Minimal Non-SUperSYmmetric Unified Model

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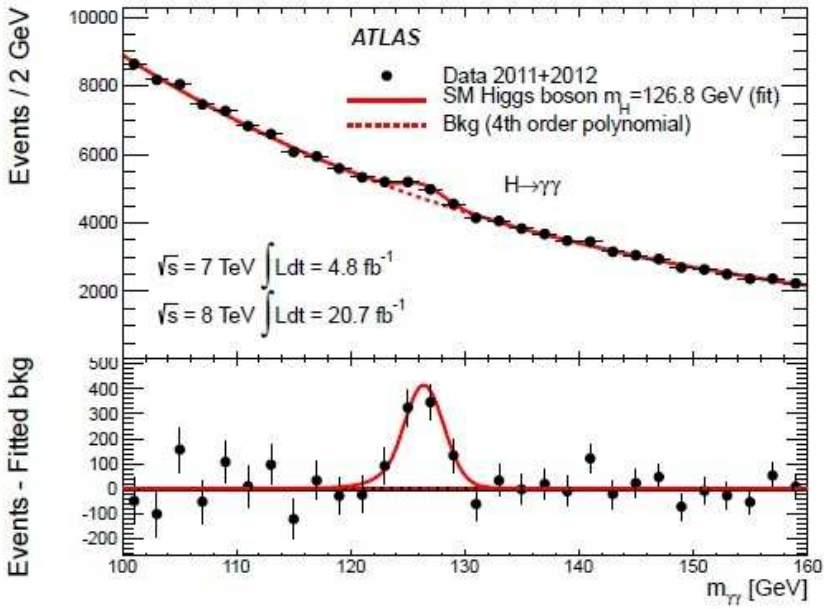
``LHC After the Higgs''

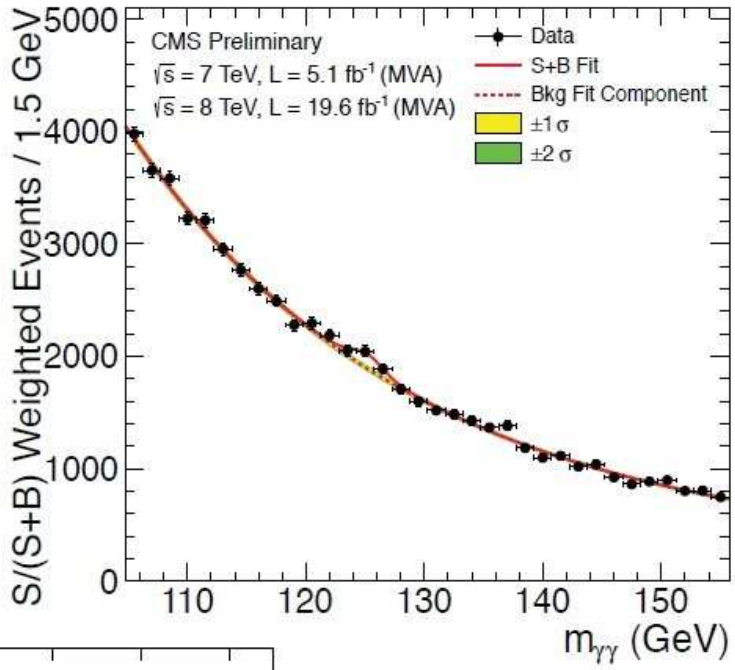
Work done with K.S. Babu

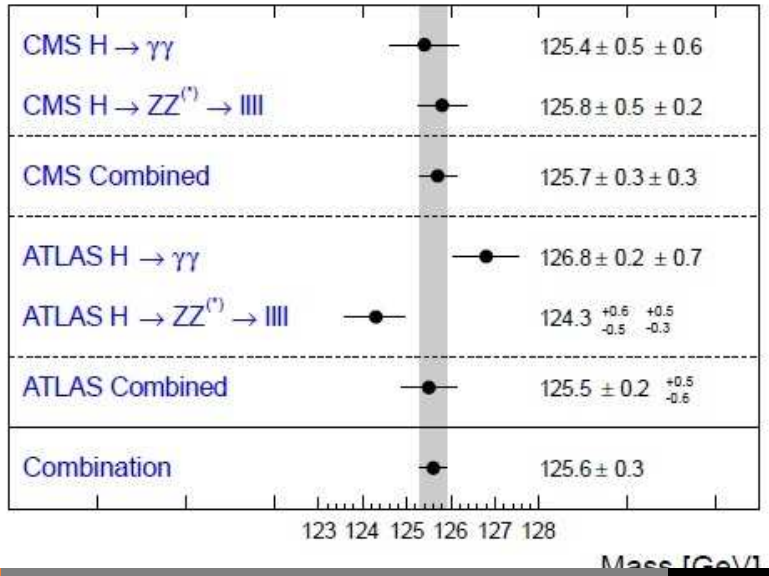
Overview

1. Discovery of Higgs at LHC
2. Motivation
3. $SO(10)$ Grand Unification
4. The Model
5. Evolution of Gauge Couplings
6. Revisit the Model
7. Scalar Mass Spectra
8. Fermion Masses, Mixings and Leptogenesis
9. Axion
10. Future Work

Discovery of Higgs at LHC







Stability of Higgs potential

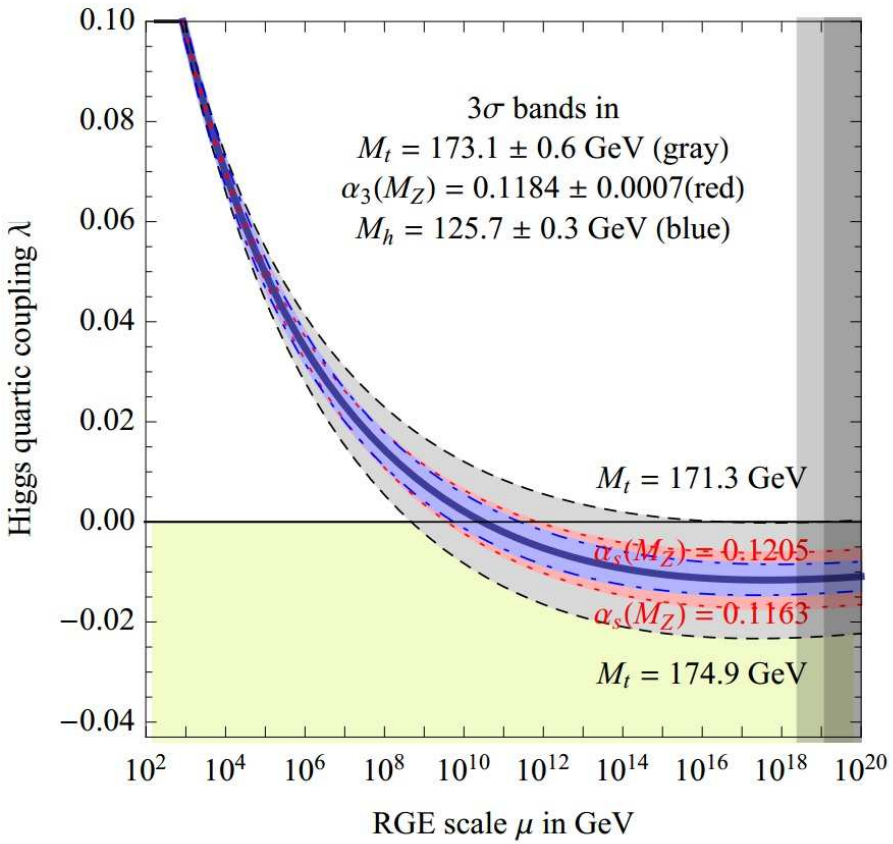


Figure : RG evolution of λ varying M_t, $\alpha_3(M_Z)$ and M_h by $\pm 3\sigma$

$$\Lambda = \mathcal{O}(10^{10} - 10^{12} GeV)$$

J. Elias-Miro et al., Phys. Lett. B709, 222 (2012)
G. Degrassi et al., JHEP 1208, 098 (2012);
D. Buttazzo et al., arXiv:1307.3536 [hep-ph]

New Physics?!?

