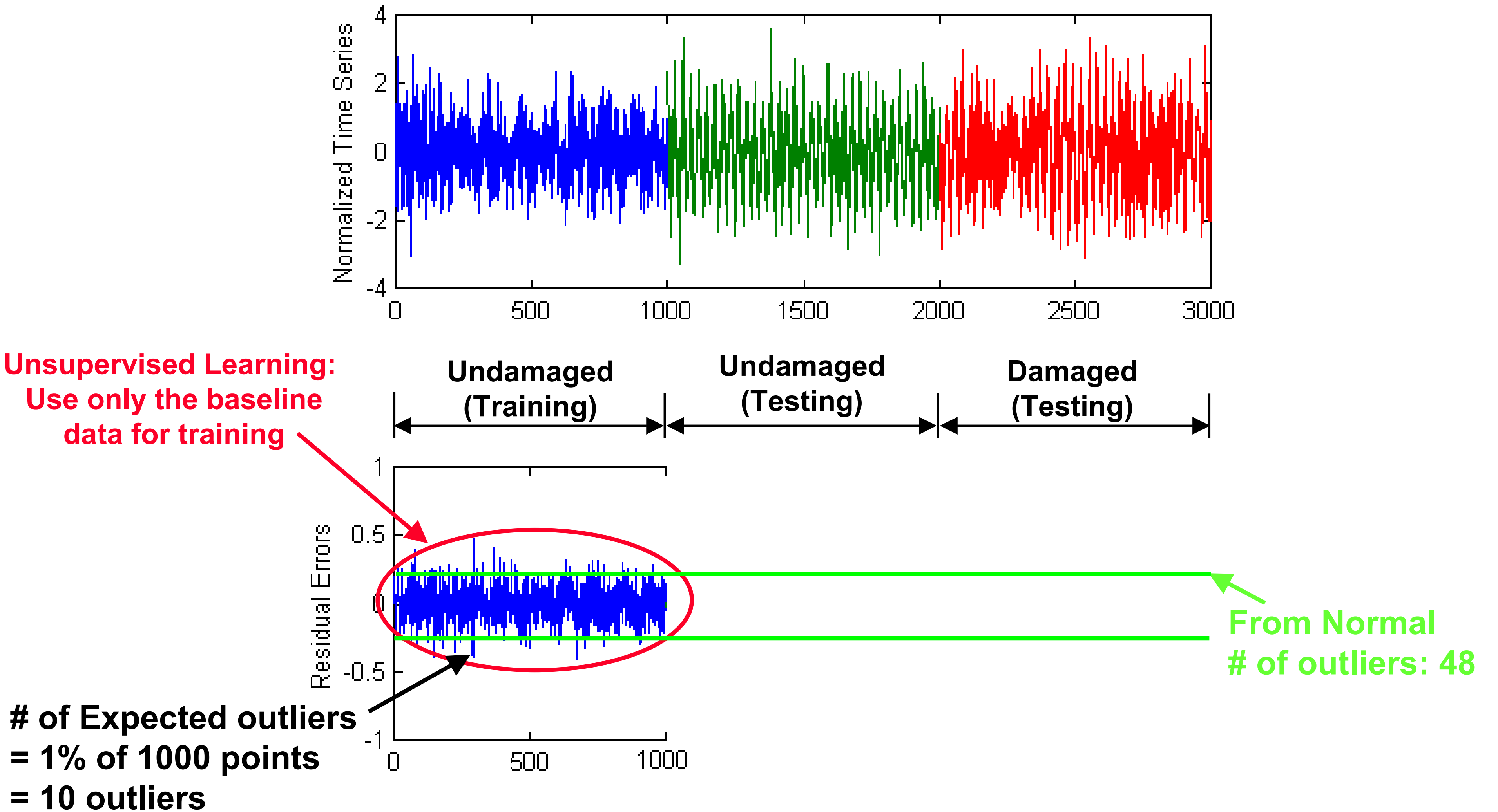


# A Moment Resisting Frame Structure Model

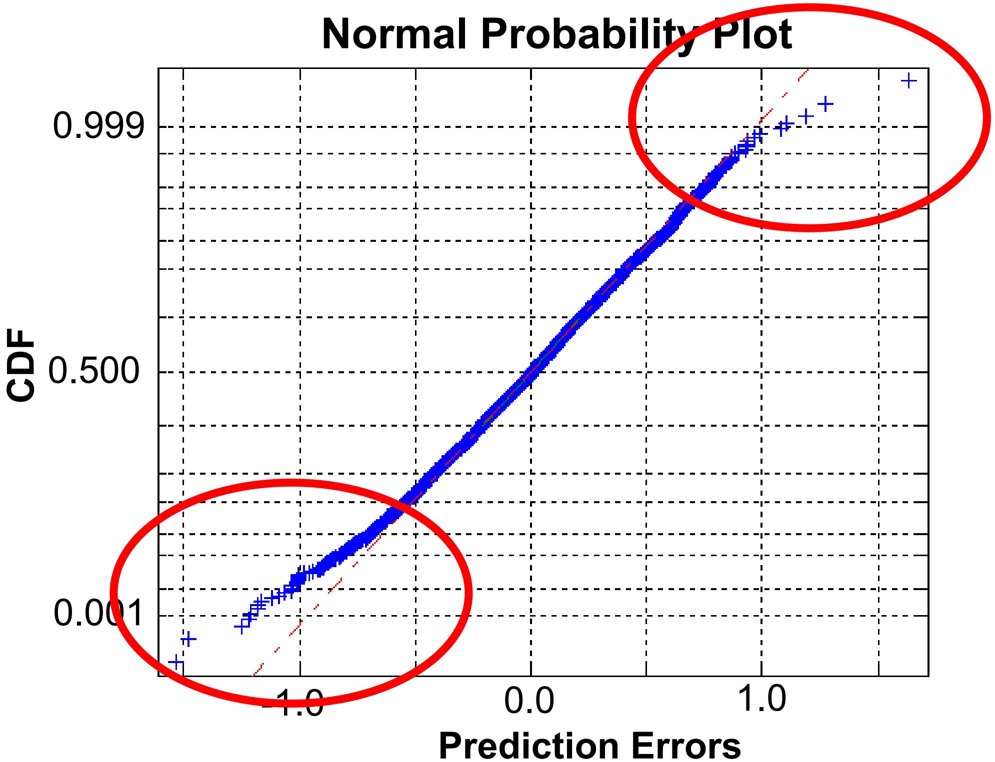


# Establishment of Decision Boundaries



# Normality Assumption of Data

- Let's look at the baseline prediction errors to see whether they have a normal distribution or not.
- A normal probability plot graphically assesses whether the data come from a normal distribution.
- If the data are normal, the plot will be linear. Otherwise, there would be curvature in the plot.
- The central population of data seems to fit to the normal distribution well, but the tails do not.



