Validation of Engineering Applications at LANL(*)

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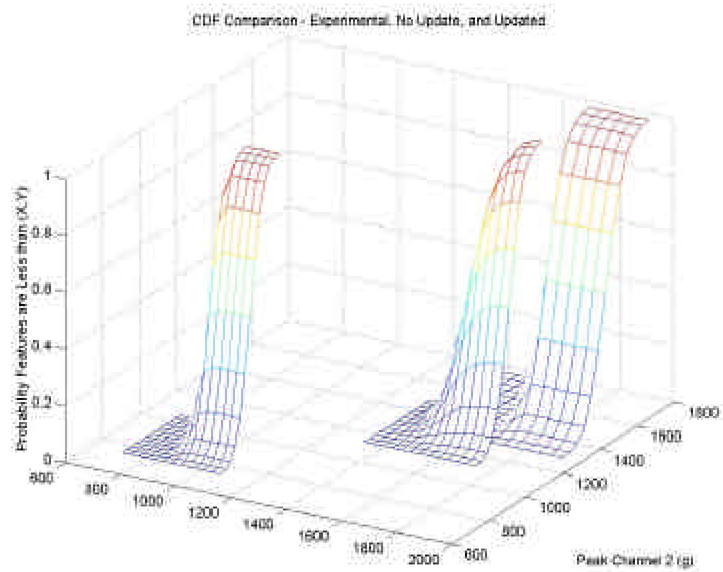

Los Alamos National Laboratory
Uncertainty Quantification Working Group
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OUTLINE

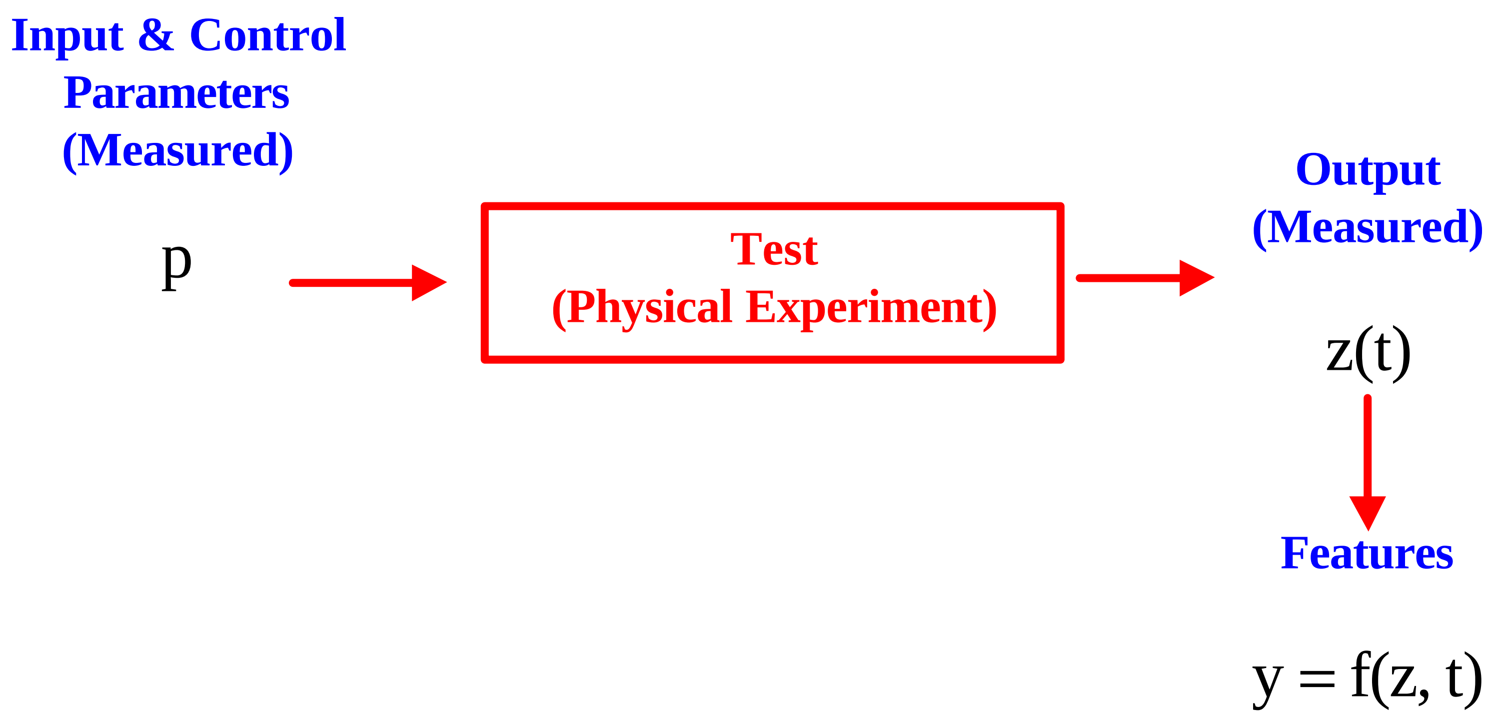
- **Notation & Definition**
- **Motivation**
- **Impact Experiment (Development of the Methodology)**
- **Forward Mount Impulse Experiment (ASCI Demonstration)**
- **Unresolved Issues & Challenges**

DEFINITION

- **Test = Physical Experiment**
- **Model = Numerical Experiment**
- **Meta-model = Fast-running, Statistical Model**
- **Feature = Quantity Synthesized From the Experiment's Output**
- **Test-analysis Correlation = Definition of a Residue or "Distance"**
- **Cost Function = Residue Expressed in a Particular Metric**
- **… Should We Define a Common Set of Notations & Definitions?**

TEST

(DEFINITION 1 OF 4)



MODEL

(DEFINITION 2 OF 4)

Input Parameters
(Fixed to a Value)

p →

Model
(Numerical Experiment)

$$z(t) = M(p, t)$$

→ **Output**
(Predicted)

$z(t)$

↓

Features

$$y = f(z, t)$$

META-MODEL

(DEFINITION 3 OF 4)

Input Parameters
(Fixed to a Value)

p

Model
(Full physics, Expensive)

$$z(t) = M(p, t)$$

Output
(Predicted)

$z(t)$

Meta-Model
(Fast-running Model)

$$y = M^{meta}(p) + e$$

Features

$$y = f(z, t)$$

TEST-ANALYSIS CORRELATION

(DEFINITION 3 OF 4)

- **Definition of a Residue or "Distance":**

Test or Physical Experiment → **Test Features** y^{test}

Analysis Features $y(p)$ ← **Analysis or Numerical Experiment**

Test-analysis Comparison

$$R(p) = y^{test} - y(p)$$

- **Definition of a Metric or "Norm":**

$$J(p) = \sum_{j=1 \cdots N_{data}} \{R(p)\}^H \left[S_{RR_j}\right]^{-1} \{R(p)\} + \{p\}^H \left[S_{pp}\right]^{-1} \{p\}$$

DEFINITIONS NEEDED?

- **How is *Uncertainty Quantification* Defined?**
- **What Constitutes a *Validated* Numerical Model?**
- **Does it Mean That it Adequately Captures the Physics *and* the Sources of Variability/Uncertainty?**
- **Do We Agree on the Difference Between *Validation* and *Verification*?**
- **…**

WHAT IS UNCERTAINTY QUANTIFICATION?

- **Characterization of Sources & Scenarios**
  - ✓ What is Uncertain?
  - ✓ What Varies?
  - ✓ How?
- **Forward Propagation**
  - ✓ Sampling Techniques;
  - ✓ Fast Probability Integration.
- **Statistical Effects Analysis**
  - ✓ Sensitivity Study;
  - ✓ Correlation Study.
- **Characterization of the Output**
  - ✓ Joint Probability Density Functions;
  - ✓ Higher-order Statistical Moments.
- **Optimization**
  - ✓ Statistical Parameter Estimation;
  - ✓ Model Improvement.
- **Characterization of "Rare" Events**
  - ✓ Reliability;
  - ✓ Tails of the Statistical Distributions.

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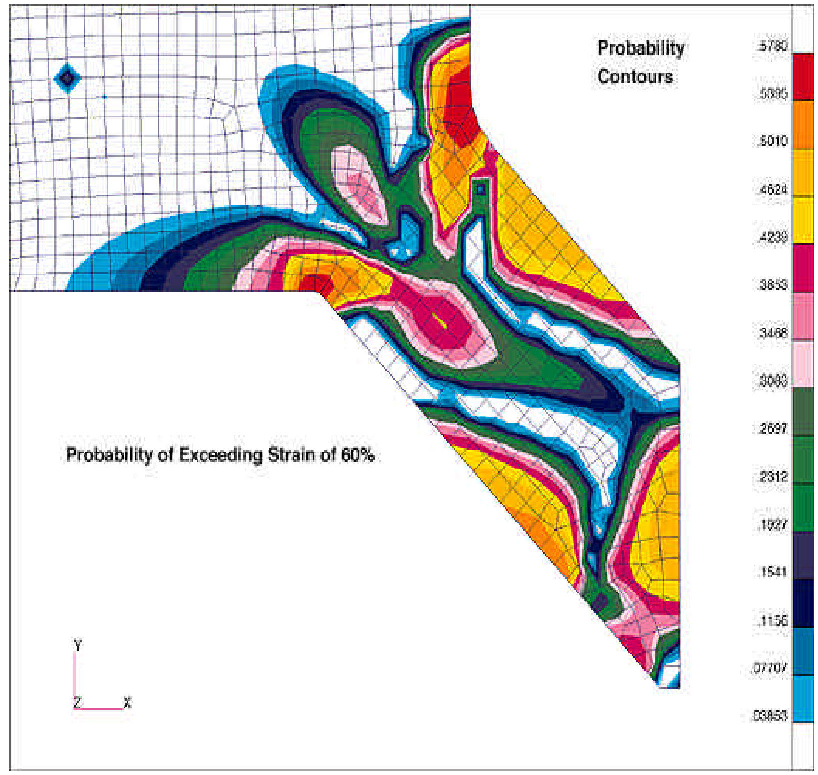
- Unresolved Issues & Challenges

ASCI — ENGINEERING APPLICATIONS

- **Two Main Objectives:**

✓ **Model and Predict Engineering Weapon System Performance in Normal and Hostile Stockpile-to-target Sequence (STS) Environments.**

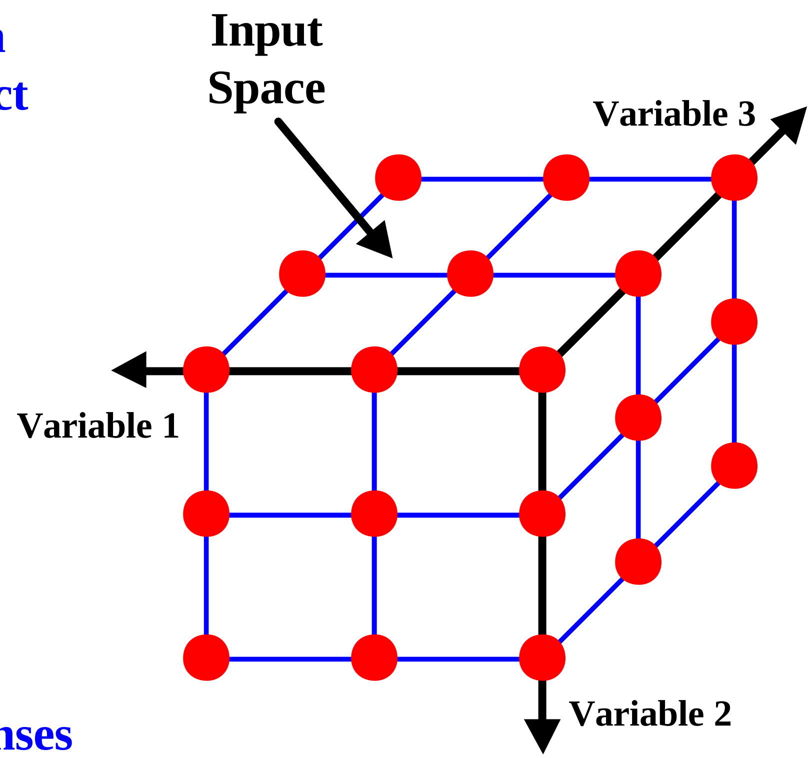
✓ **Predict the Deformed State (Geometric, Structural, Thermal, Material Damage) of the Warhead at Detonation.(*)**



(*) Characterization of the Deformed State at Detonation is the "Initial & Boundary Condition" for the Physics Designers.

