Spectral analysis of non-stationary CT noise

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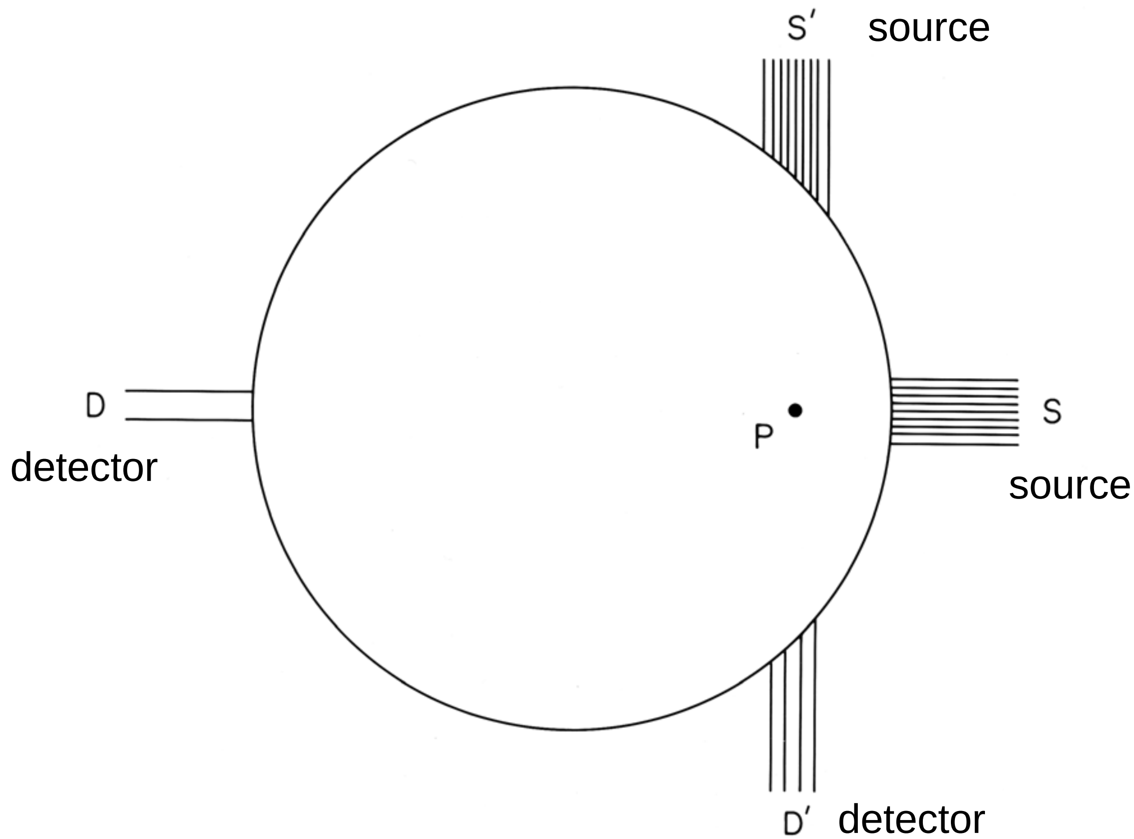
NPS and NEQ

- For CT reconstructions, the noise power spectrum (NPS), or Wiener spectrum, is given by