# Why Extreme Value Statistics ?

- In general, the distribution type of the parent data is unknown, and there are infinite numbers of candidate distributions.
- There are only three types of distributions for extreme (maximum or minimum) values regardless the distribution type of the parent data [Fisher and Tippett, 1928].
- That means, the model selection for the extreme values becomes much easier, because there are only three models to choose.(Gumbel, Weibull, Frechet distributions)

# Feasible Cumulative Density Functions for Maxima

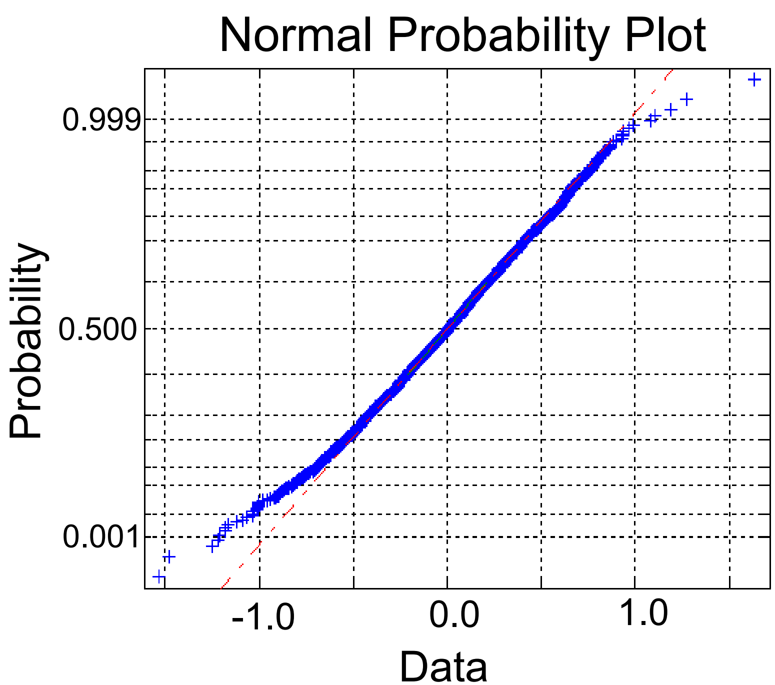
From Castillo [1988]:

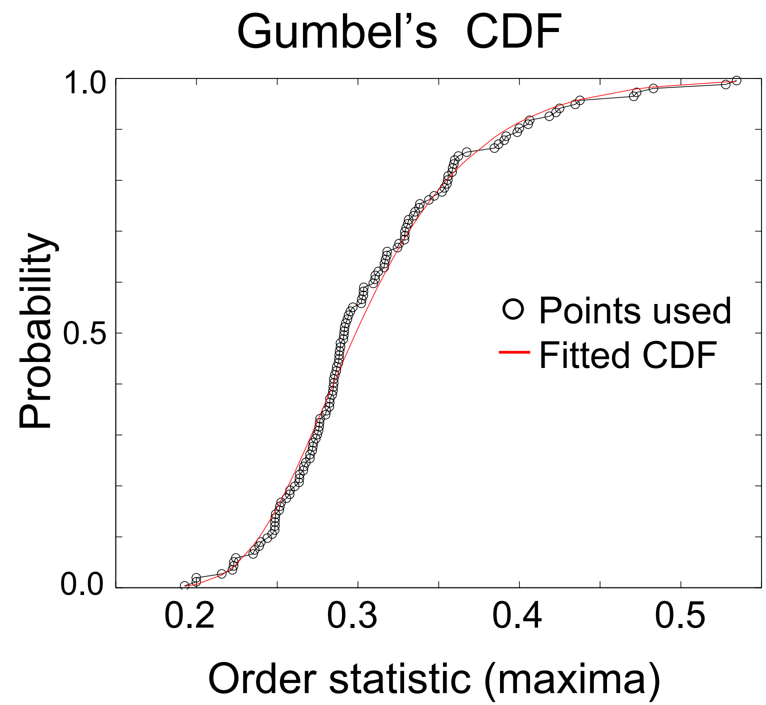
- Gumbel $$F(x) = \exp\left[-\exp\left(-\frac{x-\lambda}{\delta}\right)\right] \quad -\infty < x < \infty,\ \delta > 0$$

- Weibull: $$F(x) = \begin{cases} 1 & \text{if } x \geq \lambda \\ \exp\left[-\left(\frac{\lambda - x}{\delta}\right)^{\beta}\right] & \text{otherwise} \end{cases}$$

- Frechet: $$F(x) = \begin{cases} \exp\left[-\left(\frac{\delta}{x-\lambda}\right)^{\beta}\right] & \text{if } x \geq \lambda \\ 0 & \text{otherwise} \end{cases}$$

