ANGULAR DISTRIBUTIONS FOR π^+ ELECTROPRODUCTION AND THE PION FORM FACTOR*

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Differential cross-section measurements for the reaction $e^- + p \to e^- + n + \pi^+$ in the region of the $N^*(1236)$ resonance are compared with the dispersion theories of Adler and Zagury at four-momentum transfers between 1.2 and 15.3 F^{-2} using the pion form factor as a free parameter. The results suggest a form factor similar to that of the proton.

Extensive angular distribution measurements for the reaction $e^- + p \to e^- + n + \pi^+$ at pion-nucleon center-of-mass energies near the $N^*(1236)$ resonance have been made at the Cambridge Electron Accelerator. The principal aims of the experiment were to investigate the dependence of the pion form factor $F_\pi(q^2)$ upon the square of the four-momentum transfer, q^2, and to estimate the rms charge radius of the π^+.

Data were obtained at four-momentum transfers near 1, 3, 6, 10, and 15 F^{-2} for pion polar angles, θ_π^*, between 0° and 50° c.m. over a wide range of pion azimuthal angles, φ_π. The experiment was performed in conjunction with the previously reported measurements[1] of the reaction $e^- + p \to e^- + p + \pi^0$.

The apparatus was identical to that described in Ref. 1. Electrons scattered from a liquid-hydrogen target were detected in coincidence with positive pions or protons and momentum analyzed in a half-quadrupole spectrometer. The positive pions and protons were detected behind a large-solid-angle sweeping magnet by a counter array consisting of three scintillation counters, a 144-bin scintillation-counter hodoscope, and a Plexiglas Cherenkov counter. Pions were separated from protons by means of pulse-height information recorded by an on-line PDP-1 computer. The details of the experimental analysis will be published elsewhere.[2]

The angular distribution for single-pion electroproduction is given by

$$\frac{d\sigma}{d\Omega_\pi} = \frac{1}{\Gamma_T}\frac{d^3\sigma}{d\Omega_\pi dw_e dE'} = \frac{d\sigma_T}{d\Omega_\pi}(\theta_\pi^*) + \epsilon\frac{d\sigma_0}{d\Omega_\pi}(\theta_\pi^*)$$
$$+ [\tfrac{1}{2}\epsilon(\epsilon+1)]^{1/2}\sin\theta_\pi^* S(\theta_\pi^*)\cos\varphi_\pi + \epsilon\sin^2\theta_\pi^* T(\theta_\pi^*)\cos 2\varphi\pi,$$

where Ω_π is the c.m. solid angle for pion detection, w_e is the electron solid angle in the laboratory, E' is the energy of the scattered electron, and ϵ is the polarization of the transverse components of the virtual photon. The kinematic factor Γ_T is defined in Ref. 1.

The first term in the cross section, $d\sigma_T/d\Omega_\pi$, is associated with the transverse components of the electromagnetic field and reduces to the corresponding photoproduction cross section at zero four-momentum transfer. The second term, $d\sigma_0/d\Omega_\pi$, is associated with the scalar (longitudinal) components of the field and is sensitive to the pion form factor because of a large contribution from the one-pion-exchange interaction. In the present measurement, this term, which is approximately proportional to F_π^2, is predicted to be between 37 and 68 % of the observed cross section, depending on the four-momentum transfer and c.m. energy.[3,4] Since $d\sigma_0/d\Omega_\pi$ is evaluated in the Born approximation with small final-state interaction corrections, its calculation is expected to be fairly reliable.

The third term, produced by interference between scalar and transverse amplitudes, is also sensitive to F_π. Because the one-pion-exchange amplitude is real, its interference with the resonant magnetic dipole amplitude is suppressed. Therefore, the size of this term is sensitive to the presence of smaller, predominantly real, transverse background amplitudes and is a useful check on the theoretical models. On the other hand, the fact that the present measurements extend over a wide range of azimuthal angles minimizes the importance of this term in the determination of F_π.

The fourth term in the cross section is due to transverse interactions and is expected to be unimportant in the region of θ_π^* covered by this experiment.

In order to reduce the model-dependent errors in the determination of the pion form factor, it would be useful to isolate the scalar cross section $d\sigma_0/d\Omega_\pi$ by varying the virtual photon polarization. Such an isolation was attempted recently by Akerlof et al.[5] who measured the cross section at $\theta_\pi^* = 0^\circ$ as a function of ϵ by changing the electron scattering angle. However, because of the extreme difficulty of large-angle electron-scattering coincidence measurements this isolation was not possible and it was necessary to rely on theoretical estimates[4] of the transverse background $d\sigma_T/d\Omega_\pi$.

In the present experiment no isolation of the scalar cross section was attempted. The electron scattering angle was kept as small as possible in order to maximize the counting rate and maintain a polarization in excess of 97 %. Although the interpretation of the data is still limited by theoretical uncertainties, this approach has permitted the measurement of the cross section over a large range of polar and azimuthal angles, and provides the first detailed test of the theories between $\theta_\pi^* = 0^\circ$ and 50°.

