USING ENERGY-PEAKS FOR MEASURING (OLD AND NEW) PARTICLE MASSES

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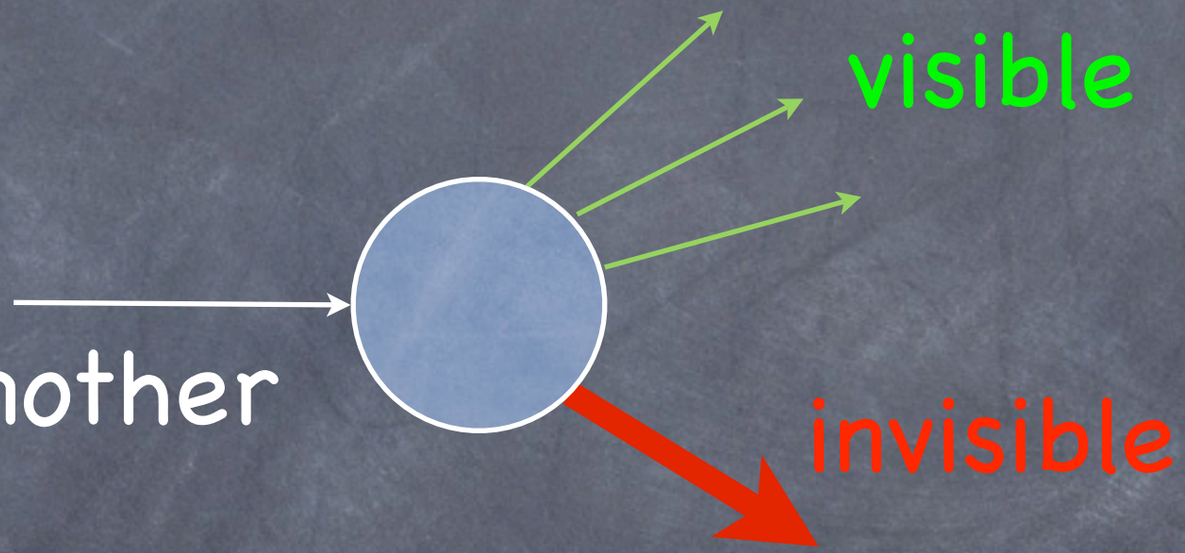
(with Roberto Franceschini, Sungwoo Hong, Doojin Kim, Kyle Wardlow: 1209.0772; 1212.5230; 1309.4776 and to appear)

Basic goal (simple!)

- determine mass of mother by measuring energy/momentum of (visible) decay products



- kinematics-based (independent of production mechanism)