$$S(f) = \frac{\pi |f| |H(f)|^2}{\mathrm{NEQ}}$$

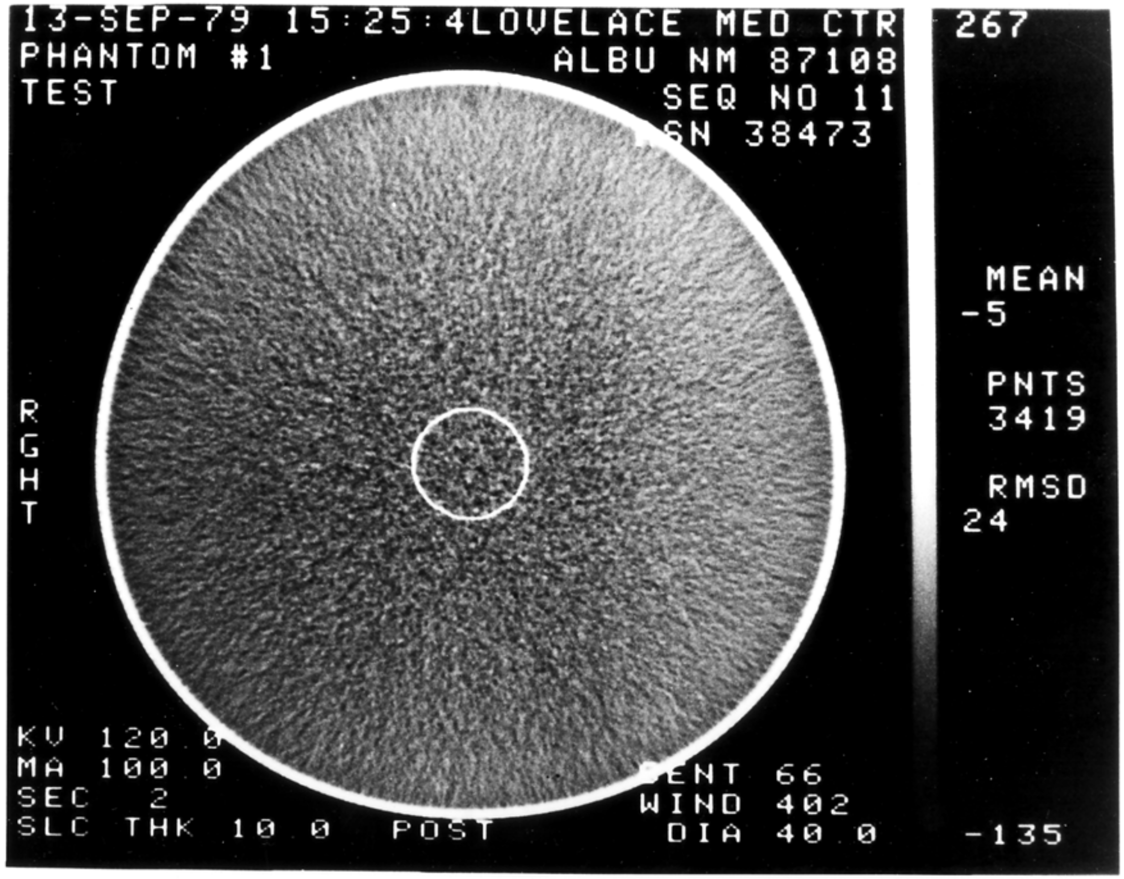
  - where f is radial spatial frequency
  - H is the effective filter used in the reconstruction algorithm
  - NEQ is the density of noise-equivalent quanta in the projection measurements
- If the NEQ is the same in all projection measurements, the NPS is stationary and isotropic
- However, if the density of detected quanta varies, the NPS is non-stationary and non-isotropic

Geometry of CT projection measurement



For the same incident x-ray fluence, the transmitted fluence is larger in the tangential direction (S' - D') than in the radial direction (S - D). Thus, one expects the noise at point P in the CT reconstruction to be anisotropic.

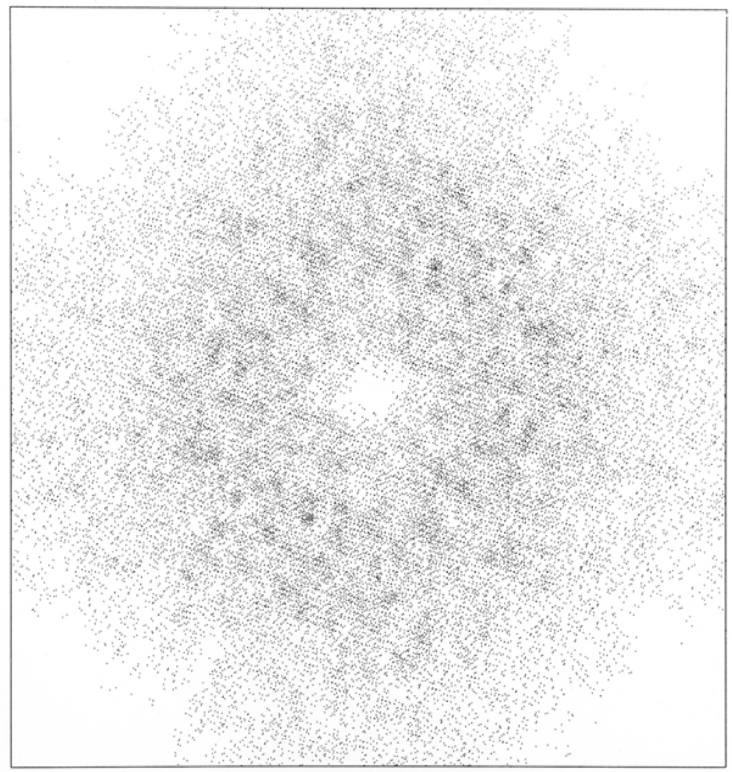
Delta 2020 scan of uniform phantom



Delta 2020 scan:
120 kVp, 200 mAs.