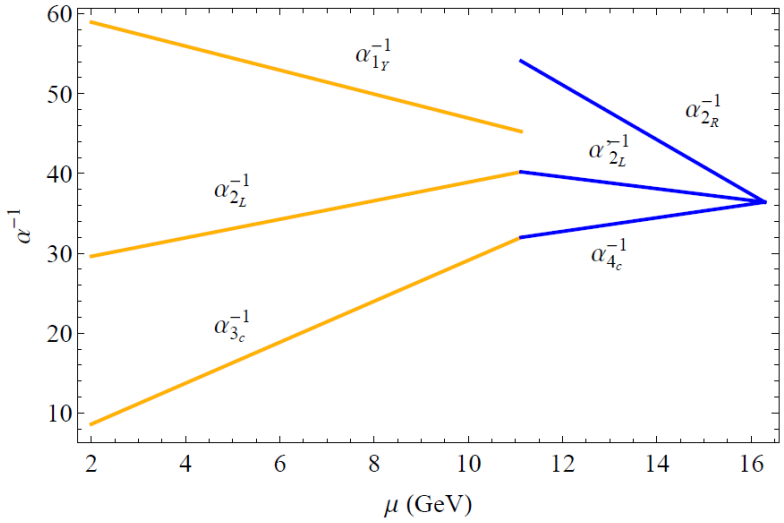
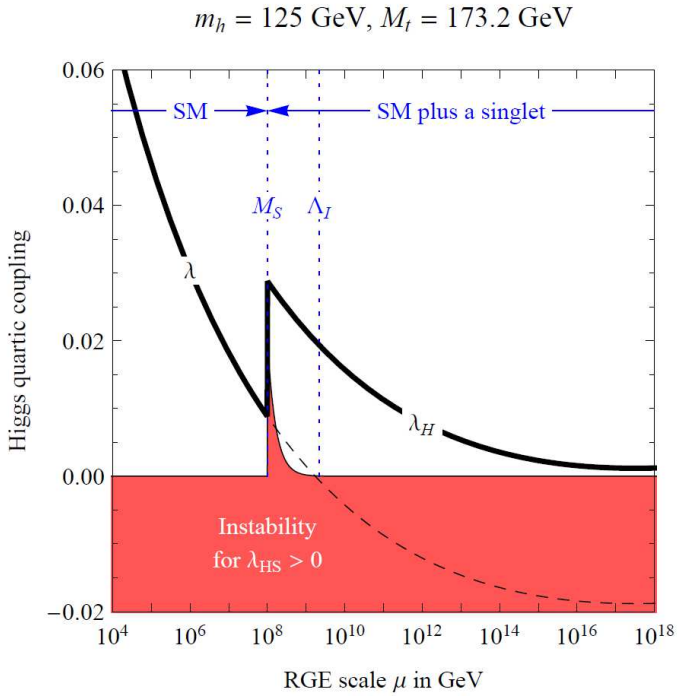


Figure 1: *Evolution of the coupling constants from M_Z to M_{GUT}.*

G. Altarelli and D. Meloni, arXiv:1305.1001 [hep-ph];



J. Elias-Miro et al. [arXiv:1203.0237 [hep-ph]].;

- A good number of GUT Models (like $SO(10)$ tend to have an intermediate scale around $\mathcal{O}(10^{10} - 10^{12} GeV)$.
- Even a single scalar with appropriate mass (ie. M_s $10^8 GeV$) and properties can change the RG run of the Higgs quartic coupling making the potential stable.

Motivation

Looking for a Minimal Realistic Unified Model with the properties:

- Unification of the coupling happens at large enough energy scale which is compatible with the proton-lifetime
- Some kind of particle spectrum which can modify the higgs quartic coupling so that stability issue of the electro weak vacuum can be addressed
- A realistic Yukawa sector which can generate realistic fermion masses and Mixings including neutrino data
- Can also generate the baryon asymmetry (most probably via leptogenesis)
- An axion suitable to so solve Strongs CP problem and account for the observed Dark Matter.

G. Altarelli and D. Meloni, 2013
Malinsky et al, 2011, 2012, 2013

$SO(10)$ Grand Unification

- 16-dimensional spinor representation of $SO(10)$ can accommodate **ALL** fermions of one generation

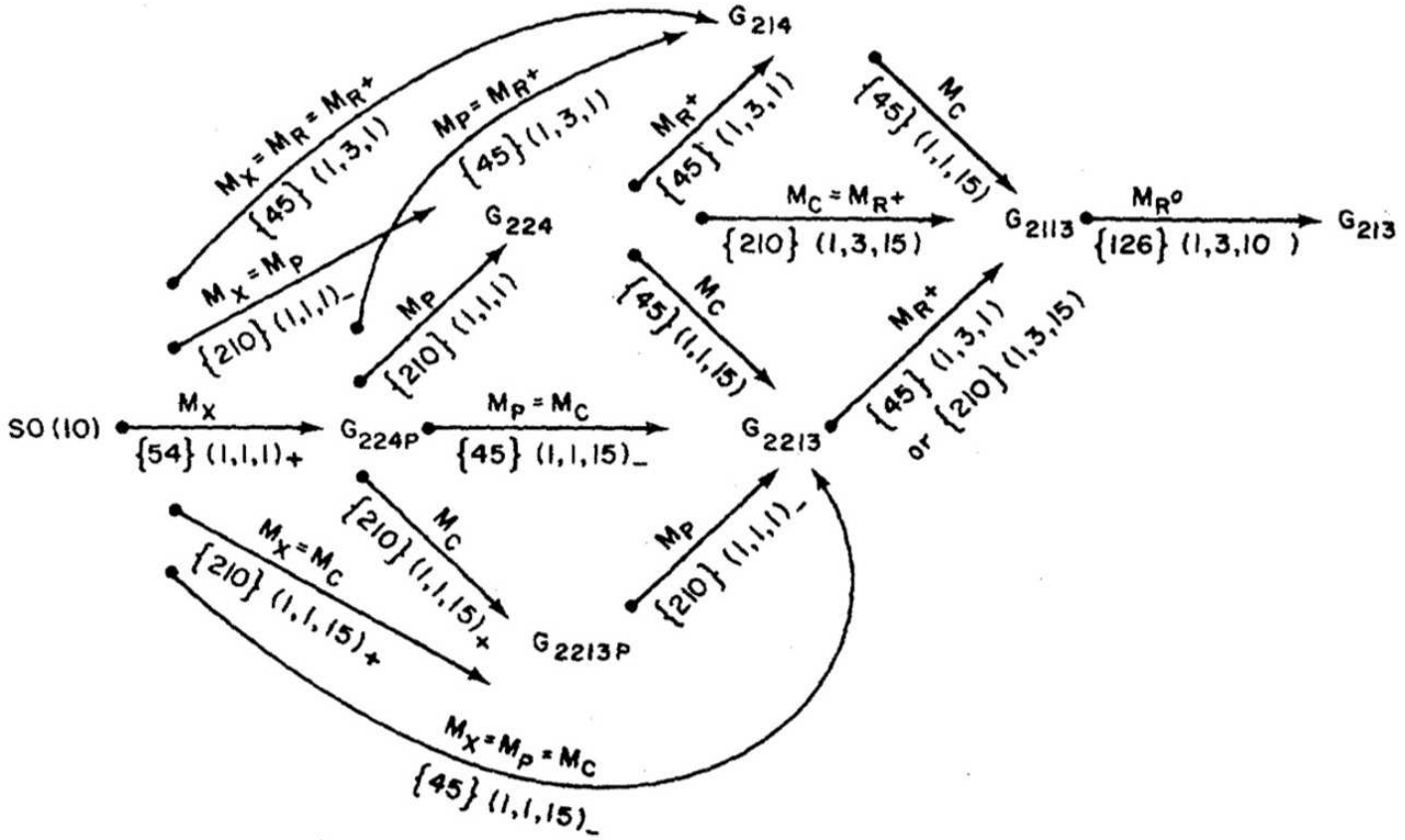
$u_r : \{-++\ +-\}$	$d_r : \{-++\ -+\}$	$u_r^c : \{+--\ ++\}$	$d_r^c : \{+--\ --\}$
$u_b : \{+-+\ +-\}$	$d_b : \{+-+\ -+\}$	$u_b^c : \{-+-\ ++\}$	$d_b^c : \{-+-\ --\}$
$u_g : \{++-\ +-\}$	$d_g : \{++-\ -+\}$	$u_g^c : \{--+\ ++\}$	$d_g^c : \{--+\ --\}$
$\nu : \{---\ +-\}$	$e : \{---\ -+\}$	$\nu^c : \{+++\ ++\}$	$e^c : \{+++\ --\}$

The first 3 indicates color spin and last two weak spin.

$$Y = \frac{1}{3}\sum(C) - \frac{1}{2}\sum(W)$$

- The gauge interactions of $SO(10)$ conserve parity thus making parity a part of a continuous symmetry.
- Unification of three couplings (α_s, α_{2L} and α_Y) into one coupling constant α_{GUT}.
- Existence of ν_R and thus neutrino mass via seesaw.
- Baryon asymmetry.