ASCI — ENGINEERING APPLICATIONS

- **15 Milli-seconds of Weapon Response Time After Impact**
- **30,000 Elements;**
  ~ 1.5 Million DOFs;
  100 Contact Pairs;
  10 Hours/Simulation
- **$(25)^3$ = 15,625 Simulations**
- **31 x 128 = 3,968 Processors**
- **6,000 Abaqus/Explicit Licenses**



- **17.8 Years of Equivalent Single-processor Computing in Just 72 Hours!**

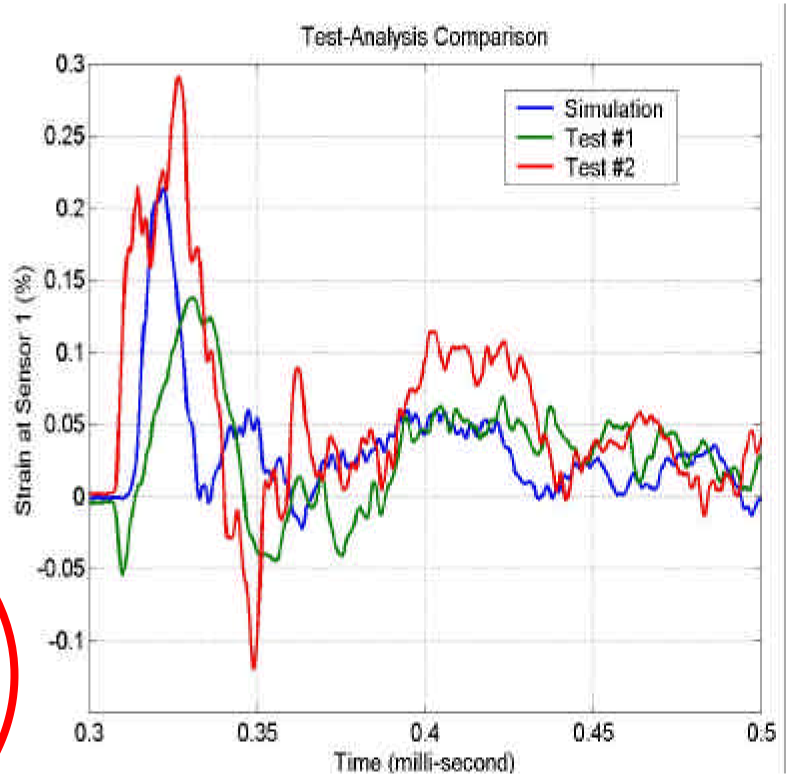
WHERE DO WE GO FROM THERE?

- Current Achievements …
  - ✓ Complex Engineering Applications Can be Numerically Simulated.
    - *ASCI Software/Hardware Platforms.*
  - ✓ Stochastic Inputs Can be Propagated Through Forward Computations.
    - *Fast Probability Integration.*
- Next …
  - ✓ Can We do it More Efficiently?
  - ✓ Can We Assess the Predictive Quality of Our Models?

PREDICTABILITY OF NUMERICAL MODELS

- **Candid Approach:**

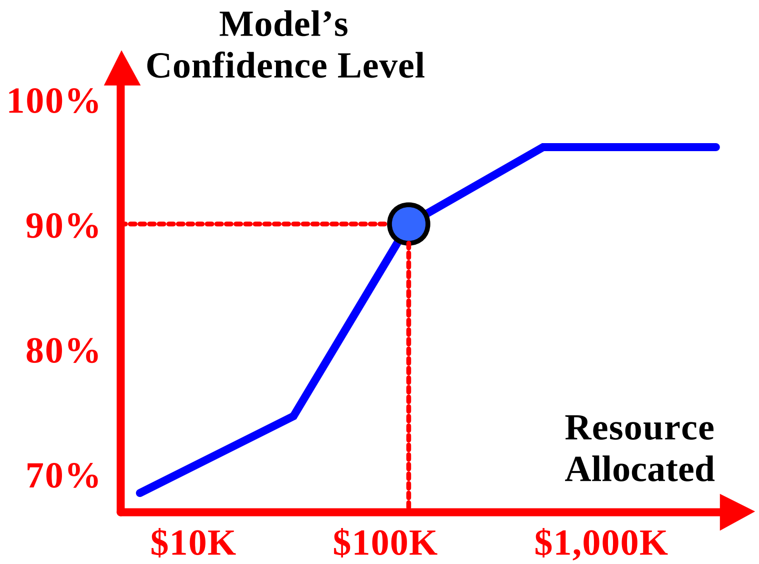
  "My Model is Valid Because it Reproduces My Test Data With Adequate Accuracy…"



- ✓ **"How Good is Good Enough?"**
- ✓ **No Probabilistic Confidence.**
- ✓ **Inappropriate For Analyzing Statistically Accurate Models.**

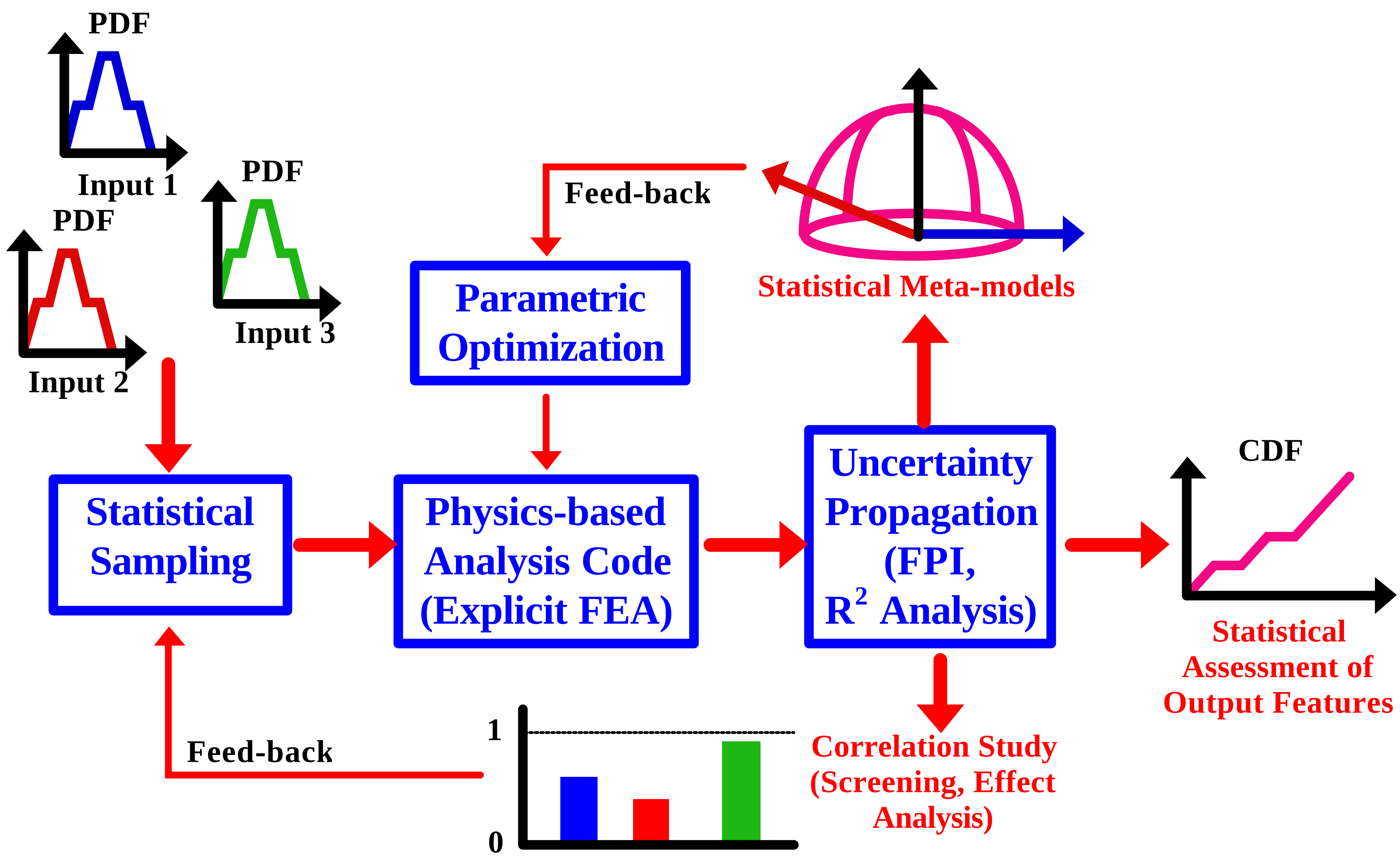
PARADIGM FOR MODEL VALIDATION

- **Are the Physical Experiments and Numerical Simulations Statistically Consistent?**



- **What is the Degree of Confidence Associated With the First Answer?**
- **If Additional Data Sets Are Available, by How Much Does the Confidence Increase?**

STEPS OF UNCERTAINTY QUANTIFICATION



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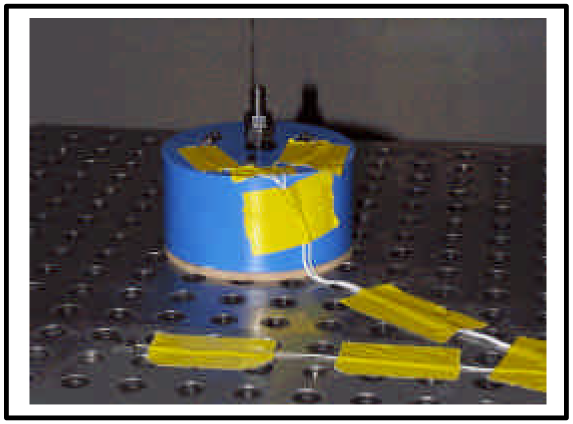
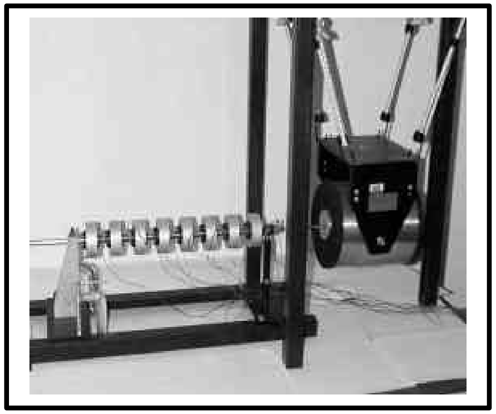
OVERVIEW OF MODEL VALIDATION (ESA-EA)

Testing of the Full-scale, Integrated System

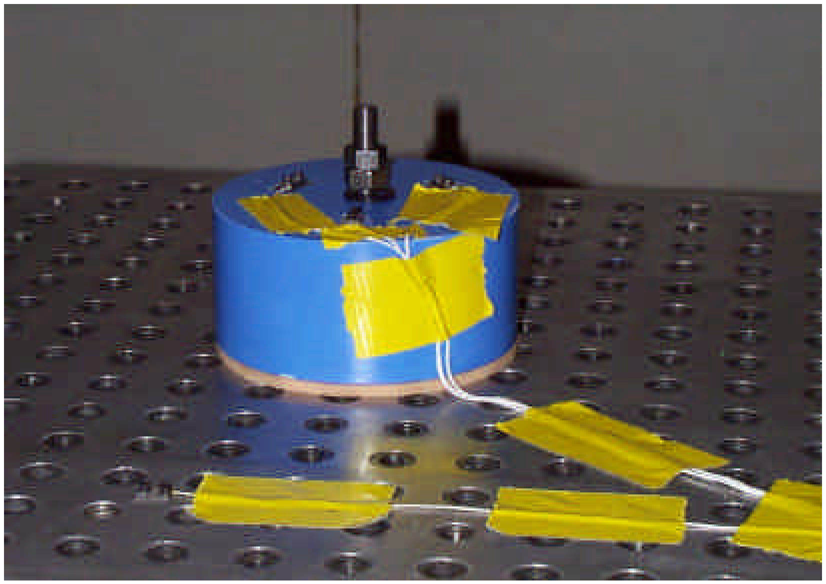


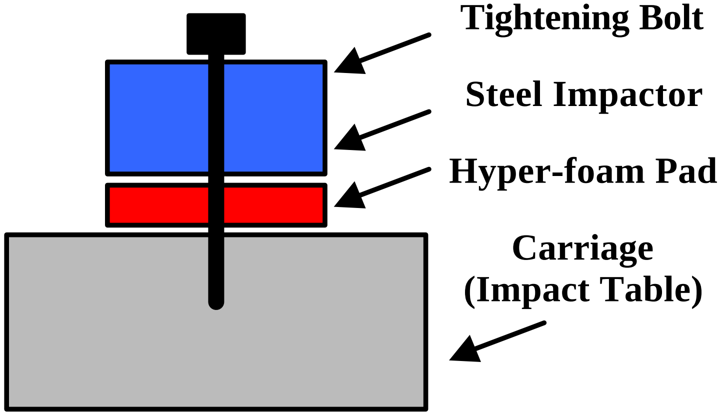
Sub-component Testing
(Validation of a Hyper-elastic Model For Low-level Impact)