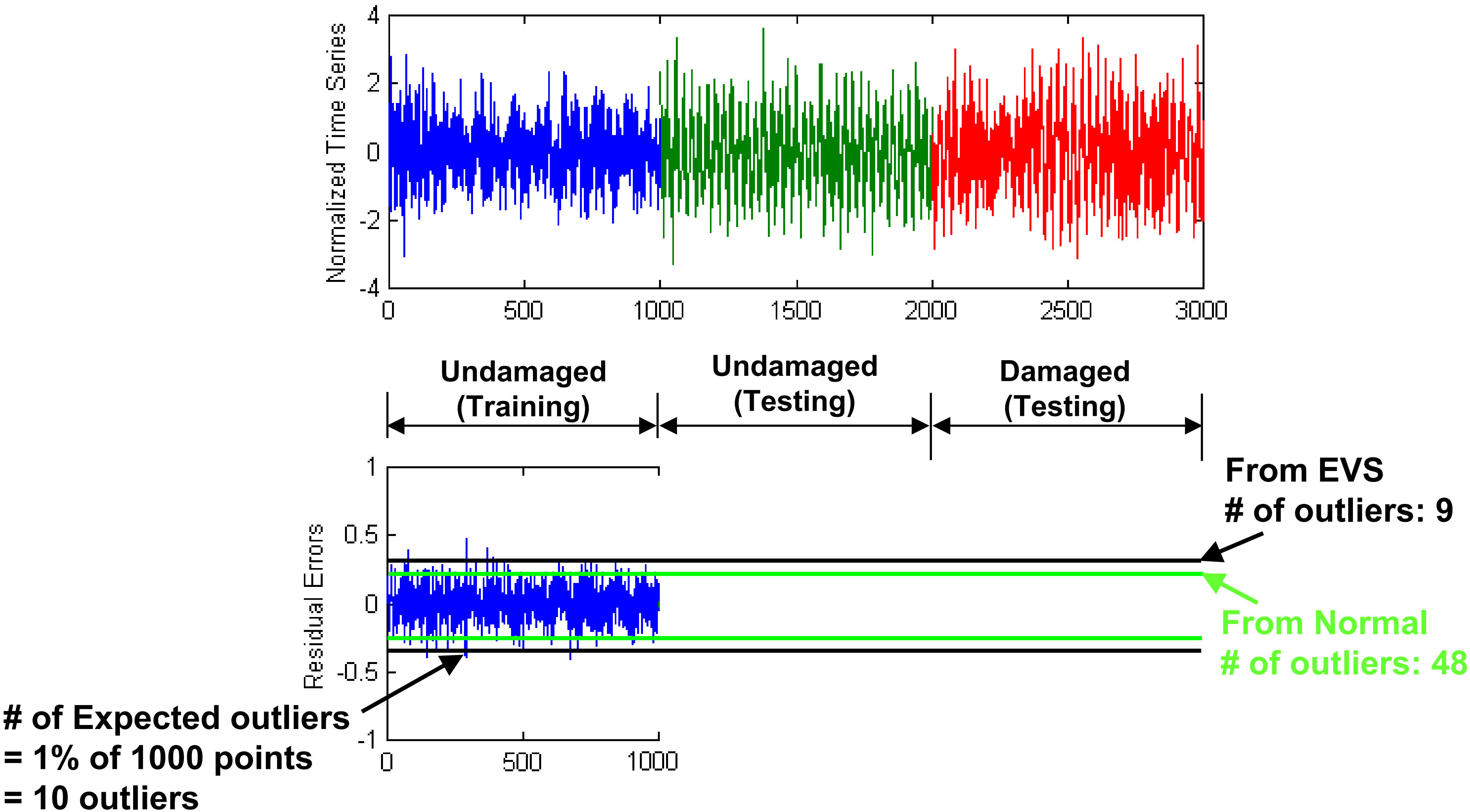
# Fitting to Gumbel Distribution



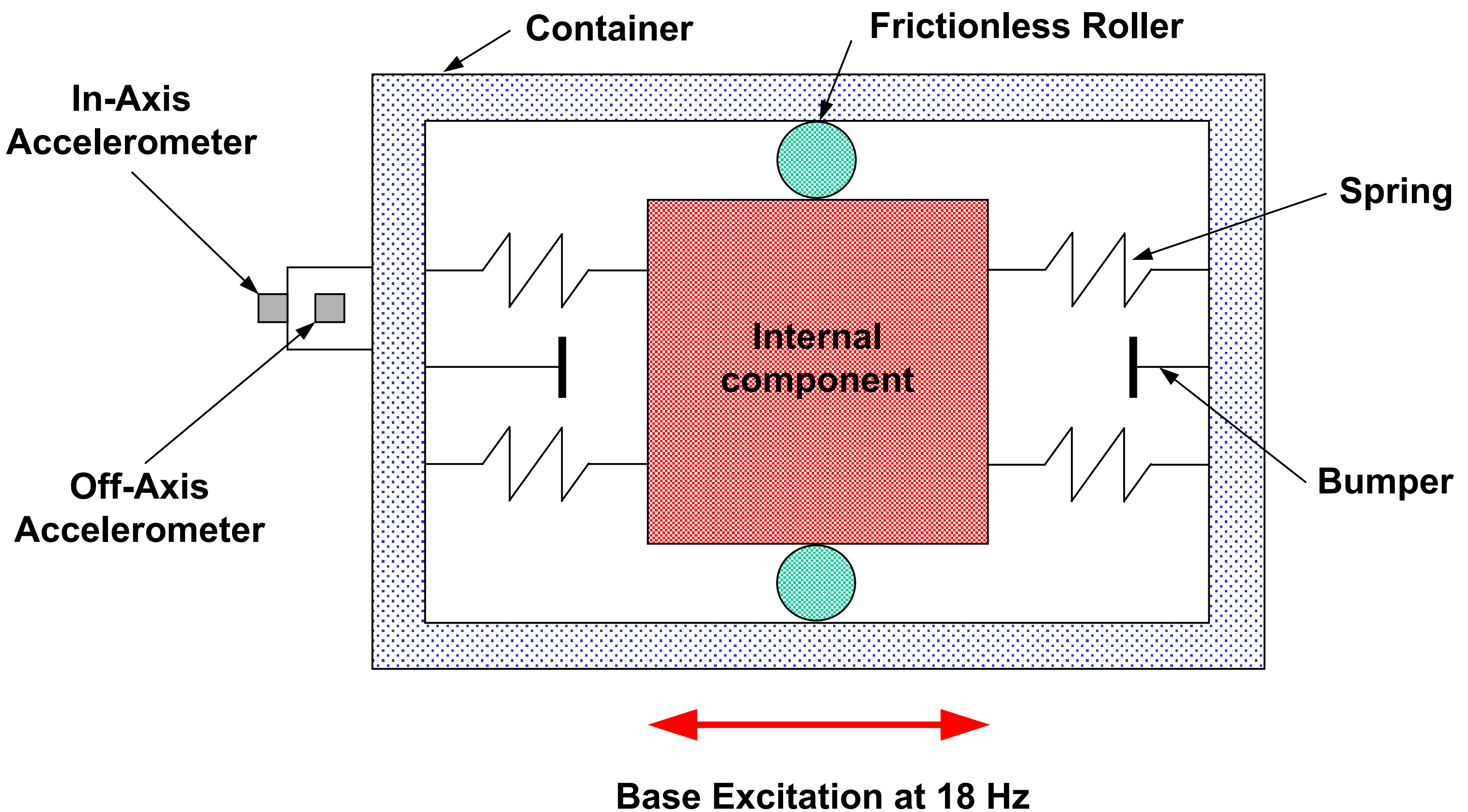




- Divide the original times series with 8192 data points into 128 time series with 64 points.
- Compute the maximum value from each block and fit the 128 maxima to a Gumbel distribution.

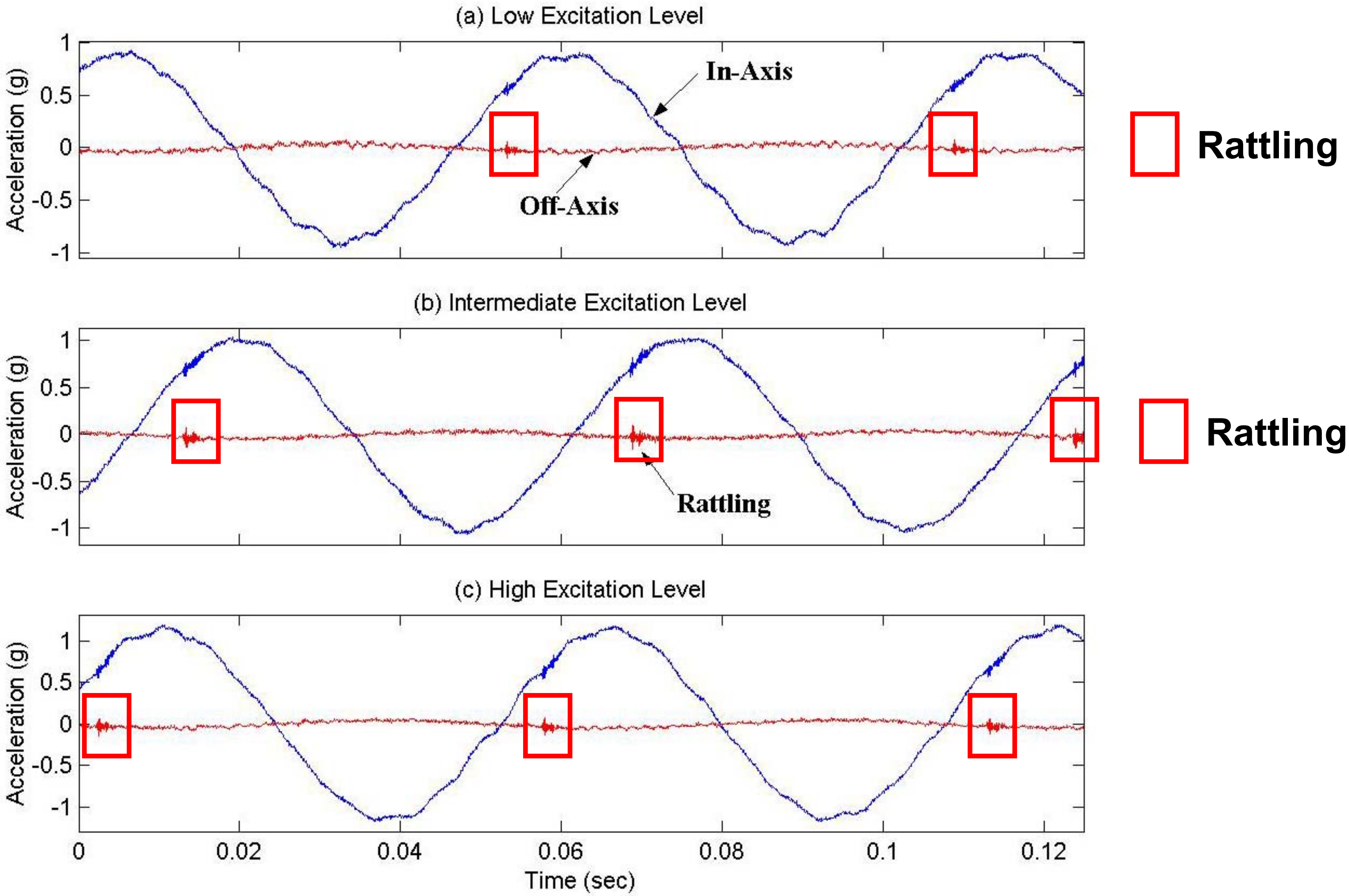
# Establishment of Decision Boundaries



# Detection of Rattling



# Acceleration Response



# Discontinuity Detection via Holder Exponent

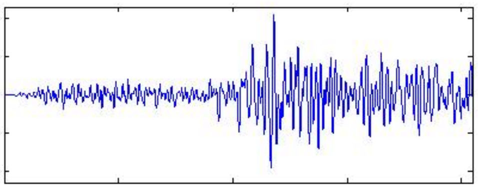
- Definition: The Holder Exponent is a measure of the regularity of the signal. The regularity of the signal is the number of continuous derivatives that the signal possesses.