CT scan of 35.5-cm-diameter water-bath phantom, done on a Delta 2020 scanner without a dose compensation wedge. Note how noise streams outward near periphery, indicating that noise characteristics depend on direction.

Standard NPS estimation

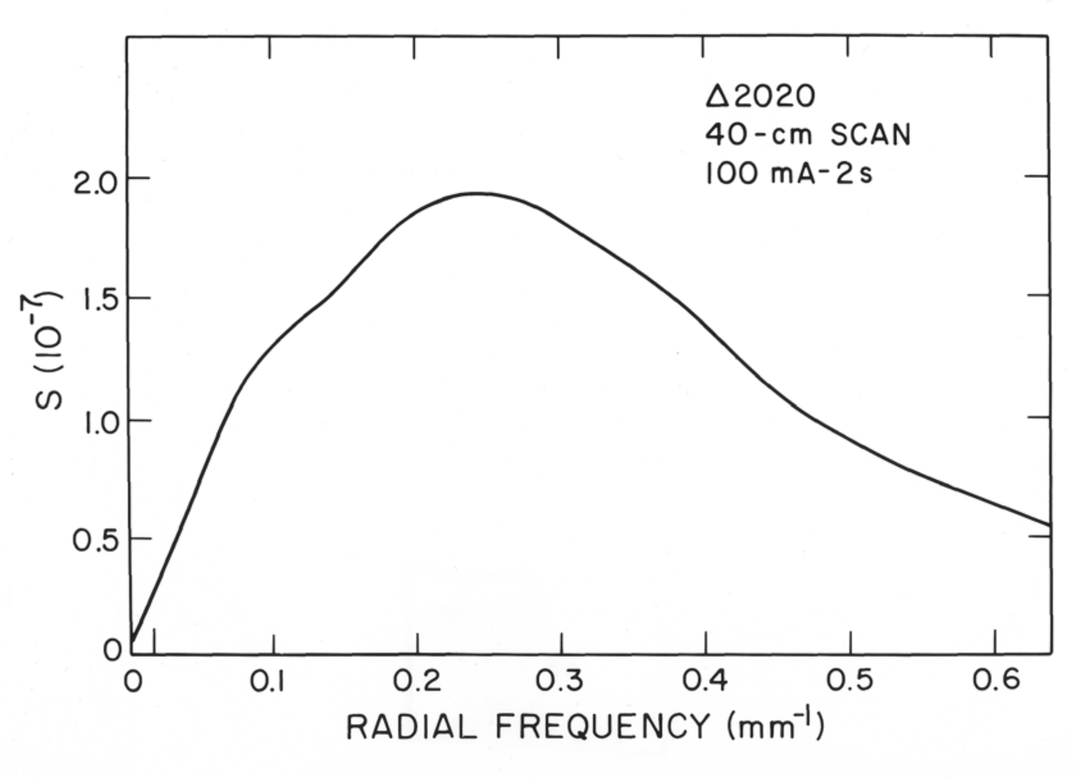


2D NPS for Delta 2020 displayed as a pseudo-grayscale image vs. horizontal and vertical spatial frequency. Zero frequency is in middle of image.

2D NPS obtained in standard manner appears to be isotropic.

NPS goes to zero at zero frequency (middle of image), as expected.