The uncertainty in the calculated transverse cross section was estimated by comparing the theories with photoproduction data.[6] The theoretical uncertainties suggested by such a comparison vary between ±4 and ±32 % depending on the c.m. energy and are approximately the same as those quoted by Akerlof et al.

The scalar-transverse interference term recently observed in the closely related reaction $e^- + p \rightarrow e^- + p + \pi^0$, which is expected to be insensitive to F_π, may indicate the presence of similar scalar interactions in the π^+ reaction which are not included in present theoretical estimates of $d\sigma_0/d\Omega_\pi$. However, the most probable explanation of this effect as an interference between the resonant $S_1{}^+$ and $M_1{}^+$ amplitudes[7] would imply a contribution of approximately 5 % to $d\sigma_0/d\Omega_\pi$. We have therefore assumed an error of ±5 % in the scalar cross section to allow for this probability.

Figure 1 shows 10 % of our differential cross-section measurements, plotted as a function of pion azimuthal angle for fixed pion polar angle. The angles are defined in Ref. 1. The polar angle, θ_π^*, is measured with respect to the direction of the three-momentum transfer $\vec{q}$, and φ_π is defined to be zero in the electron scattering plane between $\vec{q}$ and the direction of the incident electron. In addition to the statistical errors shown on the figure, each data set at fixed q^2 and c.m. energy has a normalization uncertainty of approximately ±10 %. Also shown in the figure are the predictions of the dispersion theories of Adler[3] and Zagury[4] for several values of the pion form factor.[8]

An examination of all the data indicates that the general features of the cross section are predicted fairly well, but that there are some significant numerical differences between the theories. Therefore, we have interpreted the data using both theories in order to obtain an estimate of

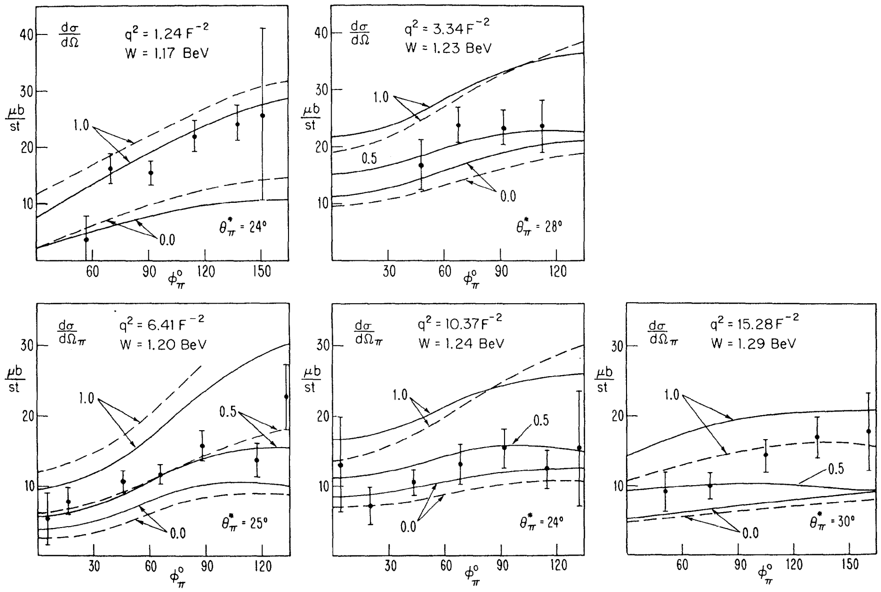

FIG. 1. π^+ azimuthal angular distributions for fixed pion polar angle. The solid and dashed curves are the theories of Adler and Zagury, respectively, evaluated for the indicated values of the pion form factor.

Table I. Measurements of the pion form factor.

$q^2 (F^{-2})$	π-N cm Energy, W (BeV)	F_π (Zagury)	ΔF_π (Zagury) Th. Error	F_π (Adler)	ΔF_π (Adler) Th. Error	Average Values F_π (Zagury)	Average Values F_π (Adler)
1.19	1.22	0.96±0.15	±0.05	0.93±0.05	±0.02		
1.22	1.20	0.94±0.09	±0.07	1.03±0.12	±0.09	0.89±0.08	0.95±0.05
1.24	1.17	0.83±0.07	±0.05	0.96±0.07	±0.05		
3.27	1.27	$0.80\pm^{0.10}_{0.12}$	±0.21	$0.69\pm^{0.09}_{0.12}$	$\pm^{0.18}_{0.24}$		
3.34	1.23	$0.61\pm^{0.09}_{0.11}$	±0.08	$0.48\pm^{0.11}_{0.18}$	$\pm^{0.09}_{0.14}$	0.54±0.08	$0.56\pm^{0.09}_{0.10}$
3.40	1.19	0.43±0.08	±0.08	0.57±0.09	±0.09		
6.29	1.26	$0.64\pm^{0.10}_{0.12}$	$\pm^{0.18}_{0.23}$	$0.45\pm^{0.12}_{0.16}$	$\pm^{0.23}_{0.31}$	0.54±0.08	0.61±0.09
6.41	1.20	0.53±0.06	±0.05	0.62±0.09	±0.05		
10.22	1.28	$0.72\pm^{0.08}_{0.10}$	$\pm^{0.22}_{0.26}$	0.55±0.09	±0.22	$0.60\pm^{0.10}_{0.16}$	$0.41\pm^{0.18}_{0.19}$
10.37	1.24	$0.56\pm^{0.08}_{0.14}$	$\pm^{0.08}_{0.14}$	$0.22\pm^{0.18}_{0.19}$	$\pm^{0.19}_{0.26}$		

model-dependent uncertainties not included in the theoretical errors discussed above.