- Objective: Identify discontinuity in signals that can be caused by certain types of damage.

- Application: Examples of damage that might induce discontinuity into the dynamic response signal include:
  - Opening and closing of cracks
  - A loose joint that is allowing contact (rattle) to occur

# Holder Exponent Analysis

**Time Signal**



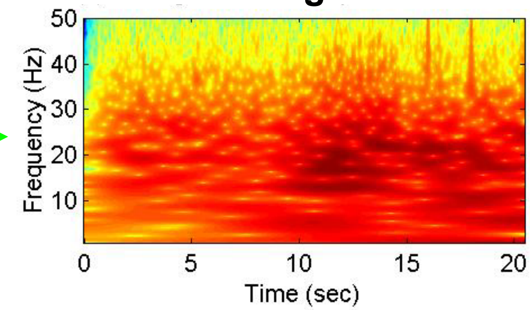
**Wavelet Transform**

$$Wf(u,s)=\int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{s}}\,\psi^*\left(\frac{t-u}{s}\right)dt$$

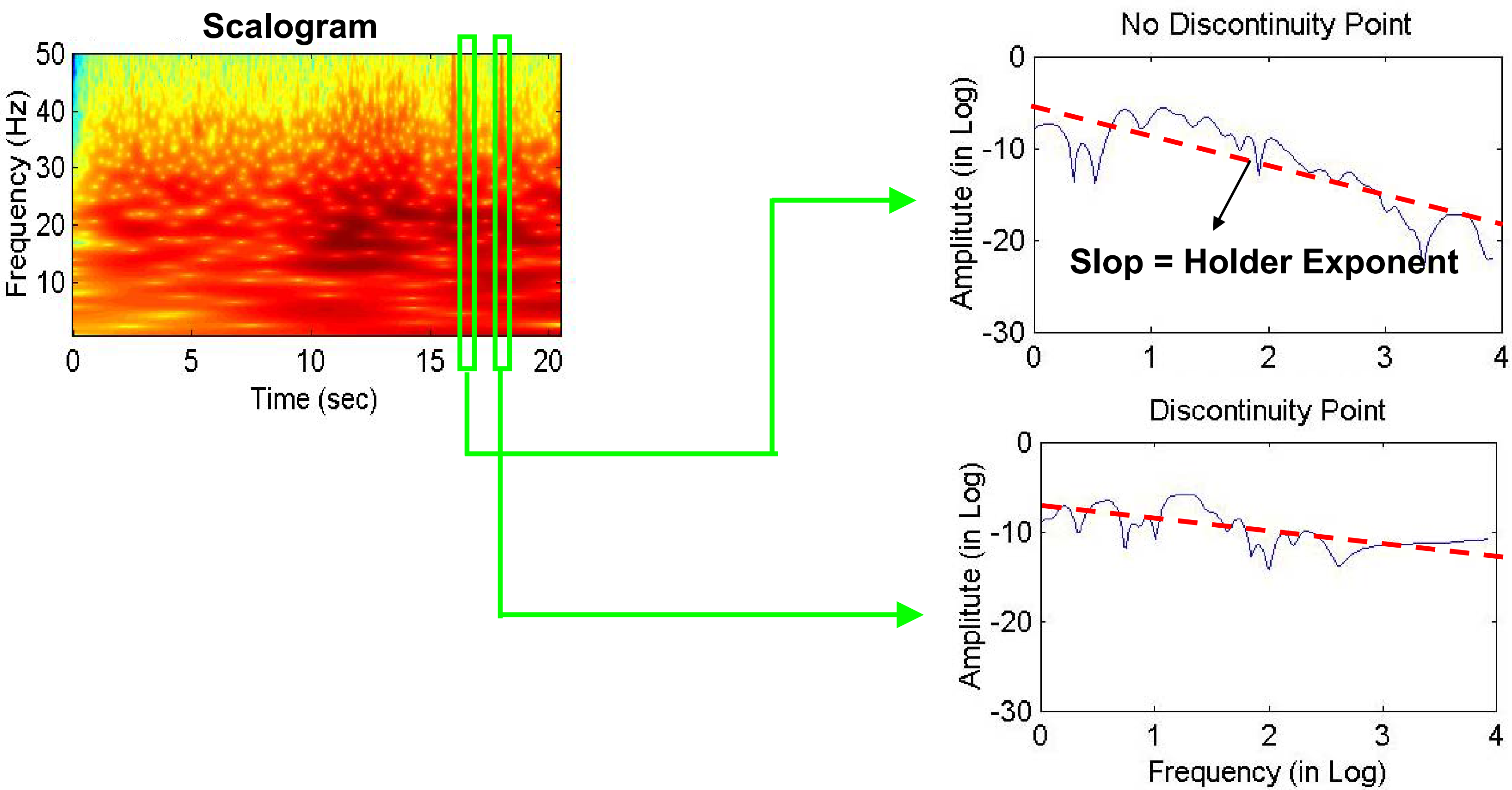
**Translation**

**Dilation**

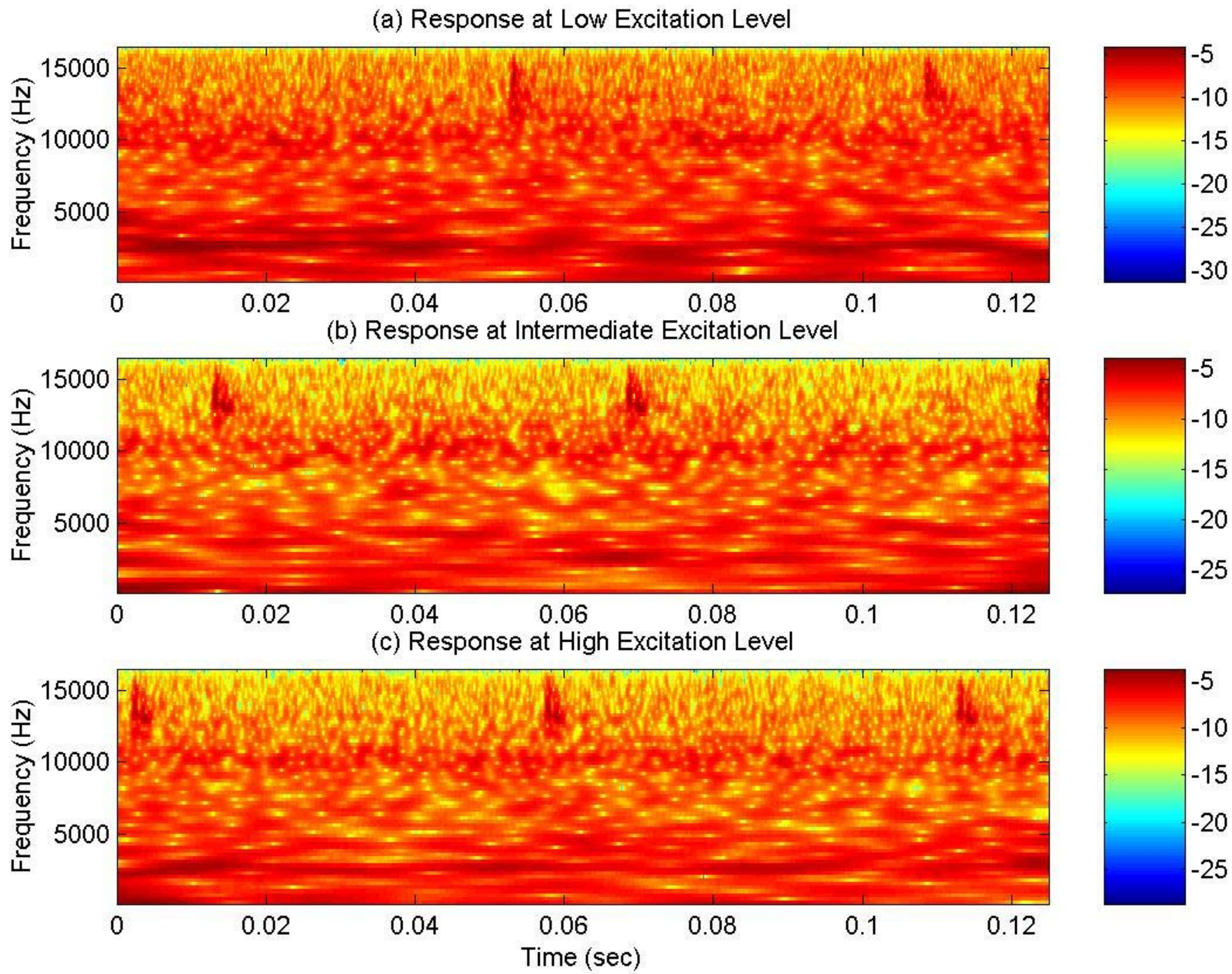
**Scalogram**



# Holder Exponent Analysis

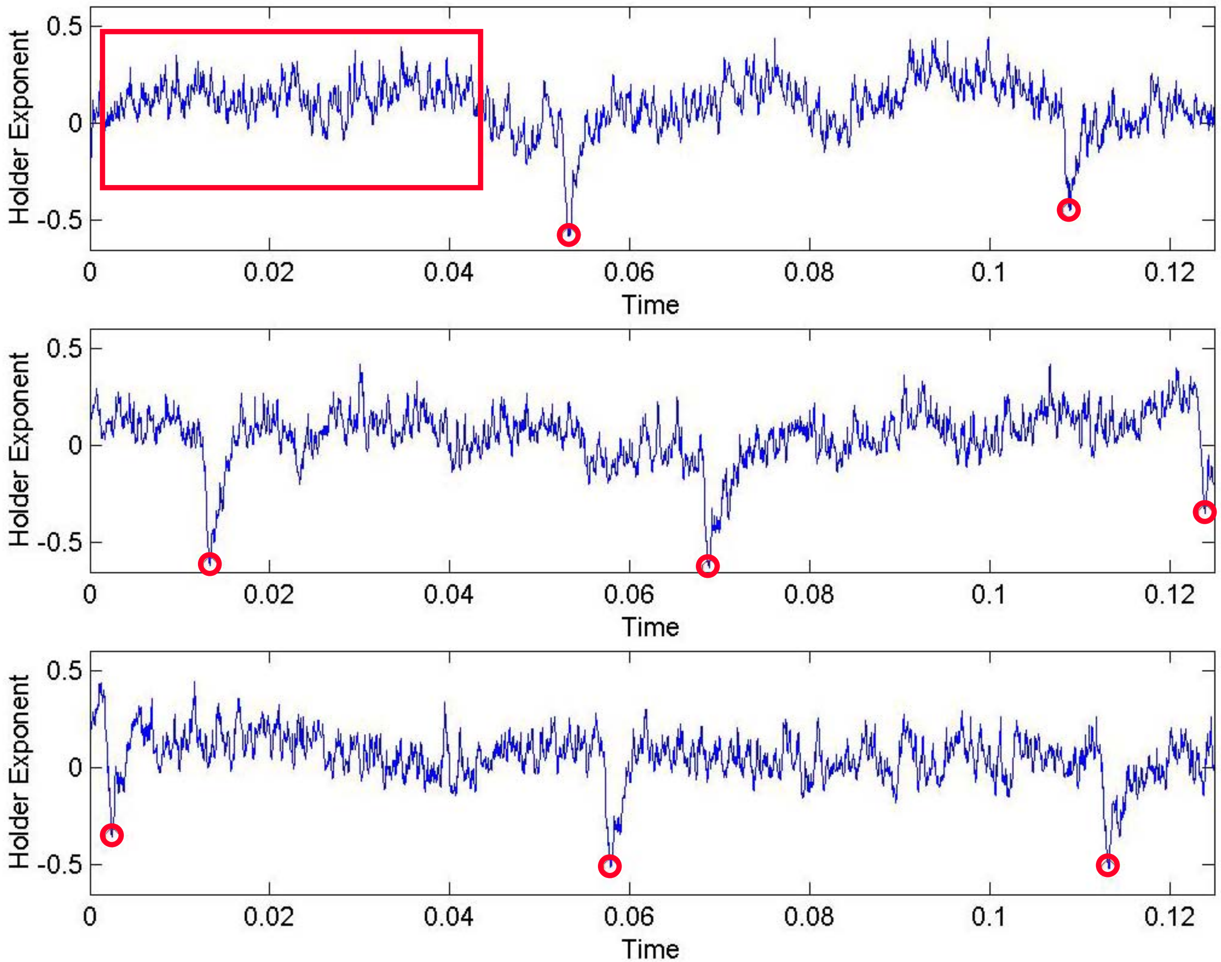


# Scalogram from Wavelet Transform



# Holder Exponent Analysis

(1) Find the max drop (2) Threshold = C x max drop (3) Discontinuity > Threshold



# Summary

- Cast structural health monitoring problems in the framework of statistical pattern recognition.
- Developed various signal-based damage detection algorithms.
- Embed damage detection algorithms into on-board microprocessors.
- Address data normalization issue explicitly.
- Decision making is based on rigorous statistical modeling.
- Provide a suite of data interrogation algorithms for structural health monitoring in the format of GUI software called DIAMOND II (patent pending).