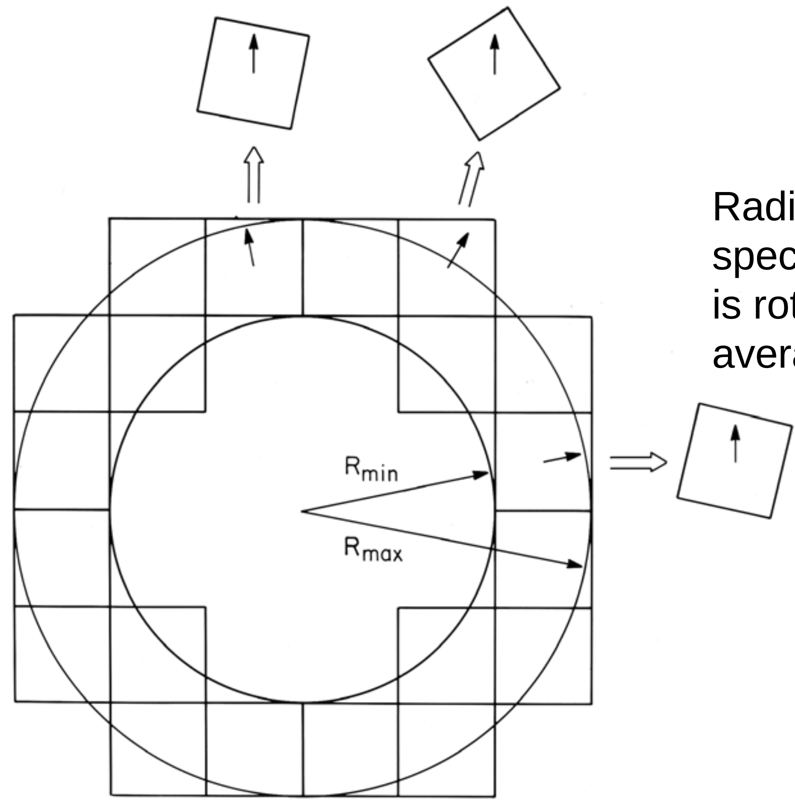
Standard NPS estimation



Nyquist frequency = 0.64 mm^{-1}

Radial frequency dependence of standard NPS, obtained by averaging 2D NPS over polar angles.
NPS is roughly proportional to frequency at low frequencies.

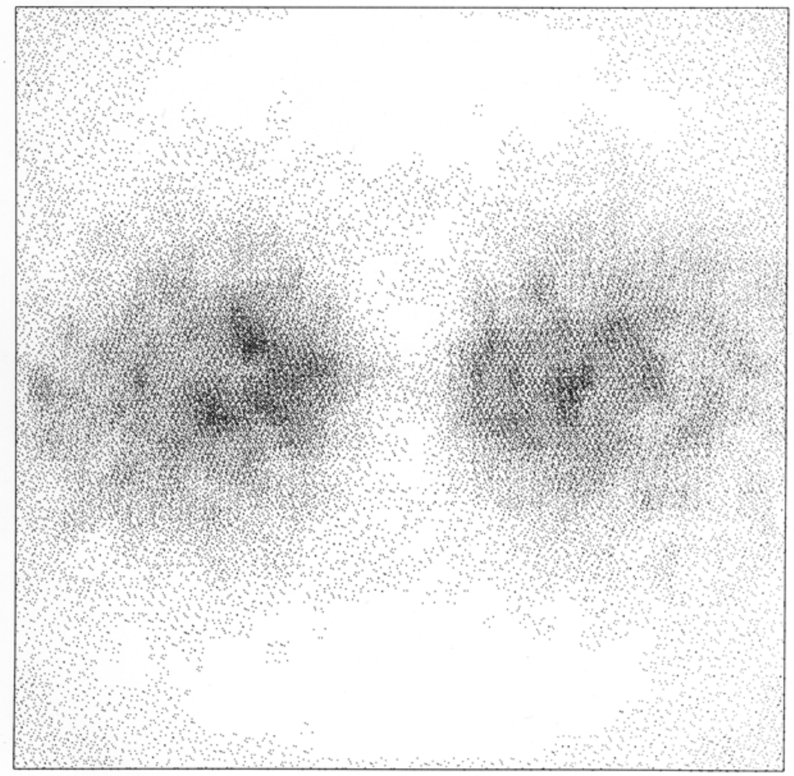
Radial-tangential NPS estimation



Radial direction of power spectrum for each subsample is rotated to vertical before averaging.

Method of estimating of radial and tangential components of NPS:
The noise power spectra are calculated for subsamples of the original noisy image whose centers lie within an annulus.
The power spectrum for each subsample is averaged after being rotated to align the radial frequency axes.