$SO(10) \to \ldots \to$ SM



Chang, Mohapatra, Gipson,Marshak,Parida 1985.
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Bertolini, Luzio, Malinsky 2009

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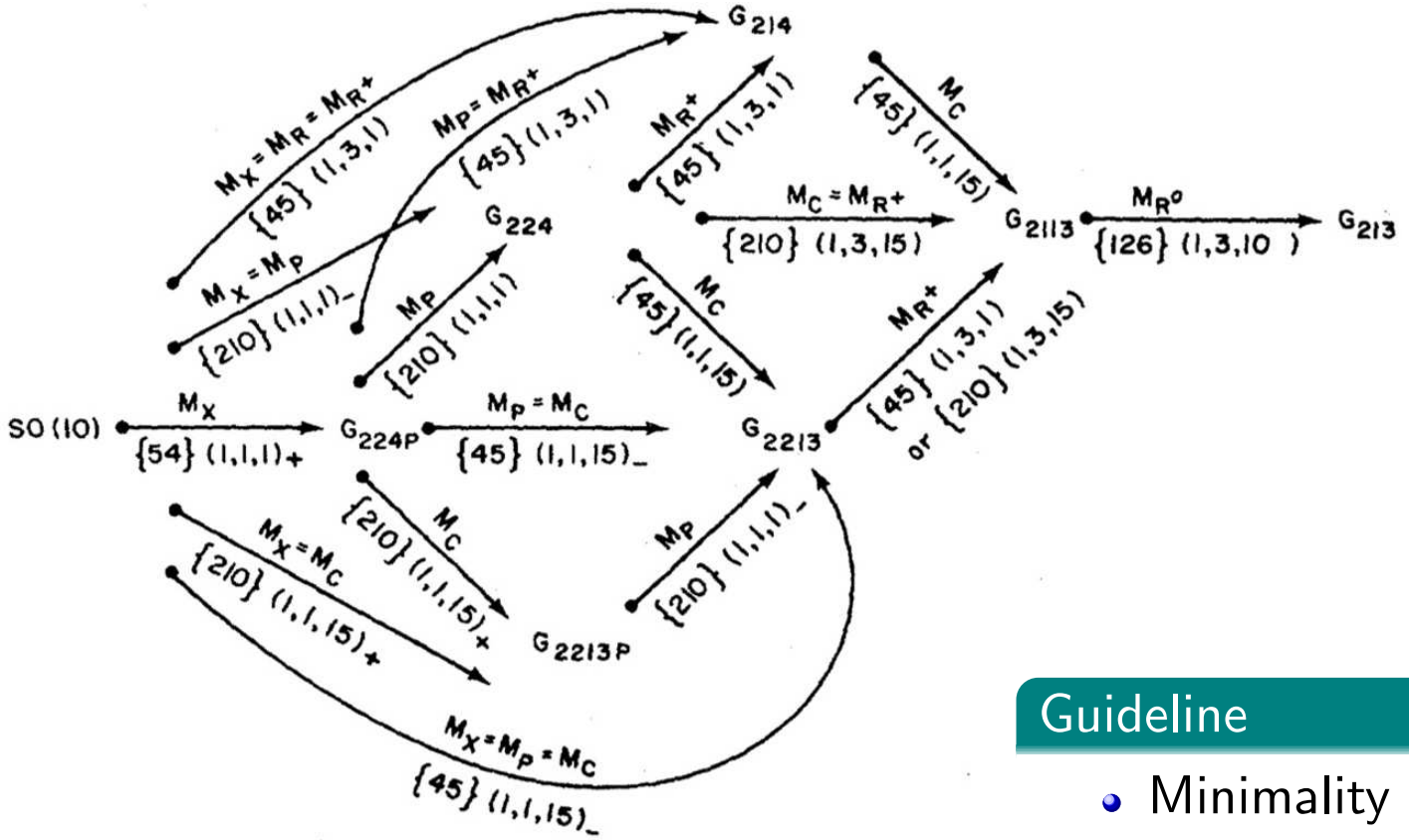

Guideline

- Minimality
- Predictivity
- Phenomenological issues

Chang, Mohapatra, Gipson,Marshak,Parida 1985.
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Energy Scales associated with $SO(10)$ GUT

Five possible energy scales in the left-right decomposition of $SO(10)$

- M_U for $SO(10) \Rightarrow SU(4) \times SU(2)_L \times SU(2)_R$
- M_C for $SU(4) \Rightarrow SU(3)_C \times U(1)_{B-L}$
- M_R for $SU(2)_R \Rightarrow U(1)_R$
- M'_R for $U(1)_R \times U(1)_{B-L} \Rightarrow U(1)_Y$
- M_W for $SU(2)_L \times U(1)_Y \Rightarrow U(1)_Q$

Condition:

$$M_U \geq M_C \geq M_R \geq M'_R \geq M_W$$

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Minimal Case: **One intermediate Scale**

- Constrained
- Predictive

Possible cases:

- $M_U = M_C \geq M_R = M'_R \geq M_W$
- $M_U \geq M_C = M_R = M'_R \geq M_W$

Possible Higgs Sector

Let's try:

- $45 + 16 \rightarrow$ wants to breaks via $SU(5)$, which is ruled out by proton lifetime. Also, disfavored by Neutrino oscillation data, as effective $B - L$ scale is suppressed and the light neutrino masses are overshoot.

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- $54 + 126 \rightarrow$ possible candidate!!!

The Model

$$SO(10) \xrightarrow{\text{Real} \quad 54} SU(4)_C \times SU(2)_L \times SU(2)_R \times D$$

$$SU(4)_C \times SU(2)_L \times SU(2)_R \times D \xrightarrow{\text{Complex} \quad 126} SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\text{Complex} \quad 10} SU(3)_C \times U(1)_Q$$

Higgs Mass Scale

Extended Survival Hypothesis(ESH): At any scale, the only scalar multiplets present are those that develop VEVs at smaller scales.
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- $126 = (10,1,3) + (\overline{10},3,1) + (15,2,2) + (6,1,1)$
  - $< (10,1,3) >$ breaks $PS \times D$ $\Rightarrow$ $(10,1,3)$ **@** M_i.
  - D-parity $\Rightarrow$ $(\overline{10},3,1)$ **@** M_i.
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- $10 = (1,2,2) + (6,1,1) = (1,2,+1/2) + (1,2,-1/2) + (3,1,-1/3) + (\overline{3},1,+1/3)$
  - $< (1,2,-1/2) >$ breaks EW $\Rightarrow$ $(1,2,-1/2)$ **@** M_w
  - ESH $\Rightarrow$ all other **@** Higher Scale

The Higgs Sector

SO(10)	$SU(4)_C \times SU(2)_L \times SU(2)_R$	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Scale
10	$H_1(6,1,1)$	$T_1(3,1,-\frac{1}{3})$	M_u
		$T_2(\overline{3},1,+\frac{1}{3})$	M_u
	$H_2(1,2,2)$	$R_1(1,2,+\frac{1}{2})$	M_i
		$R_2(1,2,-\frac{1}{2})$	M_w
54	$\zeta_1(1,3,3)$	$\phi_1(1,3,+1)$	M_u
		$\phi_2(1,3,0)$	M_u
		$\phi_3(1,3,-1)$	M_u
	$\zeta_2(6,2,2)$	$\phi_4(3,2,+\frac{1}{6})$	M_u
		$\phi_5(3,2,-\frac{5}{6})$	M_u
		$\phi_6(\overline{3},2,+\frac{5}{6})$	M_u
		$\phi_7(\overline{3},2,-\frac{1}{6})$	M_u
	$\zeta_3(20',1,1)$	$\phi_8(\overline{6},1,+\frac{2}{3})$	M_u
		$\phi_9(6,1,-\frac{2}{3})$	M_u
		$\phi_{10}(8,1,0)$	M_u
	$\zeta_0(1,1,1)$	$\phi_0(1,1,0)$	M_u

Contd...