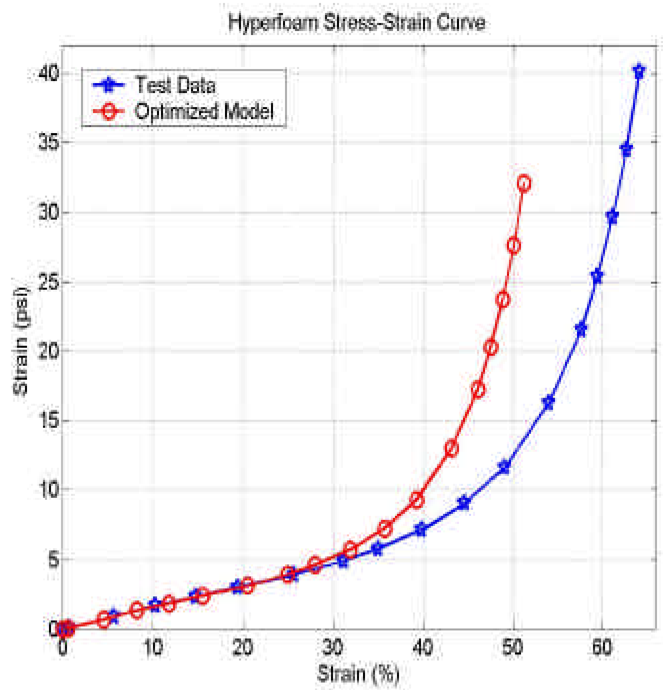


Phenomenological Testing
(Validation of a Friction Model For Nonlinear Vibrations)

LANL IMPACT EXPERIMENT

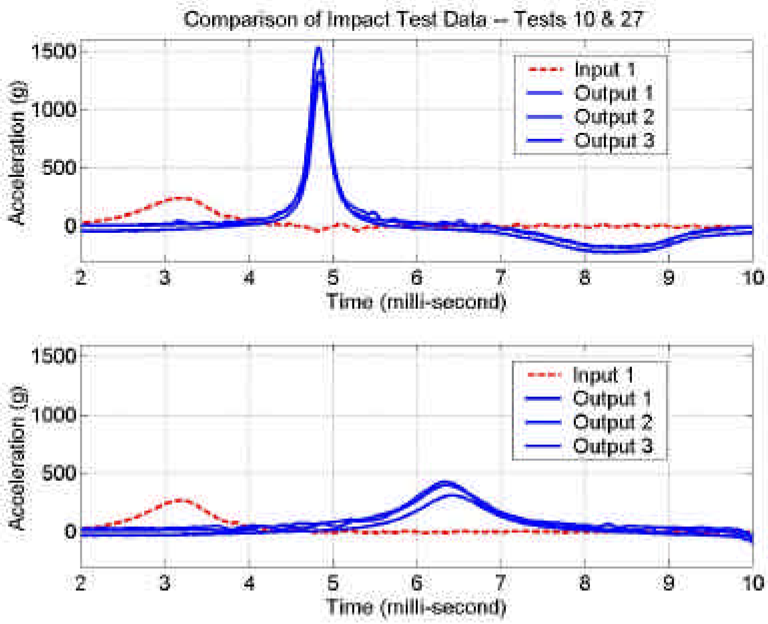


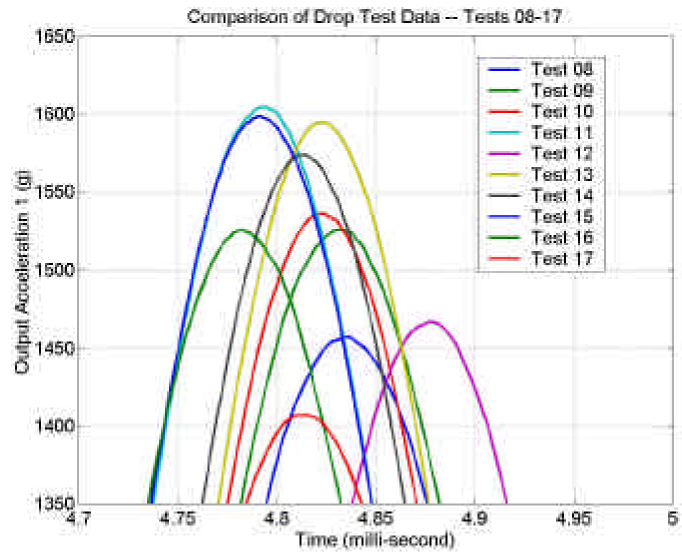




EXPERIMENTAL VARIABILITY

(LANL Impact Test)

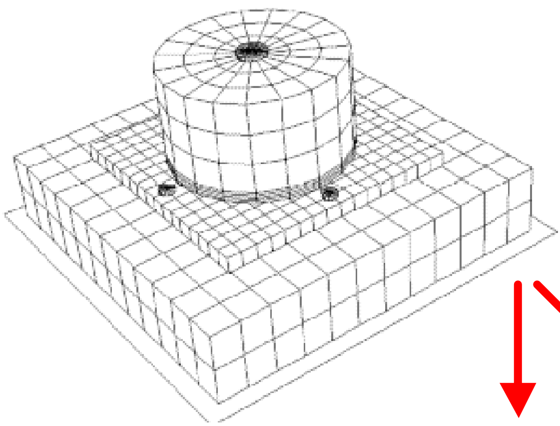




Number of Data Sets Collected	Low Velocity Impact (13 in./0.3 m Drop)	High Velocity Impact (155 in./4.0 m Drop)
Thin Layer (0.25 in./6.3 mm)	10 Tests	5 Tests
Thick Layer (0.50 in/12.6 mm)	10 Tests	5 Tests

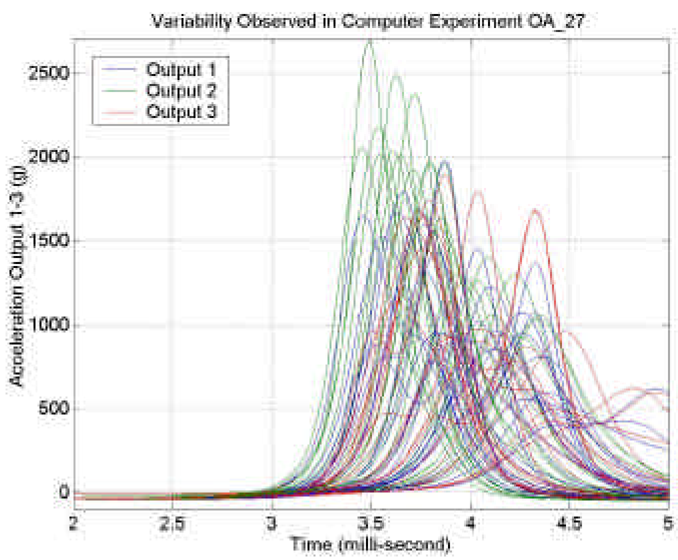
VARIABILITY OF THE NUMERICAL EXPERIMENTS

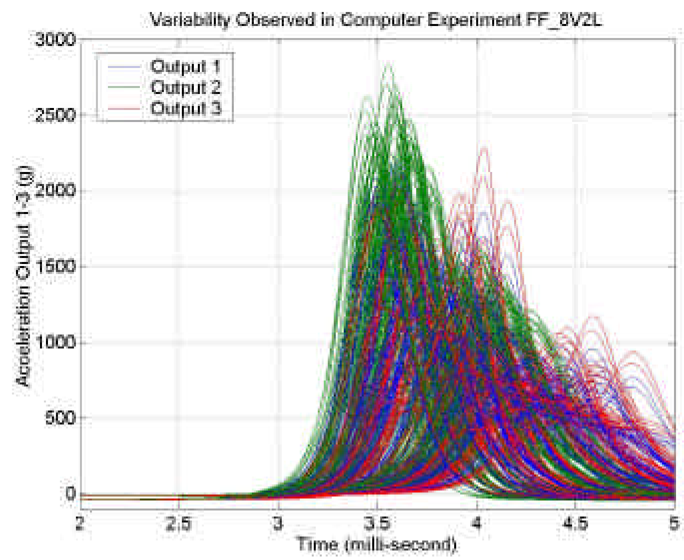
(LANL Impact Test)



- Angles of Impact (2)
- Bolt Pre-load (1)
- Material (2)
- Input Scaling (1)
- Friction (1)
- Bulk Viscosity (1)

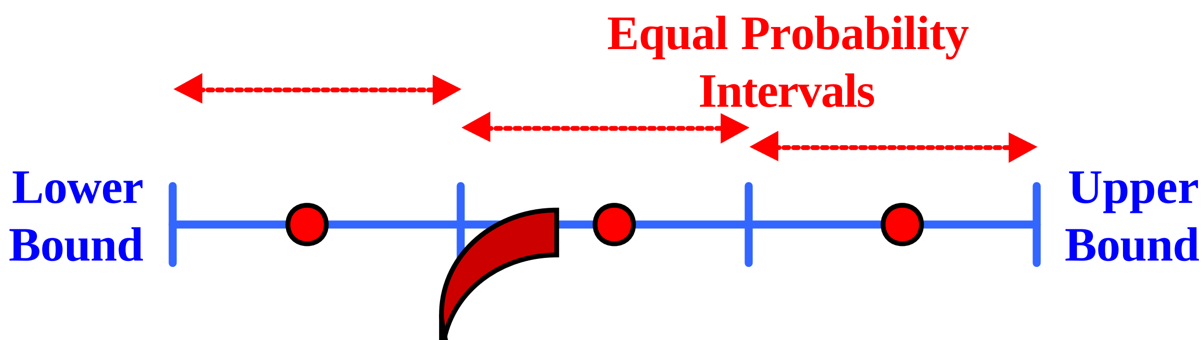
8 Variables





DESIGN OF COMPUTER EXPERIMENTS

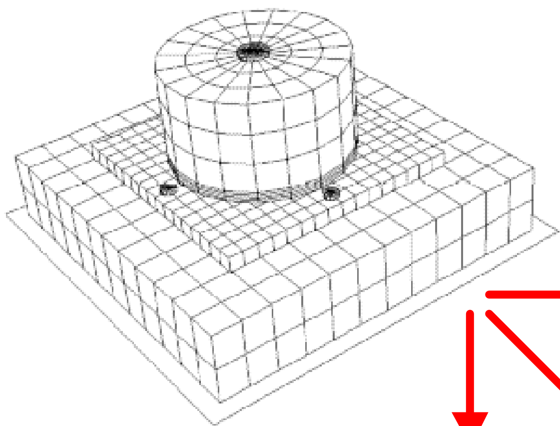
(LANL Impact Test)

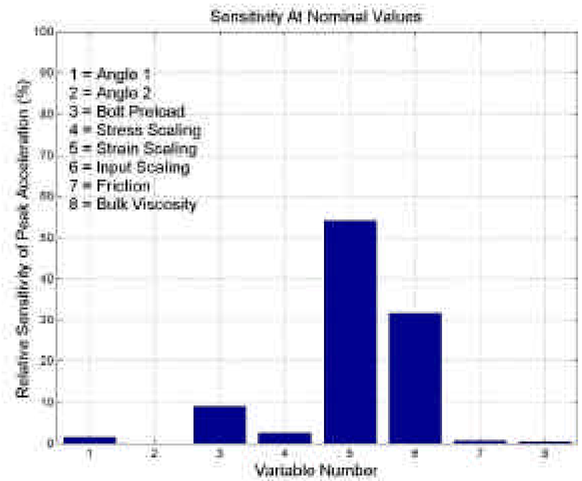


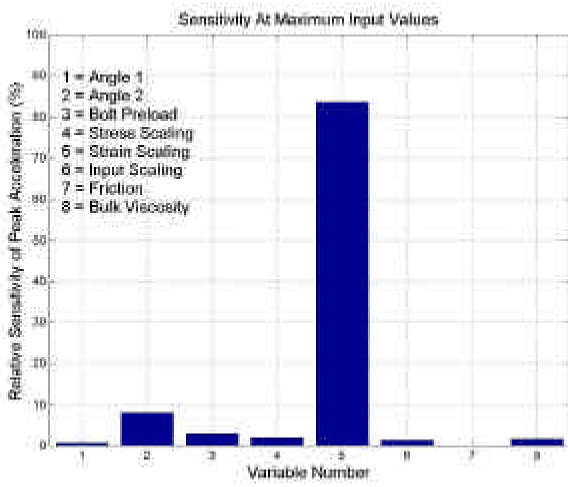
Variable	Interval 1	Interval 2	Interval 3
1	(0.0; **0.167**; 0.333)	(0.333; **0.5**; 0.667)	(0.667; **0.883**; 1.0)
2	(0.0; **0.167**; 0.333)	(0.333; **0.5**; 0.667)	(0.667; **0.883**; 1.0)
3	(0.0; **83.3**; 167.0)	(167.0; **250.0**; 333.3)	(333.3; **416.7**; 500.0)
4	(0.8; **0.867**; 0.933)	(0.933; **1.0**; 1.067)	(1.067; **1.133**; 1.2)
5	(0.8; **0.833**; 0.867)	(0.867; **0.9**; 0.933)	(0.933; **0.967**; 1.0)
6	(0.9; **0.933**; 0.967)	(0.967; **1.0**; 1.03)	(1.03; **1.07**; 1.1)
7	(0.0; **0.167**; 0.333)	(0.333; **0.5**; 0.667)	(0.667; **0.883**; 1.0)
8	(0.0; **0.167**; 0.333)	(0.333; **0.5**; 0.667)	(0.667; **0.883**; 1.0)