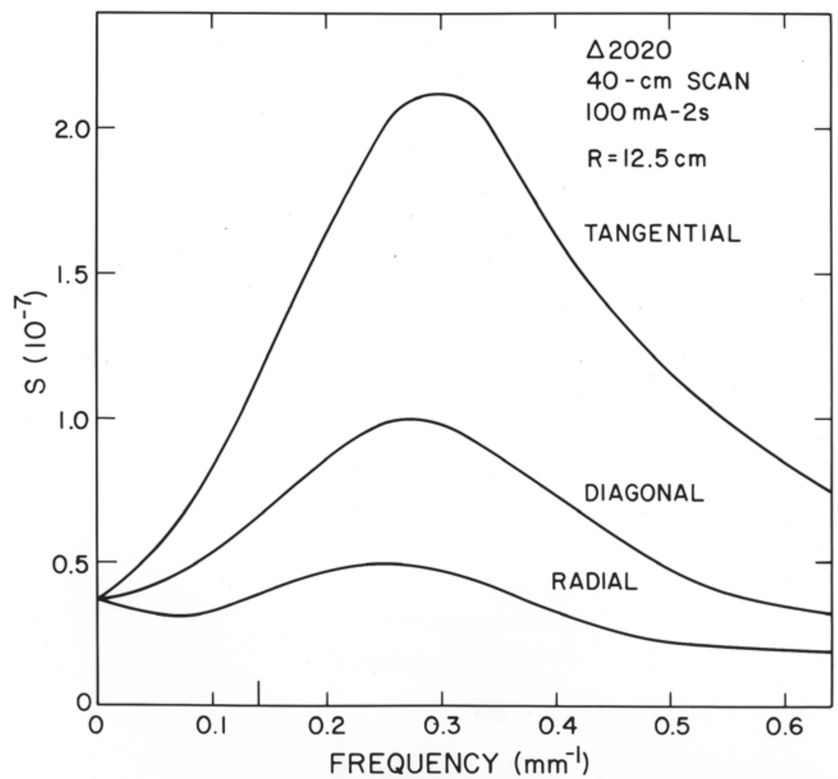
Radial-tangential NPS



Delta 2020 NPS displayed as a pseudo-grayscale image displayed in terms of the radial (vertical axis) and tangential frequency (horizontal axis). Zero frequency is in middle of plot.

2D NPS calculated for an average radius of 12.5 cm from the scan of a 35.5-cm-dia water phantom.
Decomposed into radial and tangential components, NPS is highly non-isotropic.
NPS lowest in radial direction, transverse to highest detected x-ray density.

Radial-tangential NPS

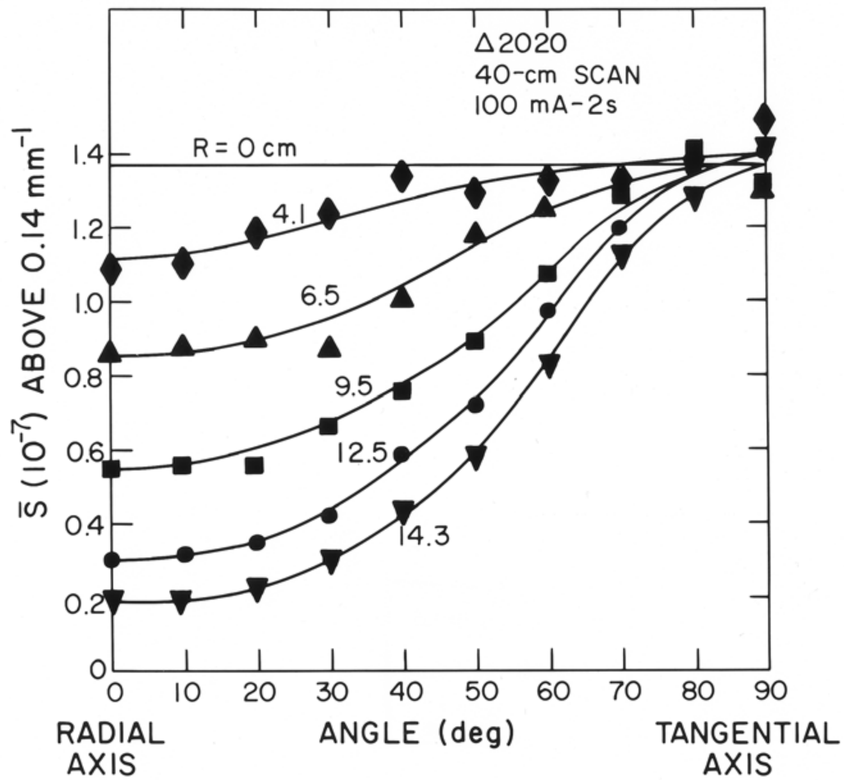


NPS analysis of a Delta 2020 scan of a 35.5-cm-dia water phantom

Nyquist frequency = 0.64 mm^{-1}

Radial-frequency dependence of NPS in radial and tangential directions, and midway between them, at average radius of 12.5 cm for Delta 2020 scan of a 35.5-cm-dia water phantom.

