Dynamics of 3D gauge theories with antisymmetric matter

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- Strongly interacting QFTs in 4d:
  - Important
  - Interesting
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- 4d SUSY theories offer a perfect laboratory for studying non-perturbative dynamics
  - Gaugino condensation, instantons, confinement with and without chiral symmetry breaking, duality...
  - Still hard!
- 3d SUSY theories: a simpler lab for strong QFT dynamics
  - Many 4d phenomena in 3d setting
  - Instanton-monopoles, Chern-Simons terms. real masses...
  - Lab for study of condensed matter systems?
  - Many results over the years, significant progress in the last year

Affleck, Harvey, Witten (82); Seiberg, Witten (96); Intriligator, Seiberg (96)

de Boer, Hori, Oz (97); Aharony, Hanany, Intriligator, Seiberg, Strassler (97)

Aharony, Razamat, Seiberg, Willet (13); Aharony, Seiberg, Tachikawa (13)

Motivation

SUSY in 3d

- Instanton-monopoles
- Fermion zero modes
- Global coordinates on the Coulomb branch

s-confinment in 3d

- $SU(4)$ with two antisymmetrics
- Matching 4d to 3d
- Consistency checks
- $SU(4)$ with a single antisymmetric

Summary and outlook

SUSY in 3 dimensions

- $\mathcal{N} = 2$ 3d theory from $\mathcal{N} = 1$ 4d theory:
  - Real scalar from dimensional reduction $A_\mu \to A_i^{(3)}, \sigma$
  - 3d photon dual is dual to a scalar $\partial^i \gamma = \epsilon^{ijk} F_{jk}$
  - Holomorphic modulus $\Phi = \sigma + i\gamma$
- r_G dimensional Coulomb branch parameterized by

$$\sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_N)\,, \quad \mathrm{Tr}\,(\sigma) = 0$$

- Work in Weyl chamber to remove gauge redundancy:

$$\sigma_1 > \sigma_2 > \ldots > \sigma_N$$

- Convenient parameterization

$$Y_i \sim \exp(\vec{\Phi}_i \cdot \vec{\alpha}/g_3^2)\,, \quad i = 1, \ldots, r_G$$

- Classical moduli space is a cylinder

Instanton-monopoles

- In 3d instantons can only exist on Coulomb branches of non-abelian theories.
- E.g.: instanton in $SU(2) \to U(1)$:
  - Symmetry breaking pattern $\mathrm{diag}(\sigma, -\sigma)$
  - Start with 4d theory on $\mathbb{R}^3 \times S^1$ and wrap monopole around compact direction: 3 dimensional instanton-monopole
  - Gauginos have 2 zero mode in the instanton-monopole background and aquire a mass

$$W = \frac{1}{Y}\,, \quad Y \sim \exp(2\sigma)$$

  - Coulomb branch is lifted, no ground state

Instanton-monopoles

- Dynamics of $SU(N)$
  - $N-1$ dimensional Coulomb branch: $Y_i \sim \exp(\sigma_i - \sigma_{i+1})$
  - $N-1$ linearly independent $SU(2)$ factors lead to $N-1$ fundamental instanton-monopoles
  - Each fundamental monopole has 2 gaugino zero modes, the full superpotential is

$$W = \sum_{i=1}^{N-1} \frac{1}{Y_i}$$

    No ground state
  - Theory on a circle, $\mathbb{R}^3 \times S^1$ has a KK monopole which winds around compact dimension.

$$Y_N = \exp(\sigma_N - \sigma_1) = \frac{1}{\prod Y_i} = \frac{1}{Y}$$

$$W = \sum_{i}^{N-1} \frac{1}{Y_i} + \eta Y$$

Matter fields in 3d

- SUSY in 3d allows real mass terms. E.g. gauge baryon number and give vev to $A_3 = \sigma_b = m_{\mathbb{R}}$:

$$K = Q^\dagger e^V Q + \bar{Q}^\dagger e^{-V} \bar{Q} \supset Q^\dagger e^{\sigma_b} Q + \bar{Q}^\dagger e^{-\sigma_b} \bar{Q}$$

- Q and $\bar{Q}$ have real masses $m_{\mathbb{R}}$ and $-m_{\mathbb{R}}$ respecively
- Additional contributions on the Coulomb branch

$$\left| \langle \sigma^a T^a \rangle^\alpha{}_\beta Q^\beta{}_i \right|^2 \quad \text{no sum over } \alpha$$