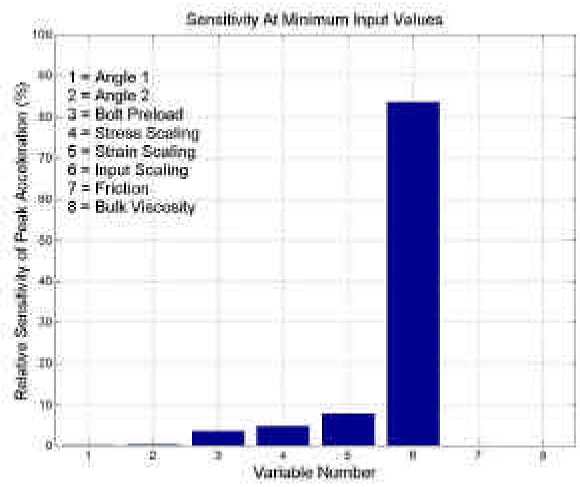
LOCAL SENSITIVITY STUDY

(LANL Impact Test)







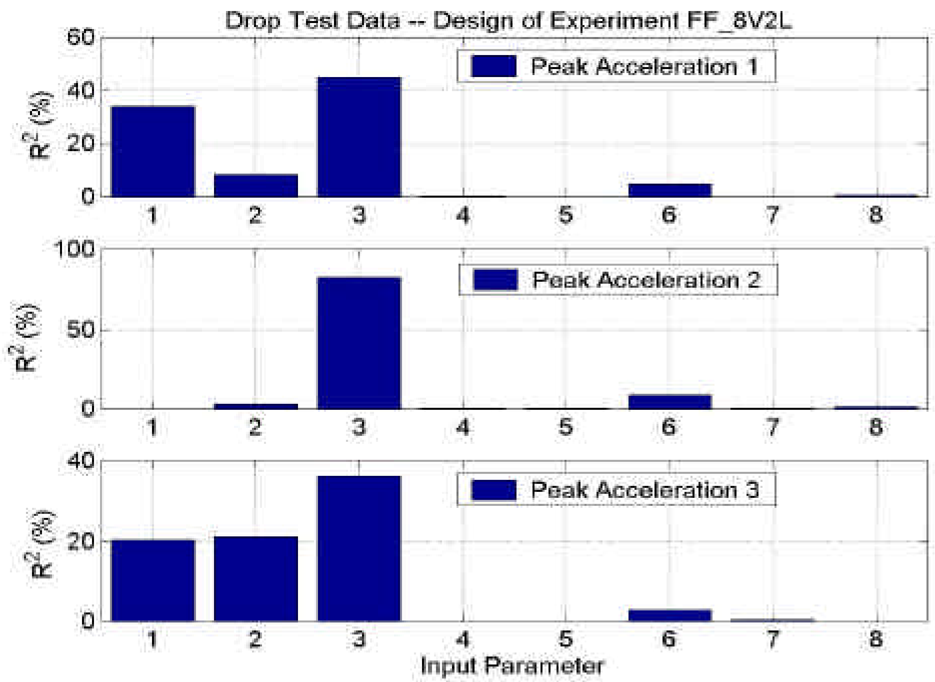


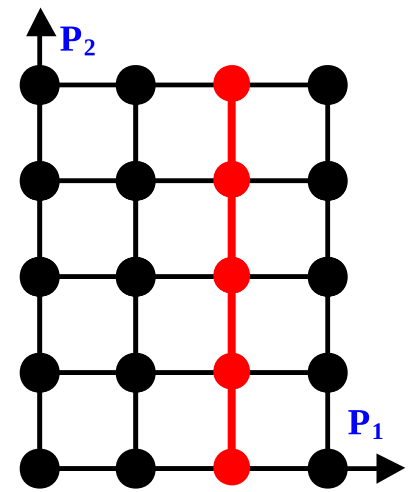
STATISTICAL EFFECTS ANALYSIS

(LANL Impact Test)

- **Which Variables or Combinations of Variables Best Explain the Total Variability?**

$$R^2 = 1 - \frac{\sum_{l=1\cdots N_{level}} \sum_{j=1\cdots N_{data}^{(l)}} \left(y_j^{(l)} - \bar{y}^{(l)}\right)^2}{\sum_{j=1\cdots N_{data}} \left(y_j - \bar{y}\right)^2}$$

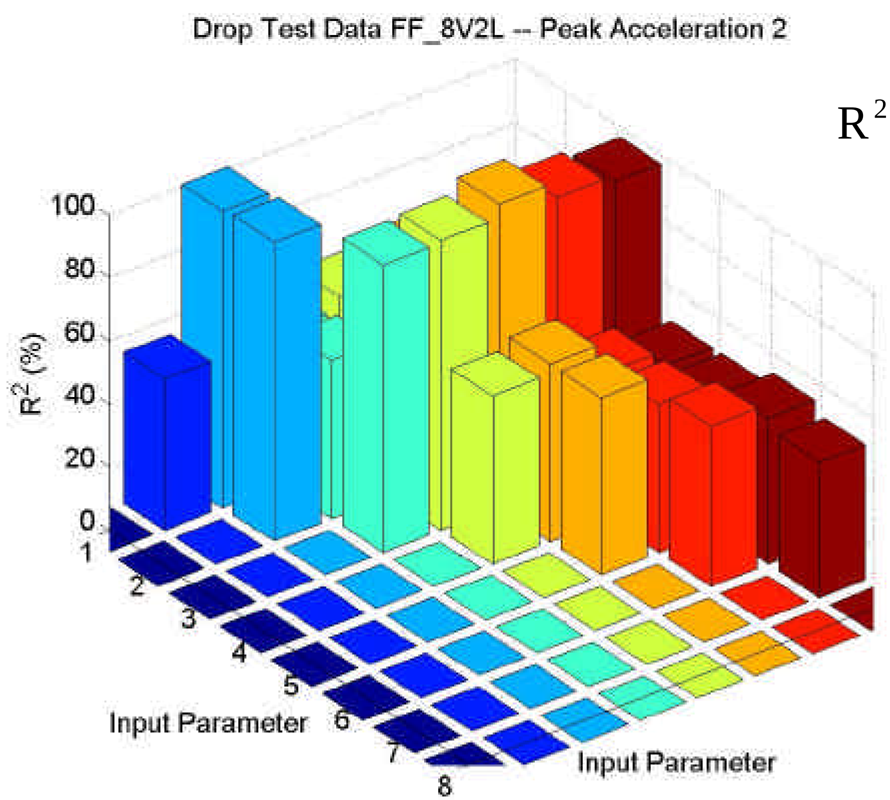




R^2 Analysis With Design of Experiments FF_8V2L (Peak Accelerations 1-3)

STATISTICAL EFFECTS ANALYSIS

(LANL Impact Test)



$$R^2 = 1 - \frac{\sum_{l=1\cdots N_{level}} \sum_{j=1\cdots N_{data}^{(l)}} \left(y_j^{(l)} - \bar{y}^{(l)}\right)^2}{\sum_{j=1\cdots N_{data}} \left(y_j - \bar{y}\right)^2}$$

R^2 Analysis With Design of Experiments FF_8V2L (Peak Acceleration 2)

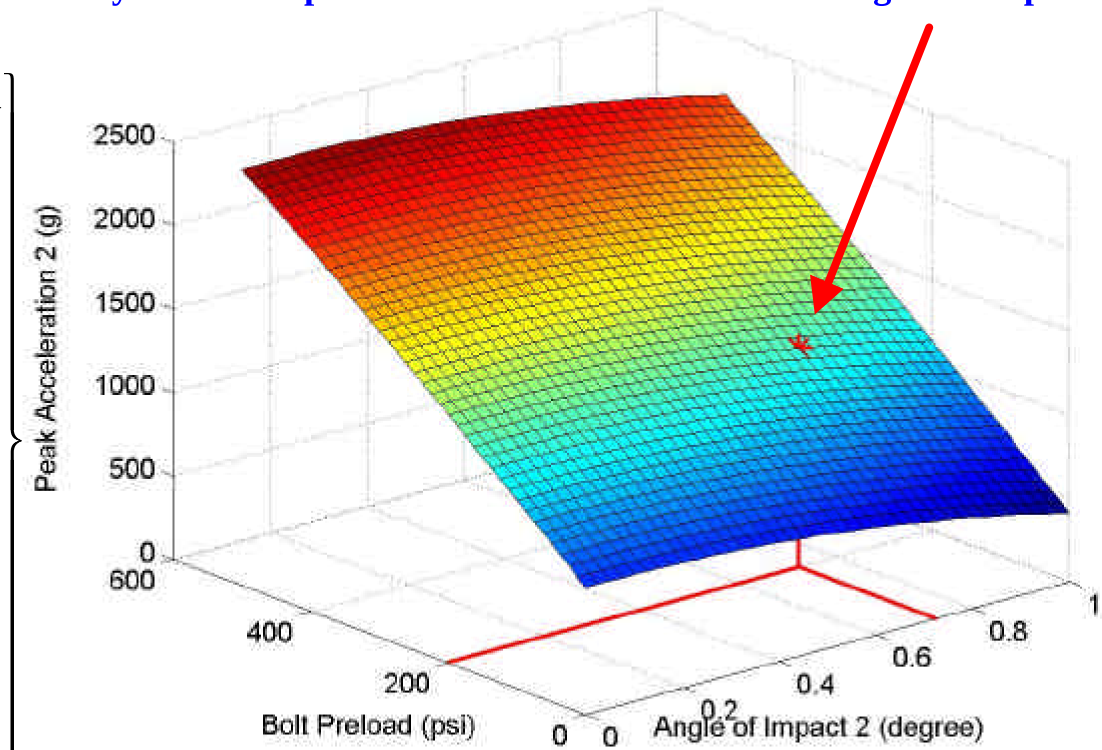
FAST RUNNING MODELS

(LANL Impact Test)

Quadratic Model Inferred From an Orthogonal Array of 81 Computer Runs

Most Probable Acceleration Measured During the Drop Tests

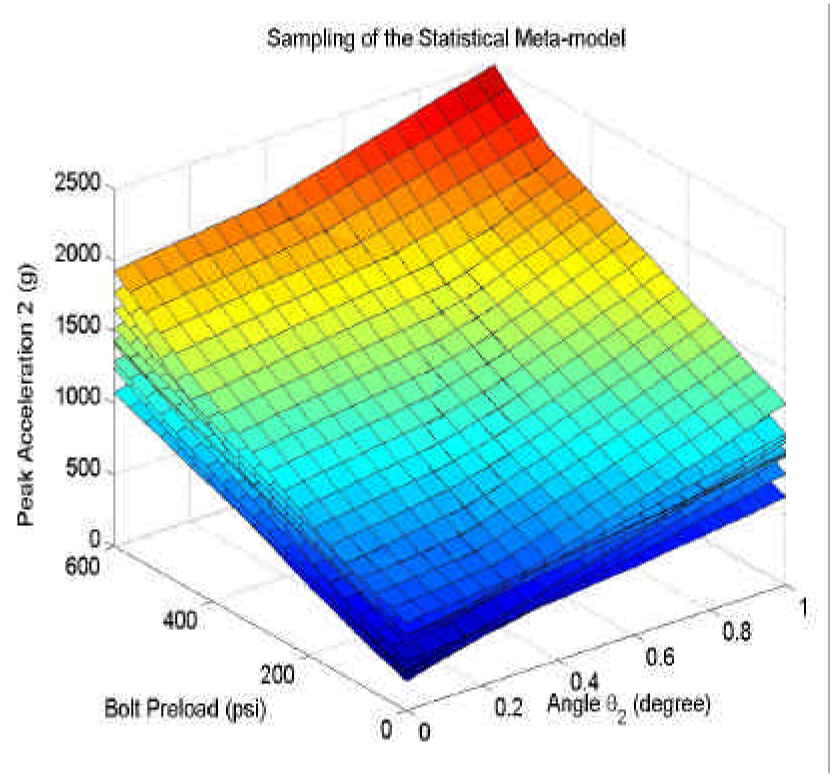
$$\ddot{x}_2^{peak} = \begin{Bmatrix} -1{,}538.2 \\ 43.6 \\ 288.4 \\ 2.4 \\ 2{,}552.8 \\ -391.3 \\ -307.1 \\ -0.0006 \\ 665.7 \\ -0.5 \\ -452.4 \\ 1.5 \end{Bmatrix}^T \begin{Bmatrix} 1 \\ a_1 \\ a_2 \\ P_{bolt} \\ s_I \\ a_1^2 \\ a_2^2 \\ P_{bolt}^2 \\ a_1 * a_2 \\ a_2 * P_{bolt} \\ a_2 * s_I \\ P_{bolt} * s_I \end{Bmatrix}$$



SAMPLING OF STATISTICAL META-MODELS

(LANL Impact Test)

Effect Kept	-95% Bound	+95% Bound	F-test Value
1	-1,597.6	-1,478.8	0.01%
a_1	11.1	76.1	0.43%
a_2	208.5	368.3	0.01%
P_{bolt}	2.3	2.6	0.01%
s_I	2,351.0	2,754.6	0.01%
a_1^2	-436.5	-346.1	0.01%
a_2^2	-352.3	-261.9	0.01%
P_{bolt}^2	-0.0008	-0.0004	0.01%
a_1*a_2	629.5	701.9	0.01%
a_2*P_{bolt}	-0.6	-0.4	0.01%
a_2*s_I	-633.4	-271.5	0.01%
$P_{bolt}*s_I$	1.1	1.9	0.01%