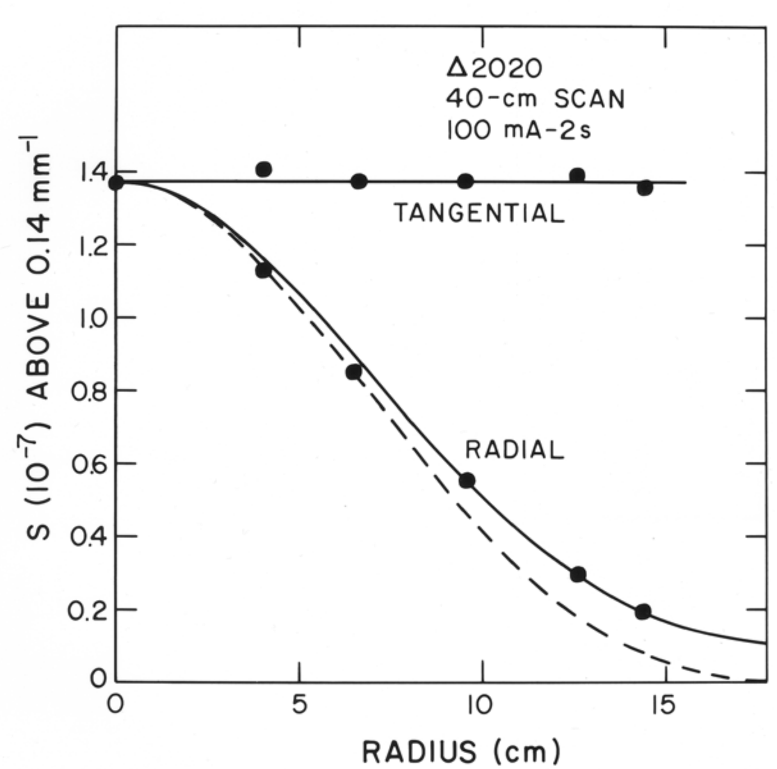
Radial-tangential NPS



NPS analysis of a Delta 2020 scan of a 35.5-cm-dia water phantom

The average noise power above a frequency of 0.14 mm^{-1} plotted versus angle wrt. radial direction for various average radii. The curves have been drawn by eye.

Radial-tangential NPS



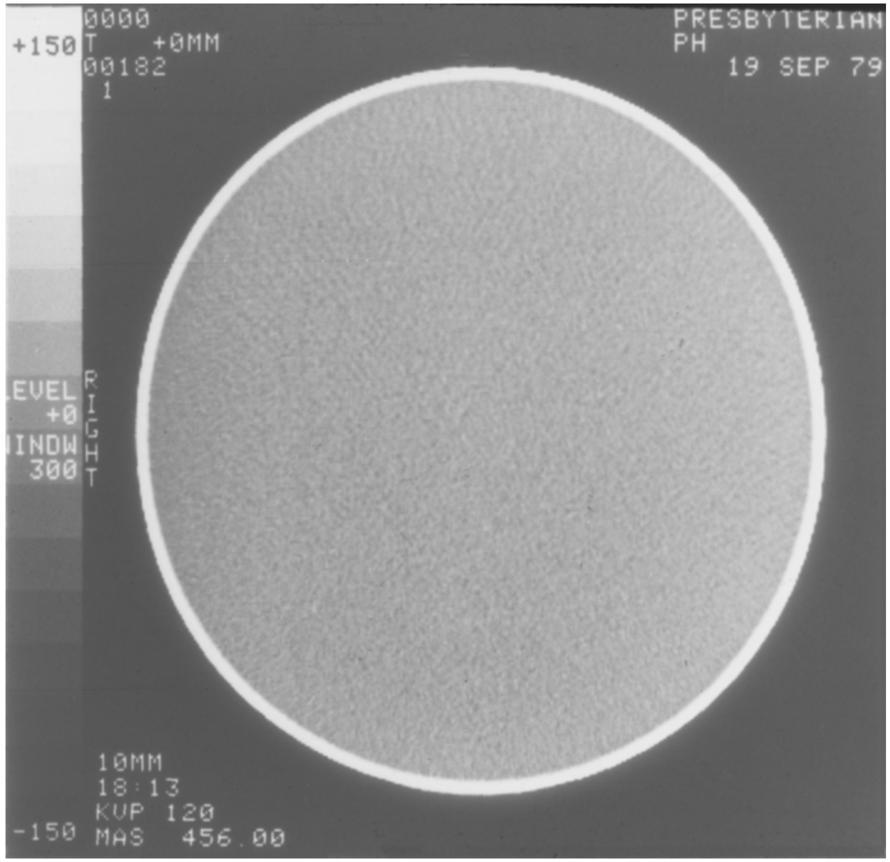
NPS analysis of a Delta 2020 scan of a 35.5-cm-dia water phantom

Average noise power above 0.14 mm^{-1} in the tangential and radial frequency directions, as a function of average radius. Tangentially, the noise power remains constant, as expected. Radially, the dashed curve is based on the dependence of the fluence calculated for the phantom geometry. The solid curve is the same, with an added uniform background of 0.11.