SO(10)	$SU(4)_C \times SU(2)_L \times SU(2)_R$	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Scale
126	$\Sigma_1(6, 1, 1)$	$\Sigma_{11}(3, 1, -\frac{1}{3})$	M_u
		$\Sigma_{12}(\overline{3}, 1, +\frac{1}{3}$	M_u
	$\Sigma_2(10, 3, 1)$	$\Sigma_{21}(1, 3, -1)$	M_i
		$\Sigma_{22}(3, 3, -\frac{1}{3})$	M_i
		$\Sigma_{23}(6, 3, +\frac{1}{3})$	M_i
	$\Sigma_3(\overline{10}, 1, 3)$	$\Sigma_{31}(1, 1, 0)$	M_i
		$\Sigma_{32}(1, 1, +1)$	M_i
		$\Sigma_{33}(1, 1, +2)$	M_i
		$\Sigma_{34}(\overline{3}, 1, +\frac{4}{3})$	M_i
		$\Sigma_{35}(\overline{3}, 1, +\frac{1}{3})$	M_i
		$\Sigma_{36}(\overline{3}, 1, -\frac{2}{3})$	M_i
		$\Sigma_{37}(\overline{6}, 1, -\frac{4}{3})$	M_i
		$\Sigma_{38}(\overline{6}, 1, -\frac{1}{3})$	M_i
		$\Sigma_{39}(\overline{6}, 1, +\frac{2}{3})$	M_i
	$\Sigma_4(15, 2, 2)$	$\Sigma_{41}(1, 2, +\frac{1}{2})$	M_i
		$\Sigma_{42}(1, 2, -\frac{1}{2})$	M_i
		$\Sigma_{43}(3, 2, +\frac{7}{6})$	M_i
		$\Sigma_{44}(3, 2, +\frac{1}{6})$	M_i
		$\Sigma_{45}(\overline{3}, 2, -\frac{1}{6})$	M_i
		$\Sigma_{46}(\overline{3}, 2, -\frac{7}{6})$	M_i
		$\Sigma_{47}(8, 2, +\frac{1}{2})$	M_i
		$\Sigma_{48}(8, 2, -\frac{1}{2})$	M_i

Evolution of the Gauge Couplings

$$\beta(g) = \mu\frac{dg}{d\mu} = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}C_2(G) - \frac{4}{3}\kappa S_2(F) - \frac{1}{6}\eta S_2(S)\right\} \tag{1}$$
$$-\frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}\left[C_2(G)\right]^2 - \kappa\left[4C_2(F) + \frac{20}{3}C_2(G)\right]S_2(F) - \left[2C_2(S) + \frac{1}{3}C_2(G)\right]\eta S_2(S)\right\} + \cdots$$

Two- Loop RG Equation:

$$\frac{d\alpha^{-1}(\mu)}{d\ln\mu} = -\frac{a_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2\alpha_j^{-1}}$$

where,

$$\begin{aligned}
a_i &= -\frac{11}{3}C_2(G_i) + \frac{4}{3}\kappa S_2(F_i) + \frac{1}{6}\eta S_2(S_i) \\
b_{ii} &= -\frac{34}{3}\left[C_2(G_i)\right]^2 + \kappa\left[4C_2(F_i) + \frac{20}{3}C_2(G_i)\right]S_2(F_i) \\
&\quad - \left[2C_2(S_i) + \frac{1}{3}C_2(G_i)\right]\eta S_2(S_i) \\
b_{ij} &= 4\kappa C_2(F_j)S_2(F_i) + 2\eta C_2(S_j)S_2(S_i)
\end{aligned}$$

Here $i \neq j$.

Contd...

β coefficients of the theory:

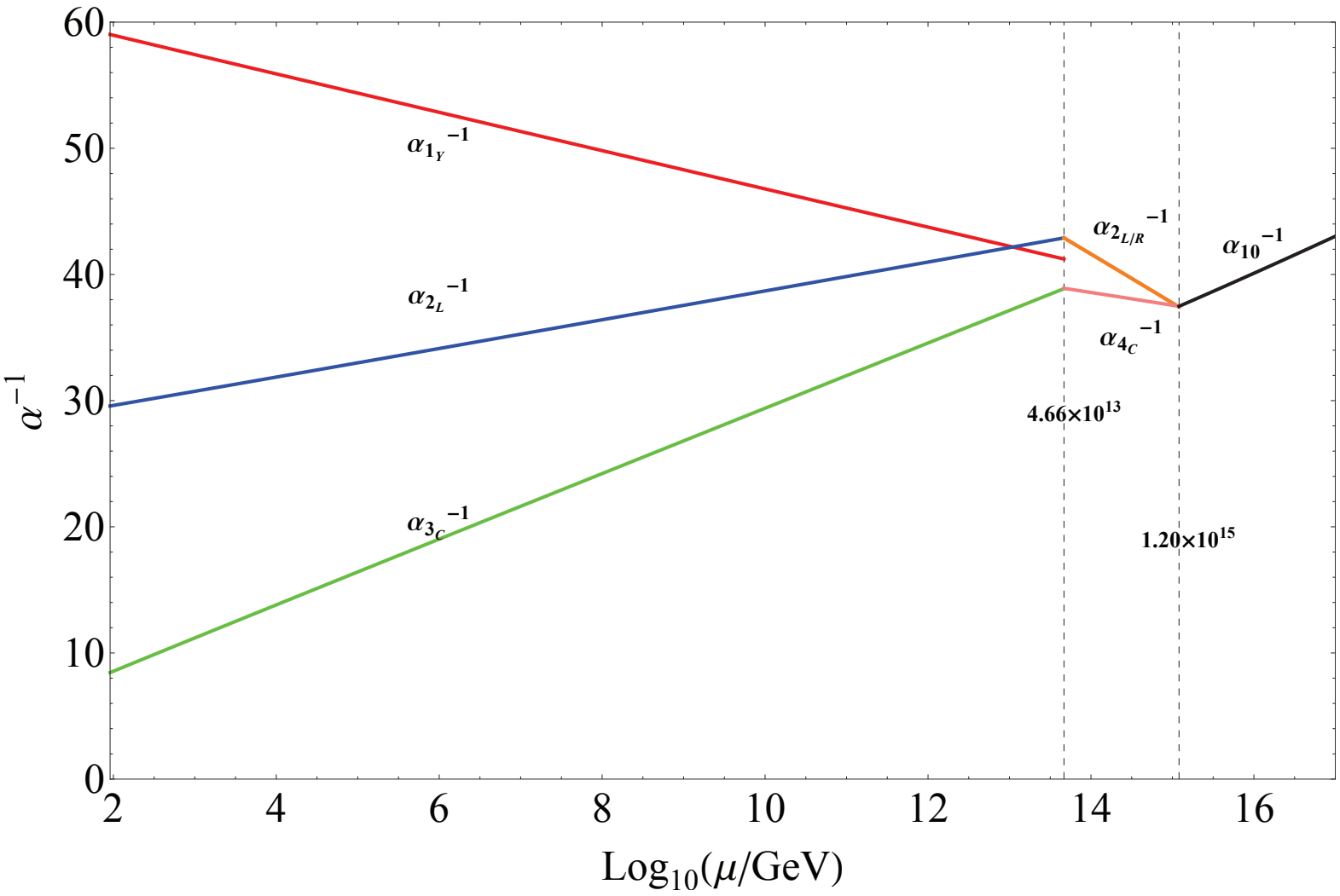
$$a_{SM}^T = \begin{pmatrix} \frac{41}{10} & -\frac{19}{6} & -7 \end{pmatrix}; \qquad b_{SM} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix};$$

$$a_{PS}^T = \begin{pmatrix} \frac{26}{3} & \frac{26}{3} & 1 \end{pmatrix}; \qquad b_{PS} = \begin{pmatrix} \frac{779}{3} & 48 & \frac{1245}{2} \\ 48 & \frac{779}{3} & \frac{1245}{2} \\ \frac{249}{2} & \frac{249}{2} & \frac{1209}{2} \end{pmatrix};$$

While the matching conditions are:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_G(\mu)} - \lambda_i(\mu) \tag{2}$$

Evolution of Gauge Couplings



Proton Lifetime

$$\tau_P \approx \frac{m_X^4}{g^4 m_P^5}$$

Here,

$$\begin{aligned} g^2 &\approx \frac{4\pi}{35} \\ m_X &\approx 1.20 \times 10^{15} GeV \end{aligned}$$