- The fermion has zero modes in i^{th} monopole background if effective real mass is

$$\sigma_i > m_{eff} > \sigma_{i+1}$$

Matter fields in 3d: examples

- Doublet of $SU(2)$ has one zero mode if $\sigma > m_{\mathbb{R}} > -\sigma$

Matter fields in 3d: examples

- Doublet of $SU(2)$ has one zero mode if $\sigma > m_{\mathbb{R}} > -\sigma$
- Massless fundamental on the Coulomb branch of $SU(4)$

$$\begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \sigma_3 & \\ & & & -\sum_i \sigma_i \end{pmatrix}$$

  - Zero mode in the 3^{rd} instanton-monopole
  - No AHW suportotential for Y_3: $W = 1/Y_1 + 1/Y_2$

Matter fields in 3d: examples

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$$\begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & 0 & \\ & & & -\sum_i \sigma_i \end{pmatrix}$$

  - Zero mode in the 2^{nd} and 3^{rd} instanton-monopoles
  - No AHW superpotential for Y_2 and Y_3: $W = 1/Y_1$

Matter fields in 3d: examples

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$$\begin{pmatrix} \sigma_1 & & & \\ & 0 & & \\ & & 0 & \\ & & & -\sigma_1 \end{pmatrix}$$

  - Zero mode in all instanton-monopole
  - No superpotential
  - One dimensional Coulomb branch: $Y = \prod Y_i$

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  - Fundamental has no zero modes in KK monopole
  - The superpotential on a circle is $W = \eta Y$

Matter fields in 3d: moduli charges

$$\begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \sigma_3 & \\ & & & \sigma_4 \end{pmatrix}$$

- Region I: $\sigma_1 > 0 > \sigma_2 > \sigma_3 > \sigma_4$
$$W = \frac{1}{Y_2} + \frac{1}{Y_3}, \quad R(Y_2) = R(Y_3) = -2,\ R(Y_1) = F - 2$$
- Region II: $\sigma_1 > \sigma_2 > 0 > \sigma_3 > \sigma_4$
$$W = \frac{1}{Y_1} + \frac{1}{Y_3}, \quad R(Y_1) = R(Y_3) = -2,\ R(Y_2) = F - 2$$
- Region III: $\sigma_1 > \sigma_2 > \sigma_3 > 0 > \sigma_4$
$$W = \frac{1}{Y_1} + \frac{1}{Y_2}, \quad R(Y_1) = R(Y_2) = -2,\ R(Y_3) = F - 2$$
- $Y = \prod Y_i$ is globally defined

Matter fields in 3d: antisymmetric of $SU(4)$

- On the Coulomb branch A_{ij} has a real mass $\sigma_i + \sigma_j$

$$\begin{pmatrix} \sigma_1 & & \mathrm{x} & \mathrm{x} \\ & \sigma_2 & \mathrm{x} & \mathrm{x} \\ & & \sigma_3 & \\ & & & \sigma_4 \end{pmatrix}$$

- Instanton in the first $SU(2)$:
  Zero modes if $\sigma_1 + \sigma_{3,4} \geq 0$, $\sigma_2 + \sigma_{3,4} \leq 0$
- Instanton in the third $SU(2)$:
  Zero modes if $\sigma_3 + \sigma_{1,2} \geq 0$, $\sigma_4 + \sigma_{1,2} \leq 0$
- Instanton in the second $SU(2)$:
  Doublets have zero modes if $\sigma_2 \to \sigma_1$ and $\sigma_3 \to \sigma_4$
- One dimensional Coulomb branch $\widetilde{Y} = \sqrt{Y_1 Y_2^2 Y_3}$

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- On the Coulomb branch A_{ij} has a real mass $\sigma_i + \sigma_j$

$$\begin{pmatrix} \sigma & & & \\ & \sigma & & \\ & & -\sigma & \\ & & & -\sigma \end{pmatrix}$$

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s-confinement in $SU(4)$

- Two antisymmetrics and two fundamental flavors

	$SU(4)$	$SU(2)$	$SU(2)_L$	$SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)'$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	1	0	0	-3	0
Q	$\square$	1	$\square$	1	1	0	2	$\frac{1}{3}$
$\overline{Q}$	$\overline{\square}$	1	1	$\overline{\square}$	0	-1	2	$\frac{1}{3}$

- Expect s-confinement
- Potentially 2-dimensional Coulomb branch, Y and $\widetilde{Y}$
- Symmetries and quantum numbers

	$U(1)_1$	$U(1)_2$	$U(1)'$	$U(1)_R$
$Y = \prod_i Y_i$	-2	2	4	$\frac{2}{3}$
$\tilde{Y} = \sqrt{Y\tilde{Y}_2}$	-2	2	-2	$\frac{2}{3}$