Delta 2020

In other studies, similar scans done on a Delta 2020 scanner **with** a dose compensation wedge show no significant noise anisotropy.

GE 8800 scan of uniform phantom



GE 8800 scan:
120 kVp, 456 mAs

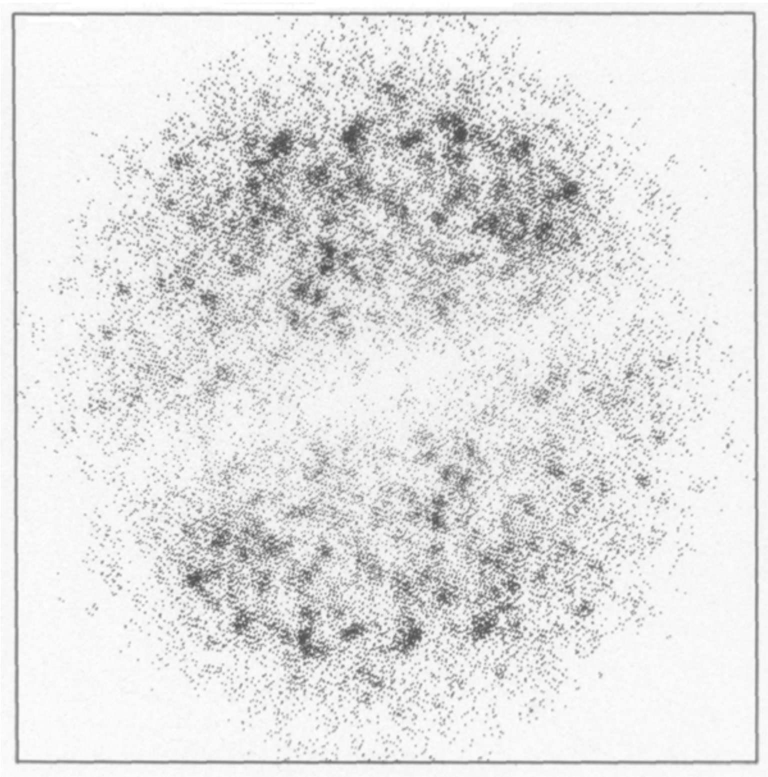
Full scan mode;
576 views / 360º

CT scan of 34-cm-diameter water-bath phantom, done on a GE 8800 scanner employing a dose compensation wedge. The noise appears to have approximately uniform characteristics everywhere and seems isotropic.

GE 8800 – third generation CT scanner

- GE 8800 is a third generation CT scanner
  - ▸ rotating fan-beam geometry
  - ▸ fan angle = α (approx. 34º)
  - ▸ source and detector rotate around patient
- Two scanning modes:
  - ▸ full scan – rotation through 360º
    - each pixel sees same ray coverage, i.e., 360º
  - ▸ half-scan mode – rotation through 180º + α
    - additional rotation by α required so all pixels have at least 180º coverage
    - but most pixels see more than 180º coverage
    - the pathlengths for the redundant rays are typically averaged together, resulting in lower noise in certain directions
    - expect to see this effect in 2D NPS