So,

$$\tau_P \approx 5 \times 10^{29} yrs$$

Current Limit on proton life time:

$$\tau_P > 1.29 \times 10^{34} yrs$$

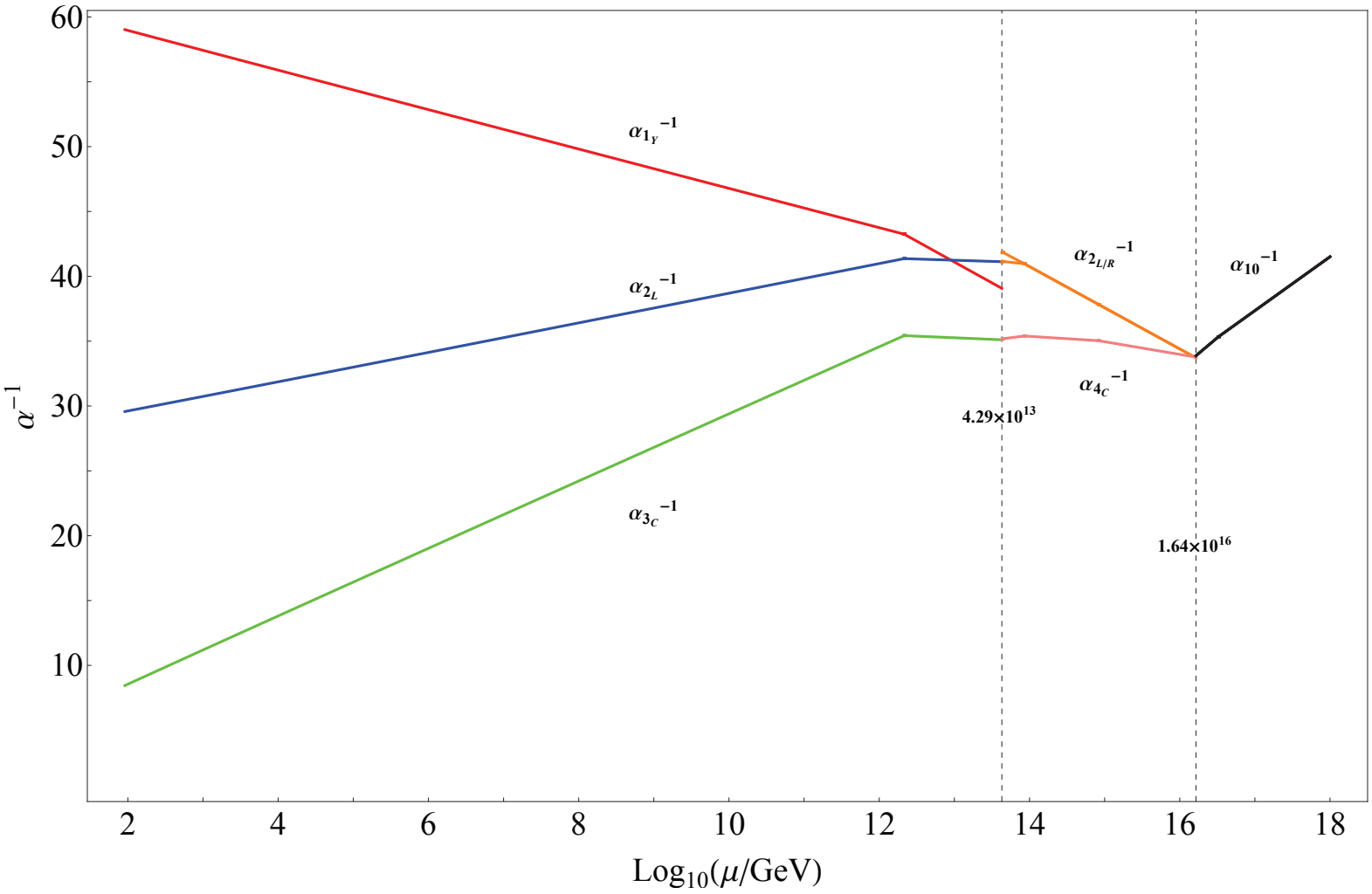
⇒ Let's be a little bit more careful!

⇒ There are lots of scalar particles.

⇒ All of them cannot be degenerate!

⇒ Proton Decay equation is too crude!

Threshold Corrections are Important!



Proton Lifetime (Revisited)

$$\tau_P \approx \frac{\pi}{4} R_L^2 (1+F+D) \frac{|\alpha|^2}{f_\pi^2} m_p \alpha_G^2 \left[A_{SR}^2 \left(\frac{1}{M_{(X,Y)}^2} + \frac{1}{M_{(X',Y')}^2} \right)^2 + \frac{4A_{SL}^2}{M_{(X,Y)}^4} \right]^{-1}$$

Here,

$$A_{SL(R)} = \prod_{i=1}^{n} \prod_{sc}^{Mz \leq m_{sc} < M_G} \left[\frac{\alpha_i(m_{sc+1})}{\alpha_i(m_{sc})} \right]^{\frac{\gamma_{L(R)i(sc)}}{b_i(m_{sc+1} - m_{sc})}}$$

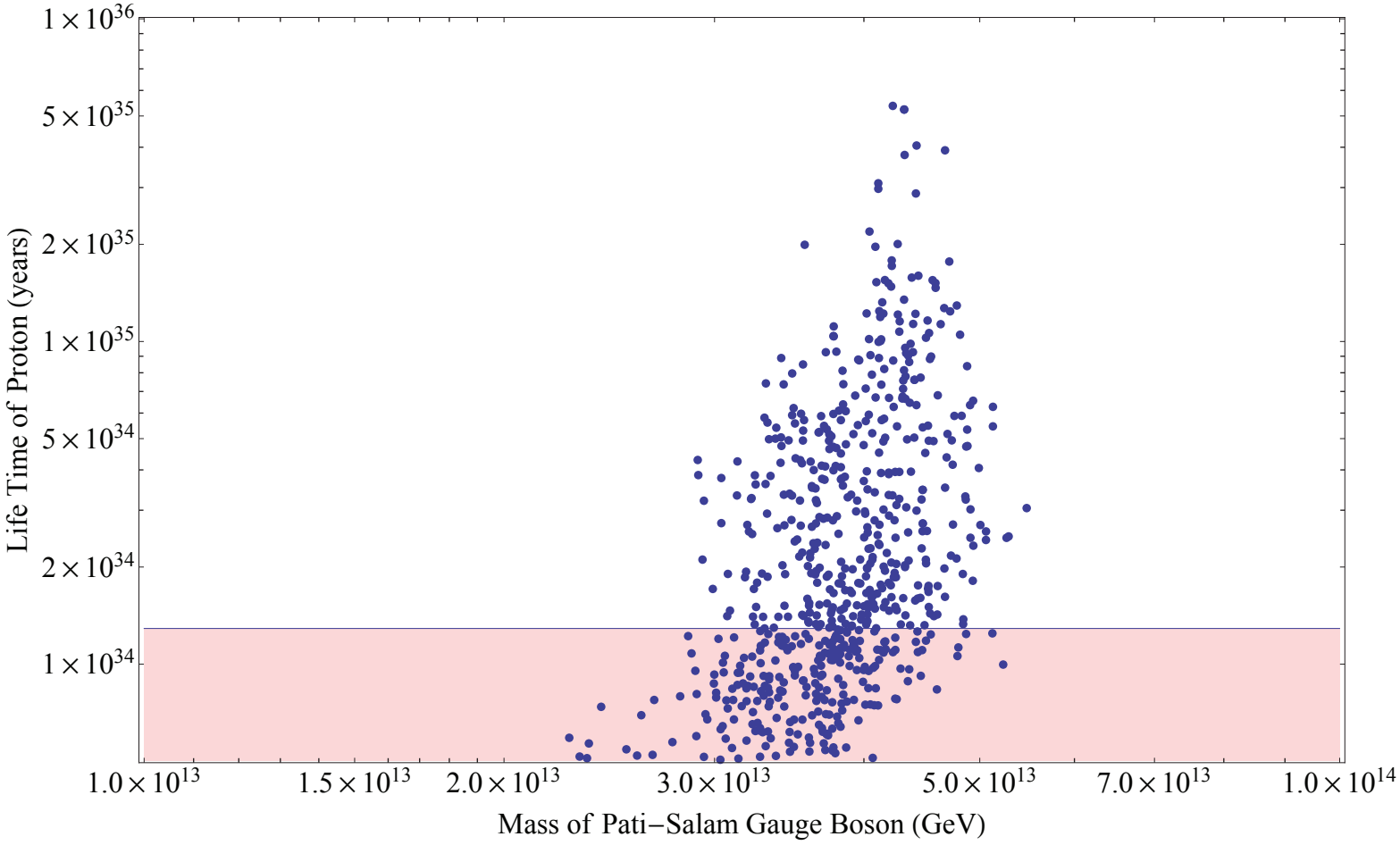
where,

$$\gamma_{L(sm)} = \left\{ \frac{23}{20}, \frac{9}{4}, 2 \right\}; \qquad \gamma_{R(sm)} = \left\{ \frac{11}{20}, \frac{9}{4}, 2 \right\}; \qquad \gamma_{L/R(ps)} = \left\{ \frac{15}{4}, \frac{9}{4}, \frac{9}{4} \right\}$$

Proton Life Time

$$\begin{aligned} \Gamma^{-1}(p \to e^+ \pi^0) &\approx (8.2 \times 10^{34} \text{yr}) \\ &\times \left(\frac{\alpha_H}{0.0122 \text{GeV}^3} \right)^{-2} \left(\frac{\alpha_G}{1/34.7} \right)^{-2} \left(\frac{A_R}{3.35} \right)^{-2} \left(\frac{M_X}{10^{16} \text{GeV}} \right)^4 \end{aligned} \tag{3}$$

Scattered plot in the parameter space



Next Step

We cannot predict scalar masses!