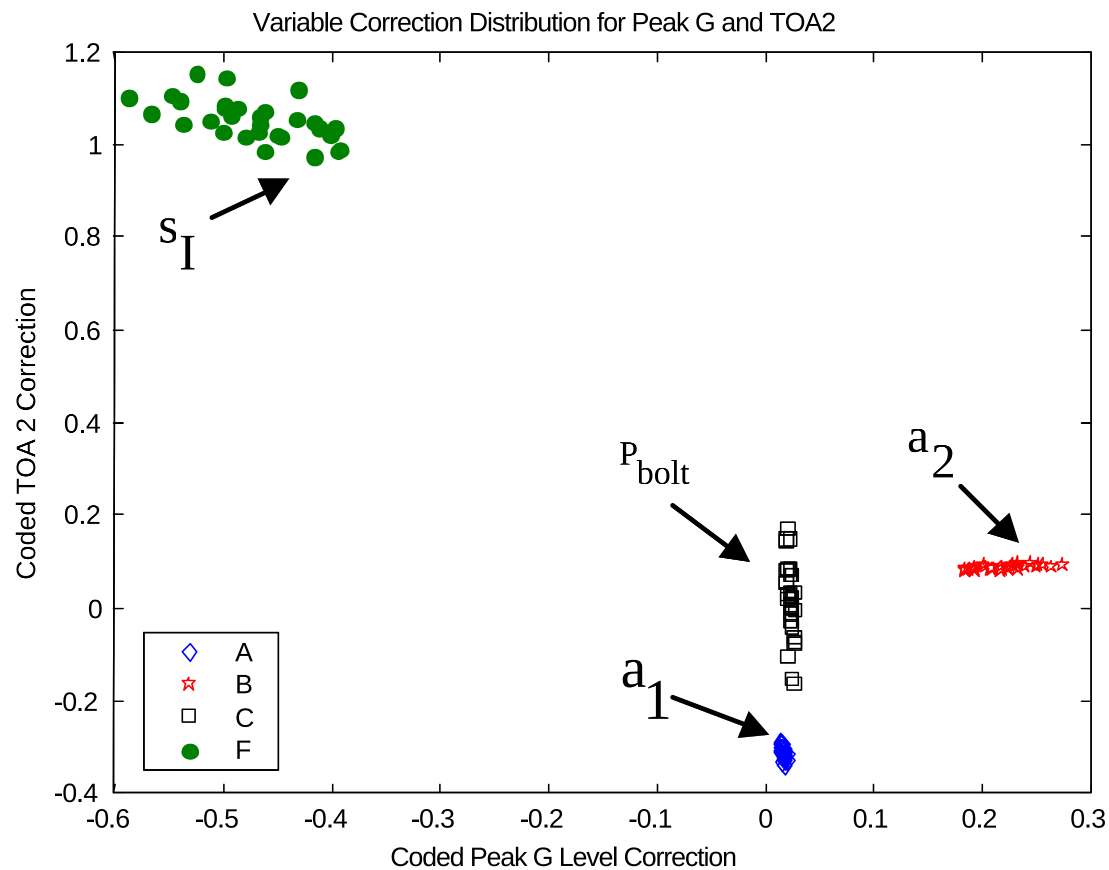


OPTIMIZATION OF INPUT PARAMETERS

(LANL Impact Test)

- **Parameter Optimizations Are Performed Using Different Features And Each Test Data Set Independently.**

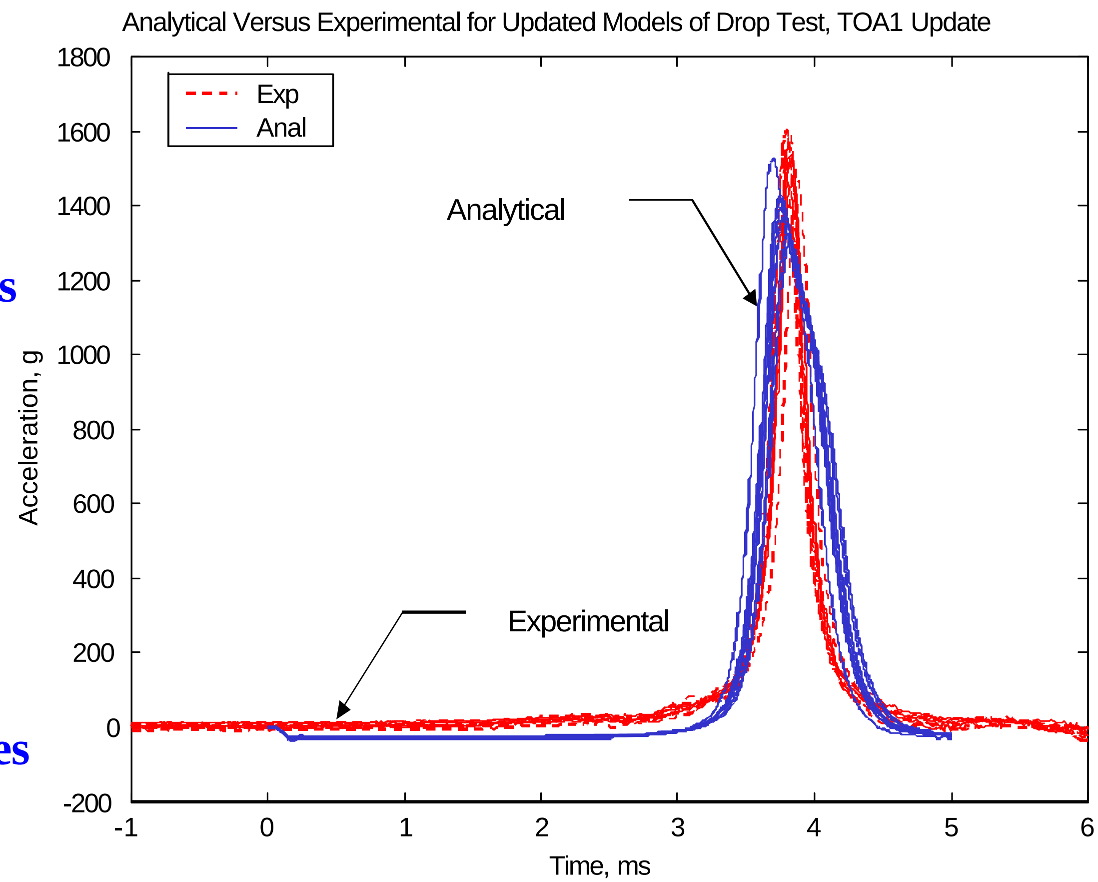
- **The Input Parameter's Variability is Assessed From These Multiple Optimizations.**



PREDICTED VS. OBSERVED VARIABILITY

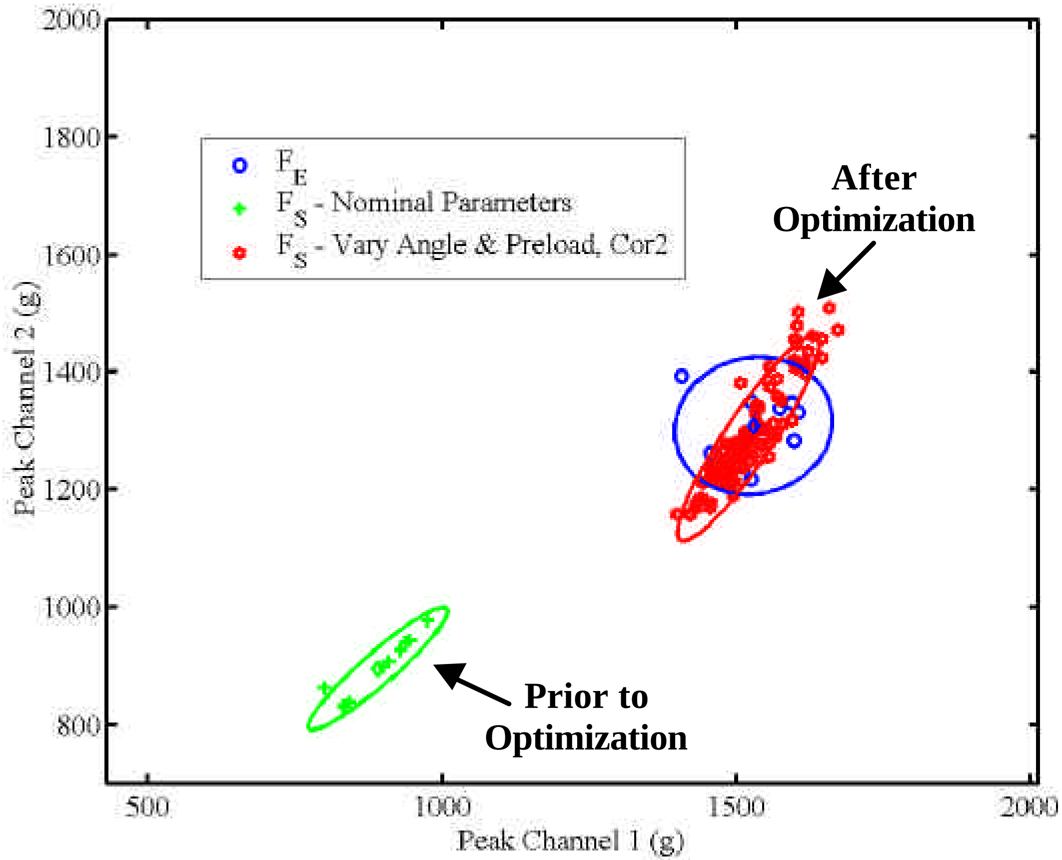
- **A Distribution of Input Parameters is Obtained Via Multiple Optimizations of the Statistical Meta-models.**

- **The Physics-based Simulation Combined With Optimized Input Parameters Reproduces the Experimental Variability.**



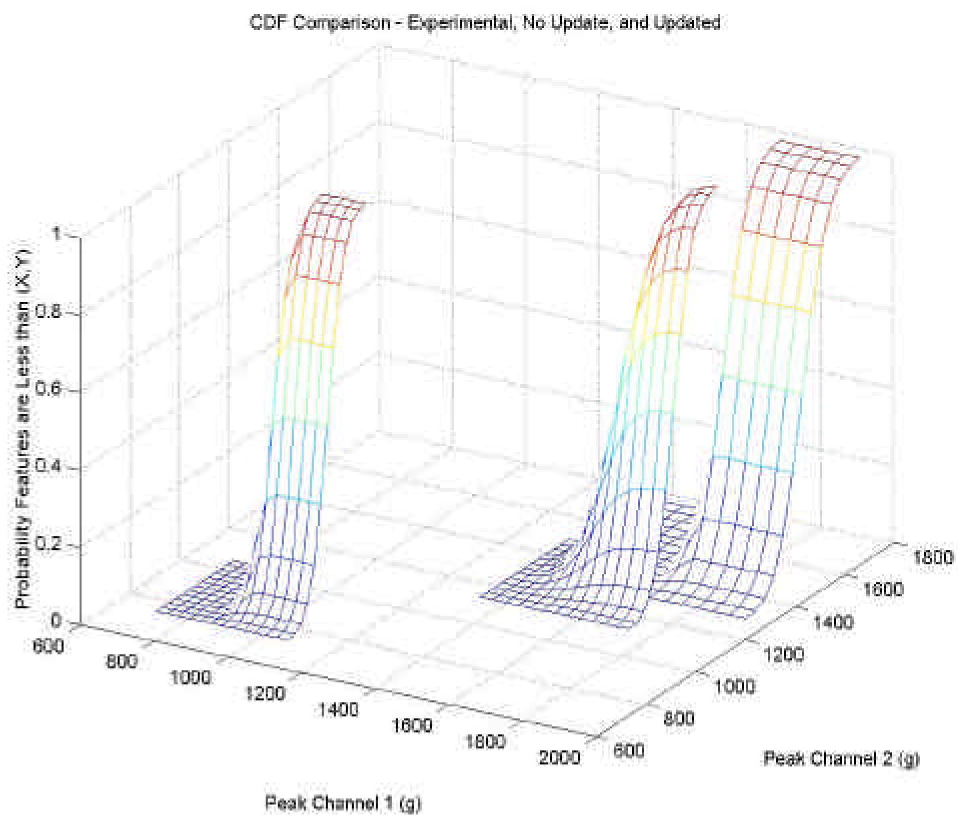
PREDICTED VS. OBSERVED VARIABILITY
(LANL Impact Test)

- **Multiple Simulations Are Analyzed by Sampling the Distributions of Input Parameters.**
- **The Mean Response is Captured With Acceptable Accuracy.**
- **Large Discrepancies Observed in Terms of Variance.**



COMPARISON OF MULTIVARIATE DISTRIBUTIONS
(LANL Impact Test)