Standard NPS estimation

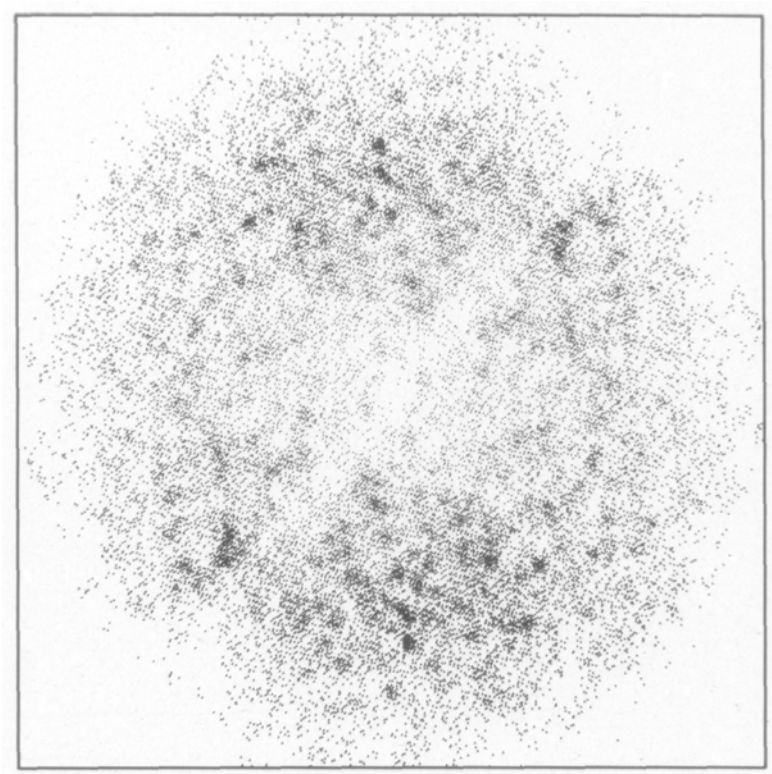


2D NPS for GE 8800 in **half-scan mode** (342 views), displayed as a pseudo-grayscale image vs. horizontal and vertical spatial frequency.

Zero frequency is in middle of image.

For a half scan (180º + 34º), 2D NPS is nonisotropic; there is a dip in NPS along a radial line, about 20º from horizontal.

Standard NPS estimation



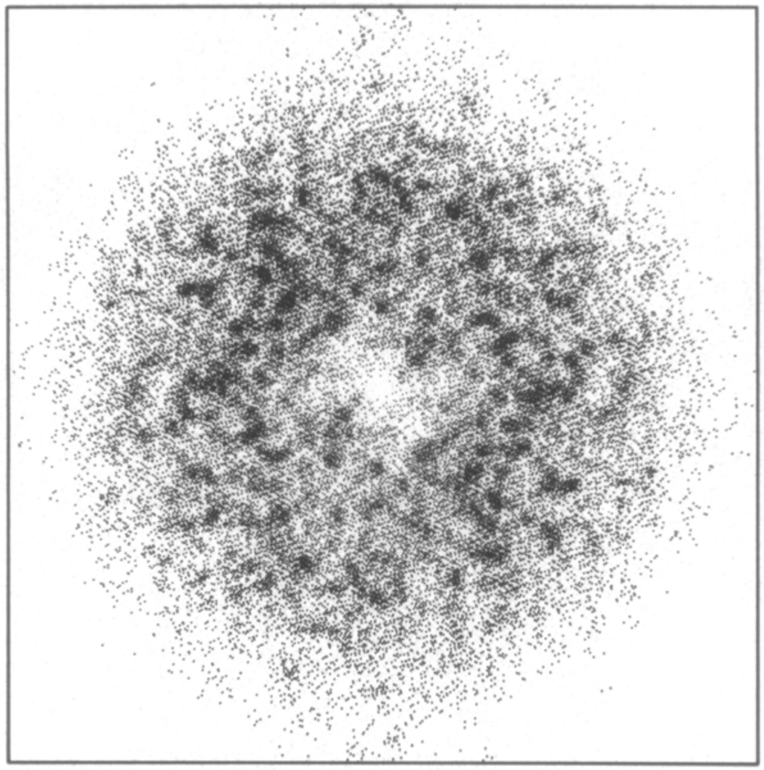
2D NPS for GE 8800 in **half-scan mode** (342 views), displayed as a pseudo-grayscale image vs. horizontal and vertical spatial frequency.

Zero frequency is in middle of image.

For a subsequent half scan, the 2D NPS is also nonisotropic; there is a dip in NPS along a radial line, about 50º from horizontal.

Note: GE 8800 starts scanning at an arbitrary angle.

Standard NPS estimation



2D NPS for GE 8800 in **full-scan mode** (576 views), displayed as a pseudo-grayscale image vs. horizontal and vertical spatial frequency.

Zero frequency is in middle of image.

For a full scan (360º gantry rotation), the 2D NPS obtained in standard manner is isotropic.

Summary

- Image noise in CT reconstructions depends on the density of detected x-rays in the projections
- Variable densities of detected x-rays implies the CT noise is non-stationary and non-isotropic
  - even so, it is possible and useful to estimate the NPS, as a function position and direction in the image
- Analysis of the noise in reconstructions from a scanner without a compensation wedge (bowtie filter) demonstrates
  - feasibility of decomposing NPS behavior into radial and tangential components
  - explanation in terms of the density of detected x-rays (NEQ)
- NPS of half-scan images from a third-generation scanner (GE8800) with compensation shows non-isotropic noise because of ray averaging

Bibliography

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