Too many parameters!

But $S0(10)$ Lagrangian has limited number of terms.

It is possible to find mass relations between the scalar masses.

It will reduce the freedom on the parameter space!

The Lagrangian

The possible invariants are:

1. $54 \cdot 54$
2. $54 \cdot 54 \cdot 54$
3. $54 \cdot 54 \cdot 54 \cdot 54$ $(\times 2)$
4. $126 \cdot \overline{126}$
5. $126 \cdot 126 \cdot \overline{126} \cdot \overline{126}$ $(\times 4)$
6. $10 \cdot \overline{10}$
7. $10 \cdot 10 \cdot \overline{10} \cdot \overline{10} \times$ $(\times 2)$
8. $S \cdot S^*$
9. $S \cdot S \cdot S^* \cdot S^*$

1. $54 \cdot 54 \cdot 126 \cdot \overline{126}$ $(\times 2)$
2. $126 \cdot \overline{126} \cdot 10 \cdot \overline{10}$ $(\times 2)$
3. $\overline{126} \cdot \overline{126} \cdot 126 \cdot 10$
4. $54 \cdot 54 \cdot 10 \cdot \overline{10}$ $(\times 2)$
5. $126 \cdot 126 \cdot 10 \cdot 10$
6. $126 \cdot \overline{126} \cdot S \cdot S^*$
7. $10 \cdot \overline{10} \cdot S \cdot S^*$
8. $54 \cdot 54 \cdot S \cdot S^*$
9. $126 \cdot 126 \cdot 54 \cdot S$
10. $10 \cdot 10 \cdot S$

Here we have assumed a PQ symmetry whose natural scale will be M_i. Under this PQ symmetry

$$\mathbf{10} \to e^{-2i\alpha}\mathbf{10}; \qquad \mathbf{126} \to e^{2i\alpha}\mathbf{126}; \qquad \mathbf{S} \to e^{-4i\alpha}\mathbf{S}$$

Scalar Potential

The most general potential for the 54 and 126 representation can be written as

$$
\begin{aligned}
V(\phi,\Sigma) \quad = \quad & -\frac{\mu^2}{2}\Phi_{i,j}\Phi_{i,j} + \frac{c}{3}\Phi_{i,j}\Phi_{j,k}\Phi_{k,i} + \frac{a}{4}\Phi_{i,j}\Phi_{i,j}\Phi_{k,l}\Phi_{k,l} + \frac{b}{2}\Phi_{i,j}\Phi_{j,k}\Phi_{k,l}\Phi_{l,i} \\
& -\frac{\nu^2}{5!}\Sigma_{i,j,k,l,m}\Sigma^*_{i,j,k,l,m} + \frac{\lambda_0}{(5!)^2}\Sigma_{i,j,k,l,m}\Sigma^*_{i,j,k,l,m}\Sigma_{n,o,p,q,r}\Sigma^*_{n,o,p,q,r} \\
& +\frac{\lambda_2}{(4!)^2}\Sigma_{i,j,k,l,m}\Sigma^*_{i,j,k,l,n}\Sigma_{o,p,q,r,m}\Sigma^*_{o,p,q,r,n} \\
& +\frac{\lambda_4}{(3!)^2\,(2!)^2}\Sigma_{i,j,k,l,m}\Sigma^*_{i,j,k,n,o}\Sigma_{p,q,r,l,m}\Sigma^*_{p,q,r,n,o} \\
& +\frac{\lambda_4'}{(3!)^2}\Sigma_{i,j,k,l,m}\Sigma^*_{i,j,k,n,o}\Sigma_{p,q,r,l,n}\Sigma^*_{p,q,r,m,o} \\
& +\frac{\alpha}{2\,(5!)}\Phi_{i,j}\Phi_{i,j}\Sigma_{p,q,r,l,m}\Sigma^*_{p,q,r,l,m} + \frac{\beta}{3!}\Phi_{i,j}\Phi_{k,l}\Sigma_{m,n,o,i,k}\Sigma^*_{m,n,o,j,l}
\end{aligned}
$$

Scalar Potential(Contd...)

The interaction part of the potential with the 10 can be written as

$$
\begin{aligned}
V(\Phi,\Sigma,\phi) \quad = \quad & -\xi_0^2\phi_i\phi_i^* + \xi_1\phi_i\phi_i^*\phi_j\phi_j^* + \xi_2\phi_i\phi_i\phi_j^*\phi_j^* + \xi_3\Phi_{i,j}\phi_i\phi_j^* \\
& + \frac{\gamma_1}{4!}\Sigma_{i,j,k,l,m}\Sigma^*_{i,j,k,l,n}\phi_m\phi_n^* + + \frac{\gamma_2}{4!}\Sigma_{i,j,k,l,m}\Sigma^*_{i,j,k,l,n}\phi_n\phi_m^* \\
& + \frac{\eta_0}{2}\Phi_{i,j}\Phi_{i,j}\phi_k\phi_k^* + \frac{\eta_1}{(3!)^2(2!)^2}\Sigma_{i,j,k,l,m}\Sigma^*_{i,j,k,p,q}\Sigma^*_{l,m,p,q,n}\phi_n \\
& + \frac{\eta_1^*}{(3!)^2(2!)^2}\Sigma^*_{i,j,k,l,m}\Sigma_{i,j,k,p,q}\Sigma_{l,m,p,q,n}\phi_n^* \\
& + \eta_2\Phi_{i,j}\Phi_{i,k}\phi_j\phi_k^* + \frac{\eta_3}{4!}\Sigma_{i,j,k,l,m}\Sigma_{i,j,k,l,n}\phi_m\phi_n \\
& + \frac{\eta_3^*}{4!}\Sigma^*_{i,j,k,l,m}\Sigma^*_{i,j,k,l,n}\phi_m^*\phi_n^*
\end{aligned}
$$

Part of the potential with the Singlet S can be written as

$$
\begin{aligned}
V(S) \quad = \quad & -\mu_s^2 SS^* + \chi_1 (SS^*)^2 + \chi_2\Sigma_{i,j,k,l,m}\Sigma^*_{i,j,k,l,m}SS^* + \chi_3\Phi_{i,j}\Phi_{i,j}SS^* \\
& + \frac{\chi_4}{4!}\Sigma_{i,j,k,l,m}\Sigma_{i,j,k,l,n}\Phi_{i,j}S + \frac{\chi_4^*}{4!}\Sigma^*_{i,j,k,l,m}\Sigma^*_{i,j,k,l,n}\Phi_{i,j}S^* \\
& + \chi_5\phi_i\phi_i^*SS^* + \chi_6\phi_i\phi_iS^* + \chi_6^*\phi_i^*\phi_i^*S
\end{aligned}
$$

Scalar Mass Spectra

From this we can find out the relevant scalar masses as,

$$
\begin{aligned}
M^2[\Phi_2(1,3,0)] &= \frac{8}{5} b\omega_s + c\omega_s \\
M^2[\Phi_{10}(8,1,0)] &= \frac{2}{5} b\omega_s - c\omega_s \\
M^2[\Sigma_{22}(3,3,-\frac{1}{3})] &= 4\left(3\lambda_2 + 3\lambda_4 + 4\lambda_4'\right)|\sigma|^2 \\
M^2[\Sigma_{23}(6,3,+\frac{1}{3})] &= 8\left(\lambda_2 + \lambda_4 + 4\lambda_4'\right)|\sigma|^2 \\
M^2[\Sigma_{33}(1,1,+2)] &= 8\left(\lambda_2 + \lambda_4 + 2\lambda_4'\right)|\sigma|^2 \\
M^2[\Sigma_{34}(\overline{3},1,+\frac{4}{3})] &= 4\left(3\lambda_2 + 3\lambda_4 + 4\lambda_4'\right)|\sigma|^2 \\
M^2[\Sigma_{37}(\overline{6},1,-\frac{4}{3})] &= 8\left(\lambda_2 + \lambda_4 + 4\lambda_4'\right)|\sigma|^2 \\
M^2[\Sigma_{38}(\overline{6},1,-\frac{1}{3})] &= 4\left(3\lambda_2 + 3\lambda_4 + 4\lambda_4'\right)|\sigma|^2
\end{aligned}
\tag{4}
$$