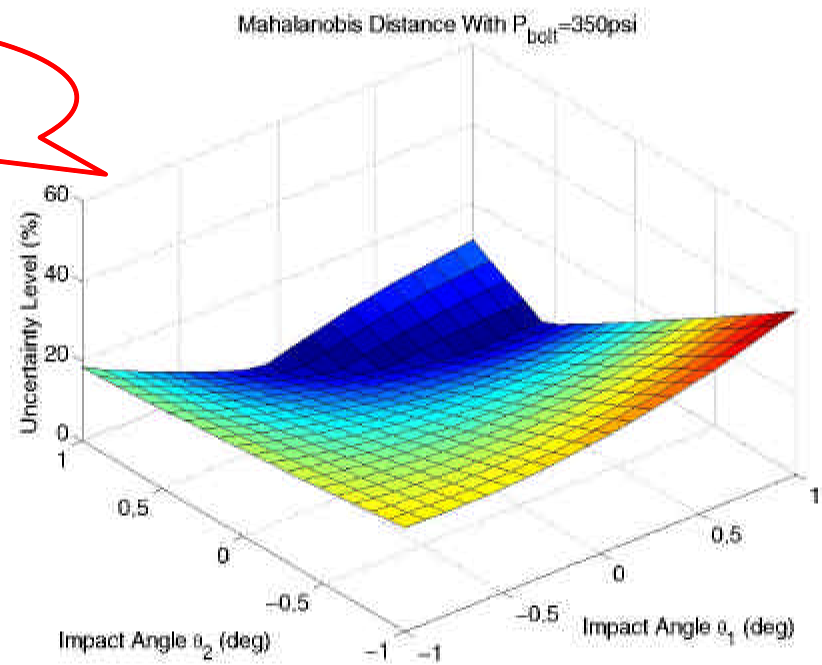
- **Pearson's Correlation Ratio**
- **Multivariate Chi^2 Analysis**
- **Kolmogorov-Smirnov Test**
- **Mahalanobis Distance**
- **Kullback-Leibler Relative Entropy**
- **… Others?**



MAHALANOBIS DISTANCE

(LANL Impact Test)

The Cost Function Becomes a Statistics.



- **The Vector of Model Features is an Estimate of the Mean Vector of Test Features to the (100-u)% Confidence Level if:**

$$\left(\overline{y}^{\,test} - y(p)\right)^T \left[S_{yy}^{\,test}\right]^{-1} \left(\overline{y}^{\,test} - y(p)\right) \leq \frac{N_y\left(N_s - 1\right)}{N_s\left(N_y - 1\right)} F_{N_y;N_s - N_y}(u)$$

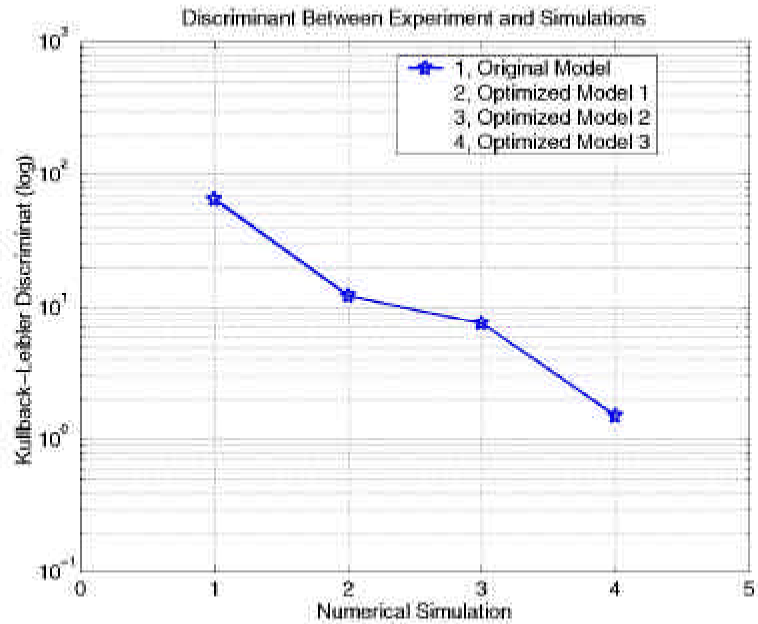
(Normal Distribution Assumed.)

KULLBACK-LEIBLER RELATIVE ENTROPY

(LANL Impact Test)

- **Expected Value of Ratio Between PDF's:**

$$J(p) = E\left[\log\left(\frac{f(y(p))}{f^{\,test}(y)}\right)\right]$$



- **If PDF's Are Normally Distributed:**

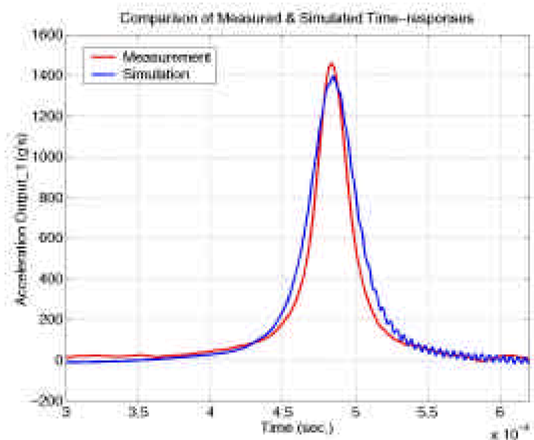
$$J(p) = \frac{1}{2}\left(\bar{y}^{test} - \bar{y}(p)\right)^{T}\left[S_{yy}^{test}\right]^{-1}\left(\bar{y}^{test} - \bar{y}(p)\right) + \frac{1}{2}\left(\mathrm{Trace}\left(\left[S_{yy}\right]\left[S_{yy}^{test}\right]^{-1}\right) - \log\left(\frac{\det\left[S_{yy}\right]}{\det\left[S_{yy}^{test}\right]}\right) - N_{y}\right)$$

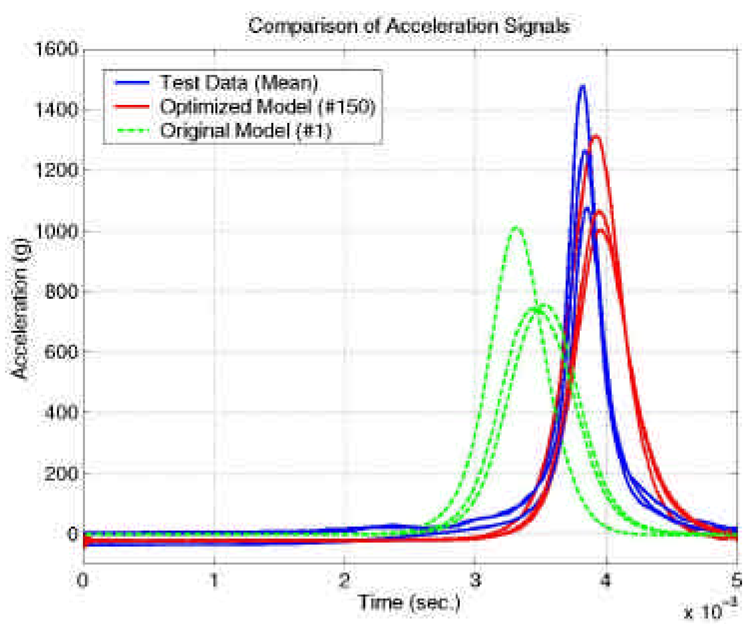
MODEL VALIDATION

(LANL Impact Test)

The Optimized Model "Replicates" the Test Data …

… Is the Model *Validated*?





Validation is Achieved When the Numerical Model Predicts the Response of a *Different* Configuration.

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- Notation & Definition
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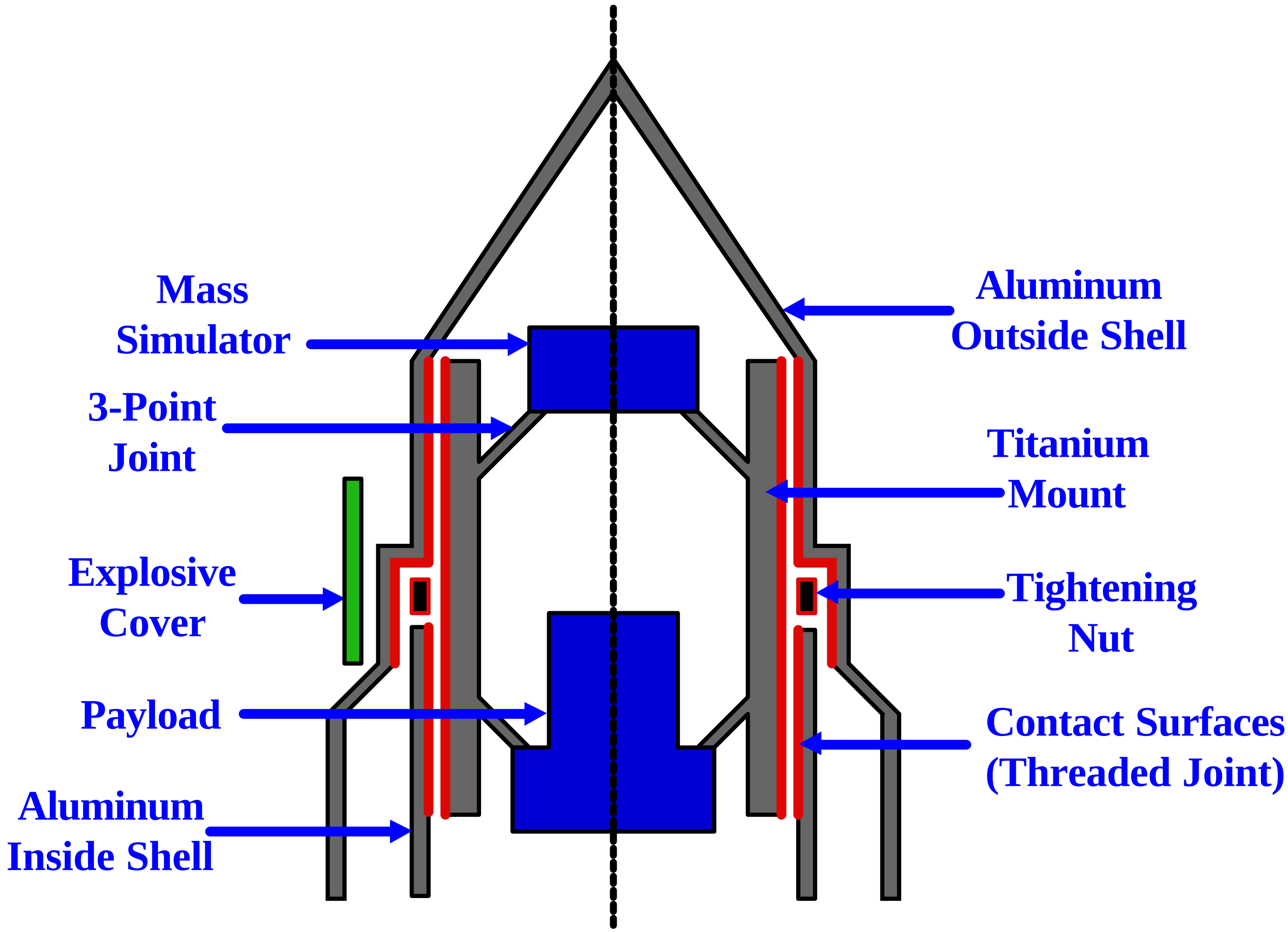
- Impact Experiment (Development of the Methodology)
- Forward Mount Impulse Experiment (ASCI Demonstration)
- Unresolved Issues & Challenges

FORWARD MOUNT SYSTEM TESTS

- **Components:**
  - ✓ **Lower Case.**
  - ✓ **Forward Mount.**
  - ✓ **Electronics.**
  - ✓ **Retaining Nut.**
  - ✓ **Upper Case.**
  - ✓ **Mass Simulators.**

ASSEMBLY OF THE FORWARD MOUNT SYSTEM



FORWARD MOUNT SYSTEM TESTS

(Low Level Impulse Tests)