From 4d to 3d: s-confinement in $\mathbb{R}^4$

- The 4d model s-confining model

	$SU(4)$	$SU(2)$	$SU(3)_L$	$SU(3)_R$	$U(1)$	$U(1)'$	$U(1)_R$
A	$\begin{array}{c}\square\\[-4pt]\square\end{array}$	$\square$	1	1	0	-3	0
Q	$\square$	1	$\square$	1	1	2	$\frac{1}{3}$
$\overline{Q}$	$\overline{\square}$	1	1	$\overline{\square}$	-1	2	$\frac{1}{3}$

- Charges of the components

	$SU(4)$	$SU(2)$	$SU(3)_L$	$SU(3)_R$	$U(1)$	$U(1)'$	$U(1)_R$
$M_0 = Q\bar{Q}$	1	1	$\square$	$\overline{\square}$	0	4	$\frac{2}{3}$
$M_2 = QA^2\bar{Q}$	1	1	$\square$	$\overline{\square}$	0	-2	$\frac{2}{3}$
$H = AQ^2$	1	$\square$	$\overline{\square}$	1	2	1	$\frac{2}{3}$
$\bar{H} = A\bar{Q}^2$	1	$\square$	1	$\square$	-2	1	$\frac{2}{3}$
$T = A^2$	1	$\square\!\square$	1	1	0	-6	0

- Exact non-perturbative superpotential

$$W_{\text{dyn}} = \frac{1}{\Lambda^7}\left(T^2 M_0^3 - 12 T H \bar{H} M_0 - 24 M_0 M_2^2 - 24 H \bar{H} M_2\right)$$

From 4d to 3d: s-confinement on $\mathbb{R}^3 \times S^1$

- KK instanton generates the superpotential

$$W = W_{\text{dyn}} + \eta Y$$

- Need to decouple KK instanton contribution:
  - Gauge a diagonal $U(1)$ in $SU(3)_L \times SU(3)_R \times U(1)$
  - Decouple the third (massive) flavor and ηY
  - Unbroken global symmetries the same as in 3d model
  - Composites without third flavor quark remain masslees
  - Composites with only one third flavor quark are heavy
  - M_0^{33} and M_2^{33} are neutral under $U(1)$: massless
  - M_0^{33} and M_2^{33} have the same charges as Y and $\widetilde{Y}$
- Claim: 3d dual desription:

$$W_{\text{dyn}} = Y\left(3T^2 \det M_0 - 12Th\overline{h} - 24 \det M_2\right) + \widetilde{Y}\left(2M_0M_2 + h\overline{h}\right)$$

Tests

- Back to $\mathbb{R}^3 \times S^1$:
  - 4d quantum modified moduli space:

$$\begin{aligned} W =& \lambda\left(3T^2 \det M_0 - 12Th\bar{h} - 24 \det M_2 - \Lambda^8\right) \\ &+ \mu\left(2M_0M_2 + h\bar{h}\right) + \eta Y \end{aligned}$$

  - Identify λ and μ with Y and $\widetilde{Y}$ ($\eta \sim \Lambda^8$)
- Coulomb branch: large $\tilde{Y}$:
  - Semiclassical symmetry breaking pattern:

$$SU(4) \to SO(4) \times U(1)$$

  - All fundamentals obtain large real mass
  - Two light $SO(4)$ vectors survive from antisymmetric
  - Low energy 3d physics is known to s-confine with one Coulomb modulus $Y_{SO} \sim Y^2/\widetilde{Y}^2$

s-confinement with a single antisymmetric

- $SU(4)$ with and antisymmetric and three flavors
- How many Coulomb branch moduli?
- Y and $\widetilde{Y}$ are not lifted by instanton-monopoles
- No candidate for $\tilde{Y}$ in $\mathbb{R}^3 \times S^1$ model
  - There is no $M_2 \sim QA^2\bar{Q}$ composite
- Expect new dynamical effects to lift $\tilde{Y}$
- The low energy dynamics

$$W = Y\left(T \det M + HM\bar{H}\right)$$

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- Compare models with one and two antisymmetrics:
  - Holomorphic mass for the fundamental flavor in theory II
  - Holomorphic mass for antisymmetric in theory I
  - $\widetilde{Y}$ decouples when integrating out antisymmetric
  - Low energy descriptions agree
- Coulomb branch again: large $\tilde{Y}$
  - $SU(4) \to SO(4) \times U(1)$
  - A single $SO(4)$ vector
  - ADS-like superpotential

Aharony, Shamir

$$W = \frac{1}{Y_{SO}^4 T}$$

  - Carefully matching moduli: $\widetilde{Y}$ lifted (preliminary).

Summary and Outlook