Contd...

$$M^2[(1,3,+1)] = \begin{pmatrix} \frac{8}{5}b\omega_s + c\omega_s + \frac{1}{2}\beta|\sigma|^2 & 2\sqrt{2}\chi_4^*\sigma^* v_s \\ 2\sqrt{2}\chi_4\sigma v_s & 8\left(2\lambda_2 + 3\lambda_4 + 2\lambda_4'\right)|\sigma|^2 \end{pmatrix};$$

$$M^2[(\overline{6},1,+\frac{2}{3})] = \begin{pmatrix} \frac{2}{5}b\omega_s - c\omega_s + \frac{1}{2}\beta|\sigma|^2 & -2i\sqrt{2}\chi_4\sigma v_s \\ 2i\sqrt{2}\chi_4^*\sigma^* v_s & 8\left(2\lambda_2 + 3\lambda_4 + 2\lambda_4'\right)|\sigma|^2 \end{pmatrix};$$

$$M^2[(3,2,+\frac{7}{6})] = \begin{pmatrix} 4\left(3\lambda_2 + 3\lambda_4 + 4\lambda_4'\right)|\sigma|^2 + \beta\omega_s^2 & -4\chi_4\omega_s v_s \\ -4\chi_4^*\omega_s v_s & 8\left(\lambda_2 + \lambda_4 + 2\lambda_4'\right)|\sigma|^2 + \beta\omega_s^2 \end{pmatrix};$$

$$M^2[(8,2,+\frac{1}{2})] = \begin{pmatrix} 4\left(3\lambda_2 + 3\lambda_4 + 4\lambda_4'\right)|\sigma|^2 + \beta\omega_s^2 & -4\chi_4^*\omega_s v_s \\ -4\chi_4\omega_s v_s & 8\left(\lambda_2 + \lambda_4 + 2\lambda_4'\right)|\sigma|^2 + \beta\omega_s^2 \end{pmatrix};$$

$$M^2[(3,2,+\frac{1}{6})] = \begin{pmatrix} 8\left(2\lambda_2 + 3\lambda_4 + 2\lambda_4'\right)|\sigma|^2 + \beta\omega_s^2 & 4\chi_4\omega_s v_s & 2\sqrt{2}\chi_4\sigma v_s \\ 4\chi_4^*\omega_s v_s & \beta\omega_s^2 & -\frac{1}{\sqrt{2}}\beta\sigma\omega_s \\ 2\sqrt{2}\chi_4^*\sigma^* v_s & -\frac{1}{\sqrt{2}}\beta\overline{\sigma}\omega_s & \frac{1}{2}\beta|\sigma|^2 \end{pmatrix};$$

The Mass Matrix of $M^2[(3,2,+\frac{1}{6})]$ has a zero eigenvalue which corresponds to the goldstone boson.

Contd...

$$M^2[(3,1,-\frac{1}{3})] = \begin{pmatrix} A1 & 8\chi_4^*\omega_s v_s & 0 & 0 & 0 \\ 8\chi_4\omega_s v_s & B1 & 16i\sqrt{2}\lambda_4'|\sigma|^2 & 4\eta_1|\sigma|^2 & 0 \\ 0 & -16i\sqrt{2}\lambda_4'|\sigma|^2 & 8\left(\lambda_2+\lambda_4\right)|\sigma|^2 & 4i\sqrt{2}\eta_1|\sigma|^2 & 0 \\ 0 & 4\eta_1^*|\sigma|^2 & 4i\sqrt{2}\eta_1|\sigma|^2 & -\frac{1}{5}\eta_2\omega_s^2-\xi_3\omega_s+2|\frac{\chi_4 v_d}{v_u}|v_s^2 & 2\chi_4^* v_s^2 \\ 0 & 0 & 0 & 2\chi_4 v_s^2 & C1 \end{pmatrix};$$

$$M^2[(1,2,+\frac{1}{2})] = \begin{pmatrix} 8\left(\lambda_2+\lambda_4-2\lambda_4'\right)|\sigma|^2+\beta\omega_s^2 & -4\chi_4\omega_s v_s & 4i\sqrt{3}\eta_1|\sigma|^2 & 0 \\ -4\chi_4^*\omega_s v_s & 4\left(3\lambda_2+3\lambda_4+4\lambda_4'\right)|\sigma|^2+\beta\omega_s^2 & 0 & 0 \\ -4i\sqrt{3}\eta_1^*|\sigma|^2 & 0 & A2 & 2\chi_6^* v_s \\ 0 & 0 & 2\chi_6 v_s & B2 \end{pmatrix};$$

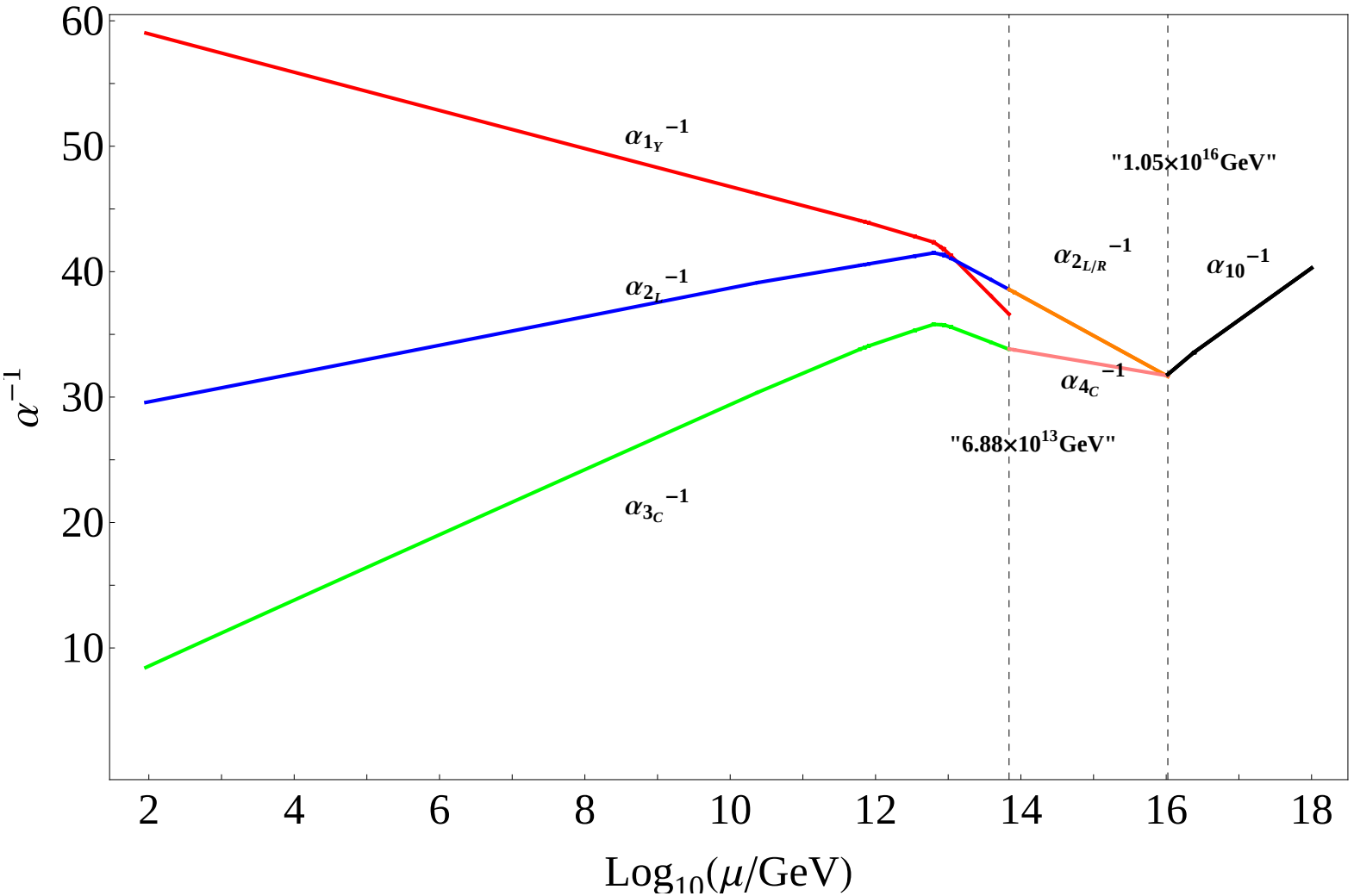
$$M^2[(1,1,0)] = \begin{pmatrix} \frac{1}{10}c\omega_s+\frac{12}{5}a\omega_s^2+\frac{14}{25}b\omega_s^2 & i\sqrt{\frac{6}{5}}(\alpha-\beta)\sigma\omega_s & \sqrt{\frac{6}{5}}\chi_3\omega_s v_s \\ -i\sqrt{\frac{6}{5}}(\alpha-\beta)\sigma^*\omega_s & \frac{1}{2}\lambda_0\sigma\sigma^* & -i\chi_2\sigma v_s \\ \sqrt{\frac{6}{5}}\chi_3\omega_s v_s & -i\chi_2\sigma v_s & 2\chi_1 v_s^2 \end{pmatrix};$$