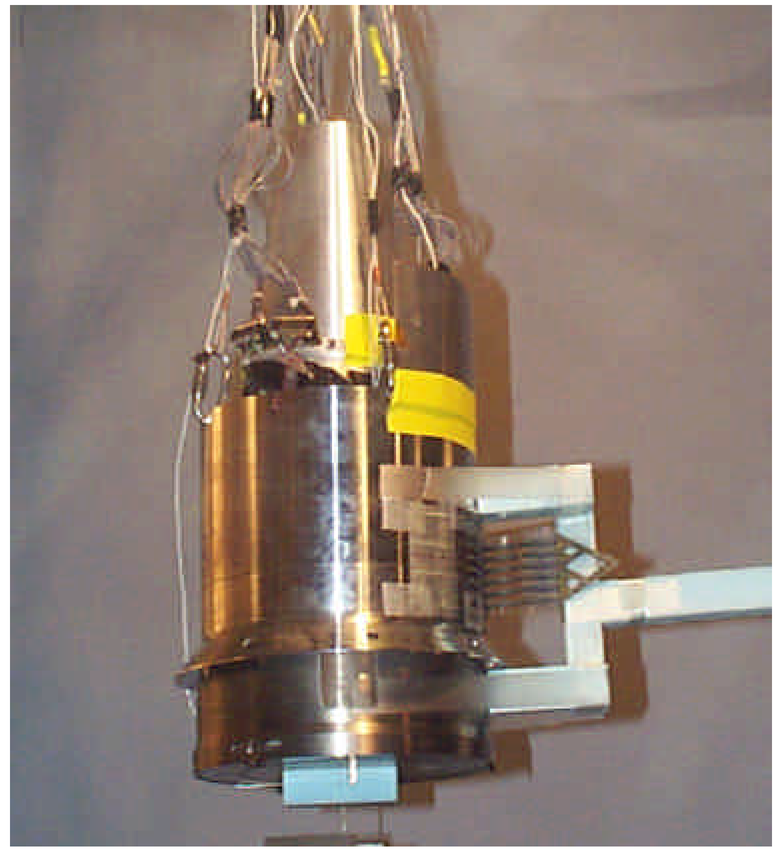
- **Useful Model Validation Requires Carefully Planned Experiments:**

✓ **Well-defined Input.**

✓ **Sufficient Instrumentation For Measuring Appropriate Response Quantities.**

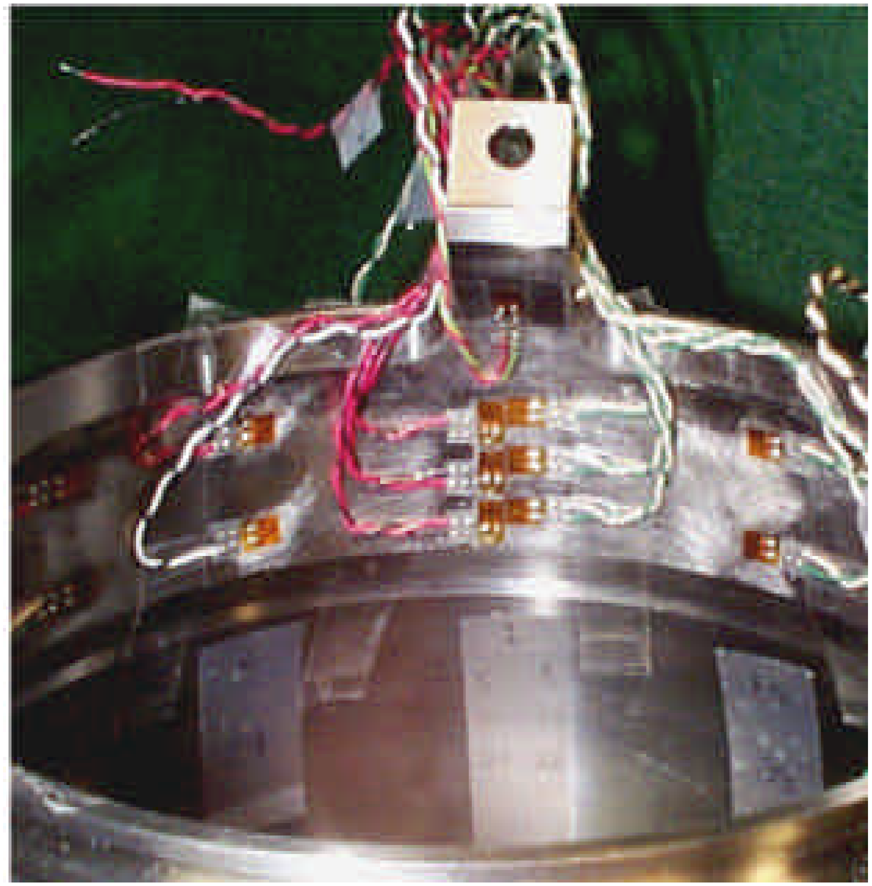
✓ **Test Matrix That Appropriately Varies Key Parameters.**

✓ **Data Resolution Capable of Capturing all Frequency Content of Interest.**



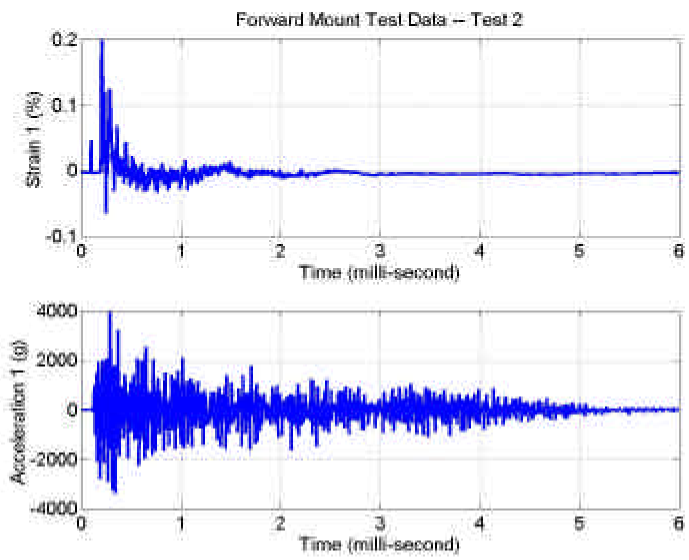
FORWARD MOUNT SYSTEM TESTS
(Instrumentation)

- **33 Strain Gages on Inside of Titanium Mount.**
- **6 Accelerometers on Mass Simulators.**
- **Fiber Optic Displacement Measurement.**
- **High Speed Photography.**
- **Sampling Rate = 50 x 10^{-9} seconds.**

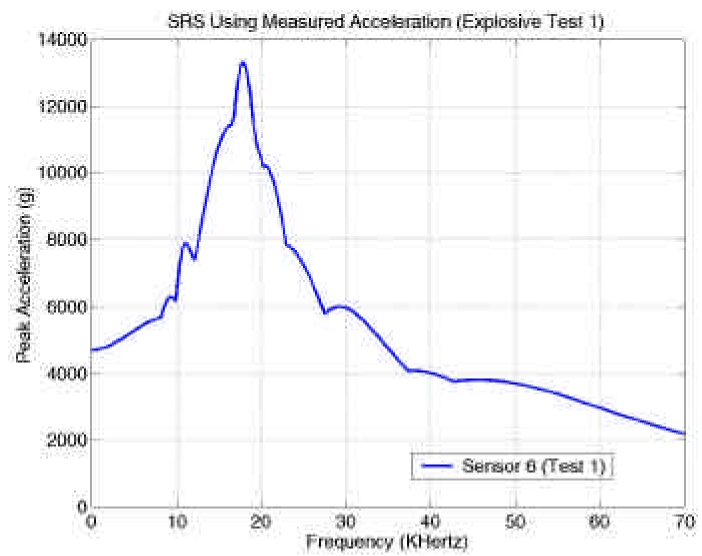


TEST DATA

(Forward Mount Low Level Impulse)

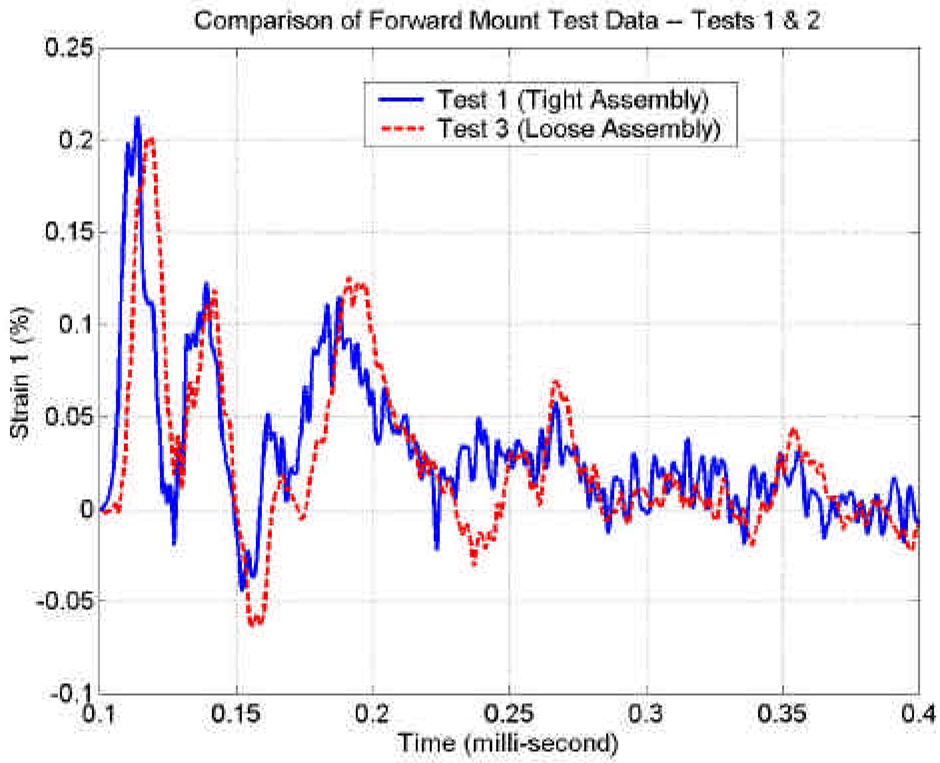


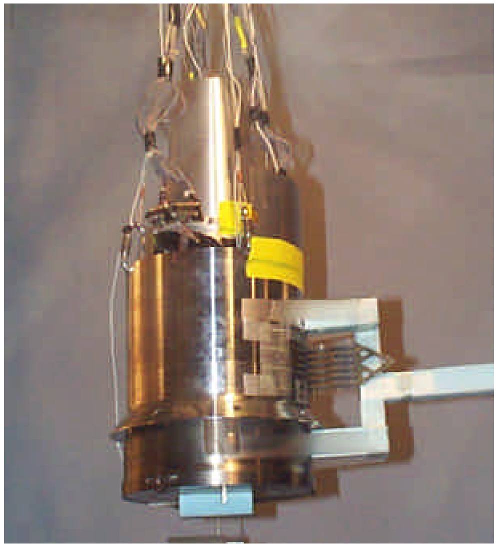
Shock Response Spectrum
(Acceleration 1, No Damping)



EXPERIMENTAL VARIABILITY

(Forward Mount Low Level Impulse)





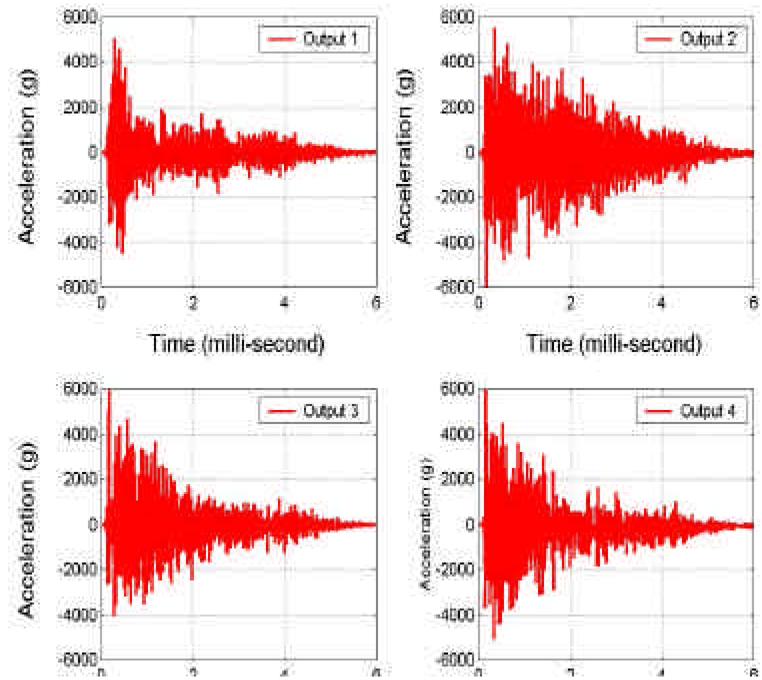
Test Matrix	Loose Assembly	Tight Assembly
Loose Manufacturing Tolerances	Test 3	Tests 1-2
Tight Manufacturing Tolerances	--	Test 4

EXPLICIT FINITE ELEMENT MODELING

(W76/Mk4)

Main Features:

- ✓ Coupled Thermal-Structure
- ✓ ParaDyn (Parallel Explicit Dyna3D)
- ✓ 1.4 Million Elements
- ✓ 480 Contact Surfaces
- ✓ Over 6 Million DOF's
- ✓ Stable Time Step = 25 Nano-second
- ✓ Platform: ASCI/BlueMountain
- ✓ 504-750 Processors
- ✓ 1.3 Hour CPU/Milli-second of Response



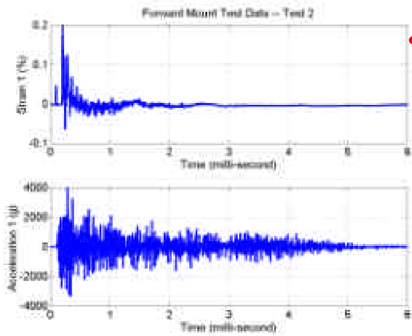
FEATURES FOR NONLINEAR DYNAMICS

- Karhunen-Loeve Decomposition
- Principal Component Analysis
- AR, ARX & ARMA Models
- Control Charts
- Shock Response Spectrum
- Spectral Density Function
- Joint Probability Density Function of the Output
- …