The results of fits to the data, using the pion form factor as a free parameter, are presented in Table I. The errors listed in columns 3 and 5 are purely experimental. Also shown are estimates of the theoretical error of each measurement, based on the comparison with photoproduction data and the assumed ±5 % uncertainty in $d\sigma_0/d\Omega_\pi$. These errors have been combined in quadrature with the experimental errors and a weighted average for F_π has been calculated at each value of q^2. The theoretical errors were assumed to be completely correlated and constitute the major error in the weighted values of F_π shown in Table I and Fig. 2.

At $q^2 = 15.3$ F^{-2} the difficulty of separating pions from protons restricted the analysis to c.m. energies near 1.29 BeV. In this region the estimated theoretical uncertainties in the transverse part of the cross section are expected to be almost as large as the scalar cross section. Therefore, no attempt was made to extract F_π above $q^2 = 10.4$ F^{-2}.

We have considered the possibility of applying first-order corrections to the data based on the discrepancies observed between theory and experiment in our π^0 results.[1] However, since the relation between large-angle π^0 production and forward-angle π^+ production is somewhat indirect, this procedure has not been applied.[9]

The data shown in Fig. 2 indicate that the pion form factor is probably similar to that of the proton. However, the results are not precise enough to rule out the form-factor dependence suggested by the ρ-dominance model.

The pion charge radius is related to the form factor by

$$r_\pi = [(-6dF_\pi/dq^2)_{q^2=0}]^{1/2}.$$

Shown in Fig. 2 are slopes corresponding to $r_\pi = 0.63$ F (ρ dominance), 0.81 F (proton), and 1.0

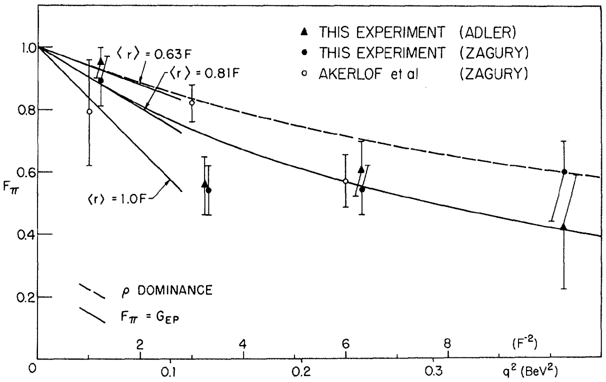

FIG. 2. Estimates of the pion form factor based on dispersion theory.

F. In spite of the fact that the precision of the form-factor values is presently limited by theoretical uncertainties, it seems very probable that the charge radius lies within the above limits.

In order to obtain a more quantitative estimate of the form-factor dependence and charge radius, the four values of $F_\pi(q^2)$ obtained in this experiment using Zagury's theory and the three measurements of Akerlof et al., which were also evaluated with Zagury's theory, have been fitted with a function of the form $F_\pi(q^2)=(1+q^2/M^2)^{-1}$ with variable mass M. The best fit value of M was 0.56 ± 0.06 BeV, which implies $r_\pi = 0.86 \pm 0.09$ F, a result consistent with the behavior of the nucleon form factors, but in rather worse agreement with the hypothesis of ρ dominance. The minimum value of χ^2 was 9 for six degrees of freedom.

The uncertainties quoted above contain estimates of theoretical errors similar to those of Akerlof et al.[5] Additional q^2-dependent errors, not considered in Ref. 5, were estimated by comparing the values of M and r_π obtained upon fitting the form-factor values predicted by each theory. The results, $\Delta M = \pm 0.05$ BeV and $\Delta r_\pi = \pm 0.11$ F, are probably underestimates of the errors, since the two theories are basically very similar, but represent the only numerical estimate available at present. Combining these errors in quadrature with those quoted above, we obtain the following results for the rms radius of the π^+ and the four-momentum transfer dependence of the pion form factor based on electroproduction experiments:

$$r_\pi = 0.86 \pm 0.14 \text{ F},$$

$$F = [1 + q^2/(0.56 \pm 0.08)^2 \text{ BeV}^2]^{-1}.$$

Estimates of the charge radius obtained by other methods are $r_\pi < 3$ F (π-e scattering),[10] and $r_\pi < 1$ F (π-α scattering).[11]

In spite of the theoretical difficulties encountered in the interpretation of electroproduction results, we feel that this method will prove to to be a reliable tool for the understanding of pion structure. The theory is being refined steadily[12] and the extension of the measurements presented in this Letter to higher four-momentum transfers is completely straightforward.[2]

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