where

$$\begin{aligned} A1 &= 4\left(3\lambda_2+3\lambda_4+4\lambda_4'\right)|\sigma|^2+4\beta\omega_s^2 \\ B1 &= 8\left(\lambda_2+\lambda_4+2\lambda_4'\right)|\sigma|^2+4\beta\omega_s^2 \\ C1 &= -|\sigma|^2(\gamma_1-\gamma_2)-\frac{1}{5}\eta_2\omega_s^2-\xi_3\omega_s+2|\frac{\chi_4 v_d}{v_u}|v_s^2 \end{aligned}$$

In the singlet mass matrix the $Im[S]$ is not included as it comes out to be zero.

Implementing the Mass relations



Fermion Masses, Mixings and Leptogenesis

The Yukawa sector of the Model looks like

$$\mathcal{L}_y = 16_F(h_{i,j}10_H + f_{i,j}\overline{126}_H)16_F \tag{5}$$

The mixture of the doublet coming from (15,2,2) of 126_H and (1,2,2) of 10_H due to the η_1 term $(126_H\overline{126}_H\overline{126}_H10_H)$ is very important.
Relevent vev's for fermion masses are:

- $\sigma = \langle(\overline{10}, 1, 3)\rangle\ \mathcal{O}(10^{13}GeV)$
- $k_{u,d} = \langle(15, 2, 2)_{u,d}\rangle \lesssim \mathcal{O}(10^2 GeV)$
- $v_{u,d} = \langle(1, 2, 2)_{u,d}\rangle\ \mathcal{O}(10^2 GeV)$

Fermion masses and mixings comes from the relations like:

$$M_u = hv_u + fk_u \qquad M_d = hv_d + fk_d$$
$$M_\nu^D = hv_u - 3fk_u \qquad M_l = hv_d - 3fk_d$$
$$M_\nu^M = f|\sigma|$$

Babu, Mohapatra (1993) , Bertolini, Frigerio, Malinsky (2004), Fukuyama, Okada (2002)
Babu, Macesanu (2005), Bajc, Melfo, Senjanovic, Vissani (2004), Bertolini, Malinsky, Schwetz (2006)
Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada (2004), Dutta, Mimura, Mohapatra (2007), Aulakh et al (2004)
Bajc, Dorsner, Nemevsek (2009), A. S. Joshipura et al (2011), Dueck, Rodejohann (2013)

Fitting the experimental data

The input at GUT Scale:

$$
\begin{aligned}
m_u &= 0.0006745 & m_c &= 0.3308 & m_t &= 97.335 \\
m_d &= 0.0009726 & m_s &= 0.02167 & m_b &= 1.1475 \\
m_e &= 0.000344 & m_\mu &= 0.0726 & m_\tau &= 1.350 \\
s_{12} &= 0.2248 & s_{23} &= 0.03278 & s_{13} &= 0.00216 \\
& & \delta_{CKM} &= 1.193 \ . & &
\end{aligned}
$$

The output for Neutrino:

$$\sin^2\theta_{12} \simeq 0.27 \ , \ \sin^2 2\theta_{23} \simeq 0.90 \ , \ \sin^2 2\theta_{13} \simeq 0.08 \ .$$

K. S. Babu and C. Macesanu, [hep-ph/0505200].

Fitting the experimental data(Contd.)

m_u (MeV)	0.495 ± 0.185	$\lvert V_{us}\rvert$	0.2254 ± 0.0006
m_d (MeV)	1.155 ± 0.495	$\lvert V_{cb}\rvert$	0.04194 ± 0.0006
m_s (MeV)	22.0 ± 7.0	$\lvert V_{ub}\rvert$	0.00369 ± 0.00013
m_c (GeV)	0.235 ± 0.035	J	$(3.16 \pm 0.1) \times 10^{-5}$
m_b (GeV)	1.00 ± 0.04	$\sin^2\theta^l_{12}$	0.308 ± 0.017
m_t (GeV)	74.15 ± 3.85	$\sin^2\theta^l_{23}$	0.3875 ± 0.0225
r	0.031 ± 0.001	$\sin^2\theta^l_{13}$	0.0241 ± 0.0025

Table 4: *Input values at the scale* $M_{GUT} = 2 \times 10^{16}$ *GeV.*

obs.	*fit*	*pull*	*obs.*	*fit*	*pull*
m_u(MeV)	0.49	0.03	$\lvert V_{us}\rvert$	0.225	0.038
m_d(MeV)	0.78	0.75	$\lvert V_{cb}\rvert$	0.042	-0.208
m_s(MeV)	32.5	-1.50	$\lvert V_{ub}\rvert$	0.0038	-0.659
m_c(GeV)	0.287	-1.49	J	3.1×10^{-5}	0.589
m_b(GeV)	1.11	-2.77	$\sin^2\theta^l_{12}$	0.318	0.611
m_t(GeV)	71.4	0.70	$\sin^2\theta^l_{23}$	0.353	-1.548
r	0.031	0.10	$\sin^2\theta^l_{13}$	0.0222	-0.758
η_B	5.699×10^{-10}	-0.001			

Table 5: *Best fit solutions for the fermion observables at the scale* $M_{GUT} = 2 \cdot 10^{16}$

light ν *masses (eV)*	*heavy* ν *masses* (10^{11} *GeV*)	*phases* (°)	m_{ee} *(eV)*
.0046	1.00	$\delta = 88.6$	5×10^{-4}
.0098	1.09	$\phi_1 = -33.2$	
.0504	21.4	$\phi_2 = 15.7$	

Table 6: *Predicted values of the light neutrino masses, of the heavy right-handed masses, of the leptonic CP-violating phases and of* m_{ee}.

$$\eta_B = 5 \times 10^{-10}$$

Altarelli and Meloni., arXiv:1305.1001v2 [hep-ph]

The Strong CP Problem and Axion

Strong interaction sector admits a term that violates both CP and P:

$$\mathcal{L}_{QCD} = \frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

This leads to a physical observable

$$\overline{\theta} = \theta + Arg[detM_q]$$

Neutron Electric Dipole Moment limits $\overline{\theta} < 10^{-10}$
Most popular solution to the problem is suggested by Peccei and Quinn which provides a dynamic solution.

DFSZ Axion:

- One complex scalar ϕ
- One uptype Higgs doublet ϕ_u
- One downtype Higgs doublet ϕ_d

Axion field is given by

$$A \approx -Im[\phi] + \frac{2v_u v_d}{v_w^2 v_s}(v_u Im[\phi_d + v_d Im[\phi_u])$$

Peccei, Quinn Phys Rev. Lett **38** (1977);
Dine, Fischler, Srednicki 1981, Zhitnitskii 1980

Axion in Minimal $SO(10)$ GUT

- One complex scalar with vev v_s
- One uptype Higgs doublet ϕ_1^u from 10_H with vev v_u
- One downtype Higgs doublet ϕ_1^d from 10_H with vev v_d
- One uptype Higgs doublet ϕ_2^u from 126_H with induced vev k_u
- One downtype Higgs doublet ϕ_2^d from 126_H with induced vev k_d

Phenomenology constraines the detemimant of the mass matrix containing Real part of the neutral component of the doublet and the complex singlet to be approximately zero ie at weak scale.
We also realize that there is enough freedom in the parameter space to have a Singlet mass to be $< 10^{10} GeV$
The axion is primarily the Imaginary part of the complex scalar and the Axion decay constant is naturally around the Intermediate Scale

$$m_A = \frac{z^{1/2}}{1+z} \frac{f_\pi m_\pi}{f_A} \approx \frac{0.60 meV}{f_A/10^{10} GeV}$$

Future Work

- Numerical Analysis of the Fermion masses and mixing needs to be checked for the constrained parameter space
- Two field inflation has a natural place here

Thank You