Analysis Techniques Specific to Linear Systems

PATTERN RECOGNITION

Data $\{y(t)\}$



Covariance of the Data Set

$$[C] = [Y(t)]^T [Y(t)]$$

Principal Component Analysis

$$[C] = \sum_{j=1 \cdots N_o} s_j \{F_j\}\{F_j\}^T$$

$$\{\tilde{y}(t)\} = [Y(t)]\{F_1\}$$

AR Model Fit

$$\tilde{y}(t) = \sum_{k=1 \cdots N_{AR}} a_k \, \tilde{y}(t-k) + e(t)$$

Output:

$$\{a\} + \{e(t)\}$$

ARX Model Fit

$$\tilde{y}(t) = \sum_{k=1 \cdots N_{ARX}} b_k \, \tilde{y}(t-k) + \sum_{k=1 \cdots N_{ARX}} c_k e(t-k) + \tilde{e}(t)$$

Output:

$$\{b\} + \{c\} + \{\tilde{e}(t)\}$$

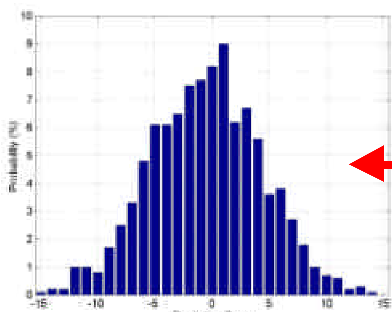
AR Model & ARX Model Fits

"New" Data Set

Output:

$$\{\tilde{e}(t)\}$$

PDF of Prediction Error $\{\tilde{e}(t)\}$

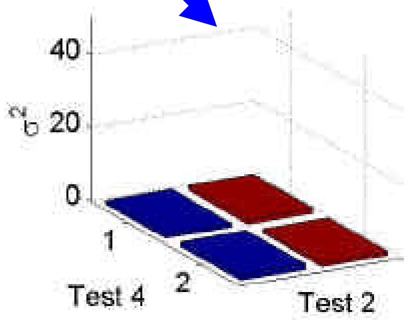


TEST-ANALYSIS CORRELATION

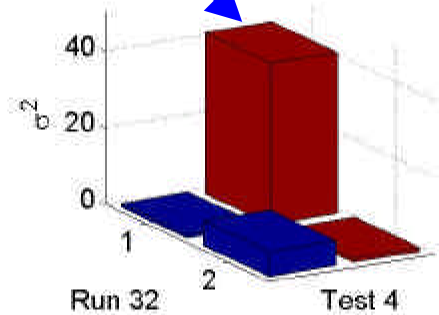
(Forward Mount Low Level Impulse)

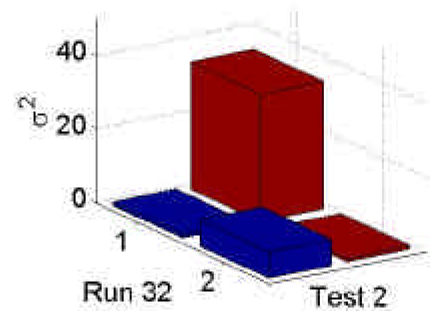
	ARX 1	ARX 2
Data 1	s^2_{11}	s^2_{12}
Data 2	s^2_{21}	s^2_{22}

Test-Test



Test-Analysis





AR Model Fit

$$\tilde{y}(t) = \sum_{k=1\cdots N_{AR}} a_k \, \tilde{y}(t-k) + e(t)$$

+ ARX Model Fit

$$\tilde{y}(t) = \sum_{k=1\cdots N_{ARX}} b_k \, \tilde{y}(t-k) + c_k e(t-k) + \tilde{e}(t)$$

+ Covariance of the Prediction Error $\{\tilde{e}(t)\}$

LIST OF INPUT PARAMETERS

(Forward Mount Low Level Impulse)

Variable Number	Physical Description	Influence Guessed?	Lower Bound	Upper Bound	Statistical Distribution
1	Tape Pre-load	Medium	100.0 N	4,000.0 N	Uniform
2	Nut Pre-load	High	250.0 N	4,000.0 N	Uniform
3	Upper Shell Pre-load	High	250.0 N	2,000.0 N	Uniform
4	Static Friction, Al/Al	Medium	0.80	4.00	Uniform
5	Static Friction, Ti/Ti	Low	0.20	2.40	Uniform
6	Static Friction, Al/Ti	High	0.50	3.00	Uniform
7	Static Friction, SS/Ti	Medium	0.20	2.40	Uniform
8	Kinetic Friction, Al/Al	Medium	0.80	4.00	Uniform
9	Kinetic Friction, Ti/Ti	Low	0.10	1.80	Uniform
10	Kinetic Friction, Al/Ti	High	0.10	3.00	Uniform
11	Kinetic Friction, SS/Ti	Medium	0.10	1.80	Uniform
12	Input Signal Scaling	Unknown	1.0	1.2	Uniform

New!

TAGUCHI ORTHOGONAL ARRAY — OA_32

(Forward Mount Low Level Impulse)

- **The Matrix of Experiments is Full Rank.**
- **Main (Linear) Effects of the Input-Output Model Are Kept Uncorrupted From Second Order Effects (No Aliasing).**

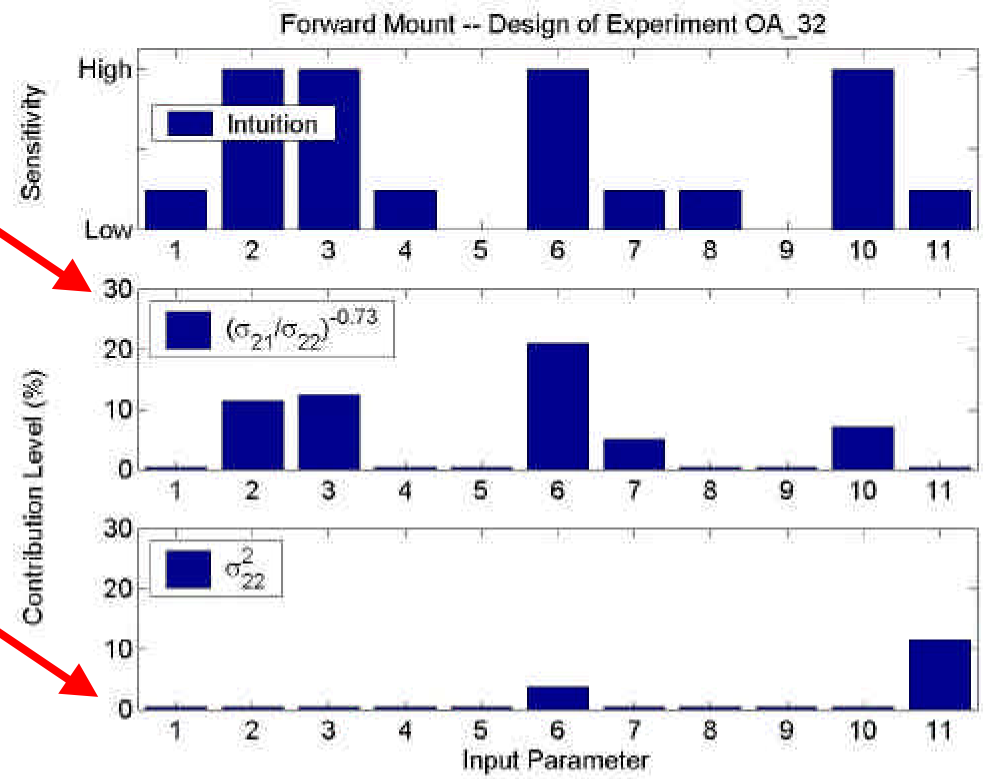
Run	1	2	3	4	5	6	7	8	9	10	11
1	20	10	10	0.8	0.2	0.5	0.2	0.8	0.1	0.1	0.1
2	220	10	10	0.8	0.2	3.0	0.2	0.8	1.8	3.0	0.1
3	15	220	10	0.8	0.2	3.0	2.4	0.8	0.1	0.1	1.8
4	220	220	10	0.8	0.2	0.5	2.4	0.8	1.8	3.0	1.8
5	20	15	120	0.8	0.2	3.0	2.4	4.0	1.8	0.1	0.1
6	220	10	115	0.8	0.2	0.5	2.4	4.0	0.1	3.0	0.1
7	15	215	125	0.8	0.2	0.5	0.2	4.0	1.8	0.1	1.8
8	220	215	125	0.8	0.2	3.0	0.2	4.0	0.1	3.0	1.8
9	20	10	10	4.0	0.2	0.5	2.4	4.0	1.8	3.0	1.8
10	220	10	10	4.0	0.2	3.0	2.4	4.0	0.1	0.1	1.8
11	15	220	10	4.0	0.2	3.0	0.2	4.0	1.8	3.0	0.1
12	220	220	10	4.0	0.2	0.5	0.2	4.0	0.1	0.1	0.1
13	20	15	120	4.0	0.2	3.0	0.2	0.8	0.1	3.0	1.8
14	220	10	115	4.0	0.2	0.5	0.2	0.8	1.8	0.1	1.8
15	15	215	125	4.0	0.2	0.5	2.4	0.8	0.1	3.0	0.1
16	220	215	125	4.0	0.2	3.0	2.4	0.8	1.8	0.1	0.1
17	20	10	10	0.8	2.4	0.5	0.2	4.0	0.1	3.0	1.8
18	220	10	10	0.8	2.4	3.0	0.2	4.0	1.8	0.1	1.8
19	15	220	10	0.8	2.4	3.0	2.4	4.0	0.1	3.0	0.1
20	220	220	10	0.8	2.4	0.5	2.4	4.0	1.8	0.1	0.1
21	20	15	120	0.8	2.4	3.0	2.4	0.8	1.8	3.0	1.8
22	220	10	115	0.8	2.4	0.5	2.4	0.8	0.1	0.1	1.8
23	15	215	125	0.8	2.4	0.5	0.2	0.8	1.8	3.0	0.1
24	220	215	125	0.8	2.4	3.0	0.2	0.8	0.1	0.1	0.1
25	20	10	10	4.0	2.4	0.5	2.4	0.8	1.8	0.1	0.1
26	220	10	10	4.0	2.4	3.0	2.4	0.8	0.1	3.0	0.1
27	15	220	10	4.0	2.4	3.0	0.2	0.8	1.8	0.1	1.8
28	220	220	10	4.0	2.4	0.5	0.2	0.8	0.1	3.0	1.8
29	20	15	120	4.0	2.4	3.0	0.2	4.0	0.1	0.1	0.1
30	220	10	115	4.0	2.4	0.5	0.2	4.0	1.8	3.0	0.1
31	15	215	125	4.0	2.4	0.5	2.4	4.0	0.1	0.1	1.8
32	220	215	125	4.0	2.4	3.0	2.4	4.0	1.8	3.0	1.8

INPUT-OUTPUT MAIN EFFECTS ANALYSIS

(Forward Mount Low Level Impulse)

- **Results of the Main Effects Analysis Match the Analyst's Intuition Perfectly.**

- **The Feature Extraction Process Does Not Interfere With the Statistical Effects Analysis.**



OUTLINE

- Notation & Definition
- Motivation
- Impact Experiment (Development of the Methodology)
- Forward Mount Impulse Experiment (ASCI Demonstration)
- Unresolved Issues & Challenges

SPECIFIC QUESTIONS & ISSUES (1/2)

- **How Can We Compare Data Sets?**
  - ✓ **Are There Features of Choice Adopted by Statisticians or Others?**
- **Are There Techniques Available For Hypothesis Testing of Multivariate Distributions?**
- **Can We Sample the Input Parameter Space Most Efficiently?**
- **We Need to Understand Better Techniques For Designing Experiments.**
  - ✓ **How to Fold-over a Design of Experiment?**
  - ✓ **How to Make Use of Runs Already Completed?**
  - ✓ **How to Add One Variable to an On-going Analysis?**

SPECIFIC QUESTIONS & ISSUES (2/2)

- **Is Our Approach For Translating Variability Observed on the Output in Terms of Statistical Distributions on the Input Correct?**
- **Are There Techniques Available For Optimizing the Variance of a Distribution?**
- **Are There Techniques Available For Replacing the Predictions of a Statistical Meta-model by Ensemble Averages?**
  - ✓ **How Can a Meta-model be Sampled?**
- **We Need to Understand Better the Differences Between Theories Available For Modeling Uncertainty.**

CONCLUSION

- **The Need For Model Validation and Uncertainty Quantification is Rapidly Growing and Expanding.**

- **Dealing With Nonlinear Systems and Uncertainty is Going to Be Very Expensive, NO MATTER WHAT YOU DO.**

- **Experience May Be Gained From Learning What is Being Achieved in Other Scientific Communities.**