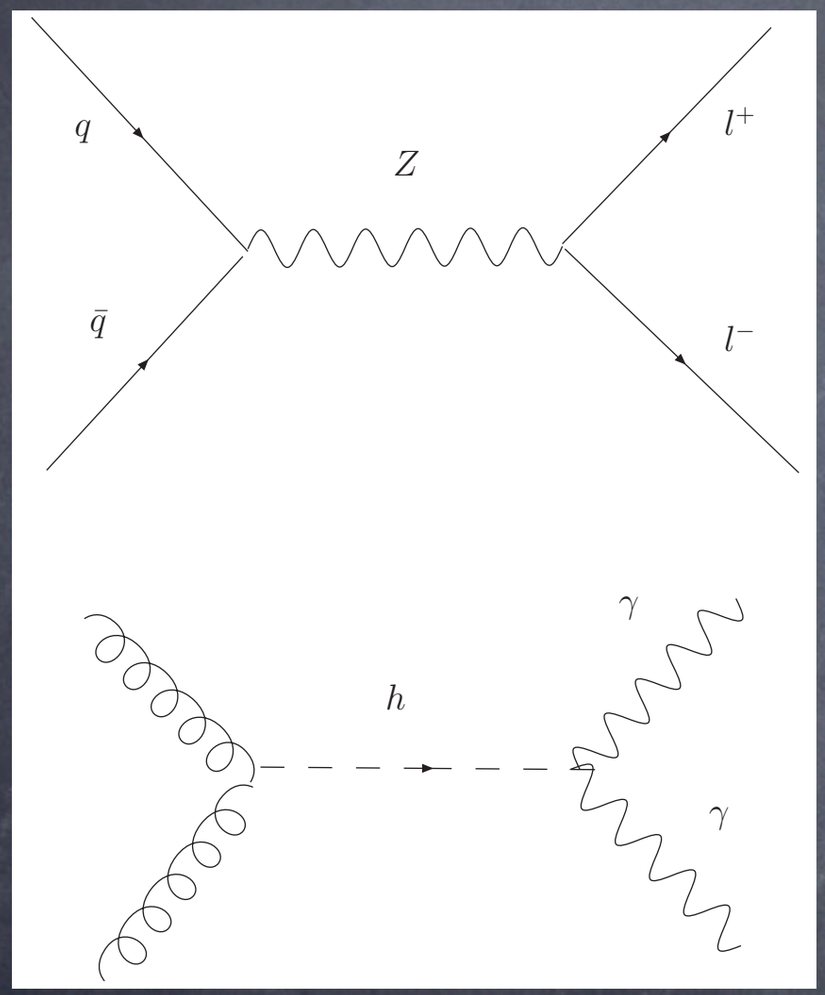
SEVERAL TECHNIQUES

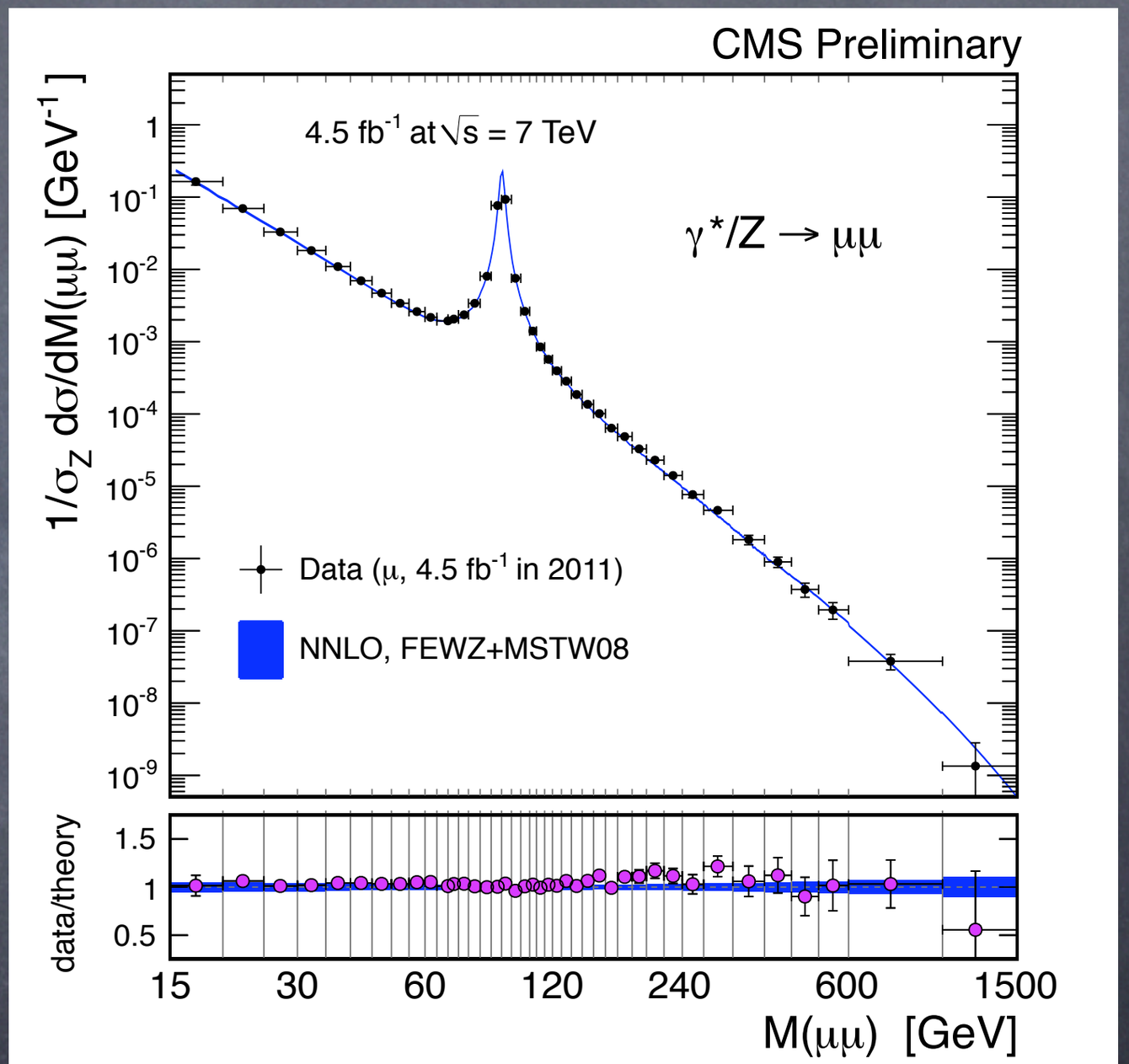
SO FAR

(MANY CASES)

Fully visible I (`golden")

- invariant mass of decay products has Breit-Wigner peak

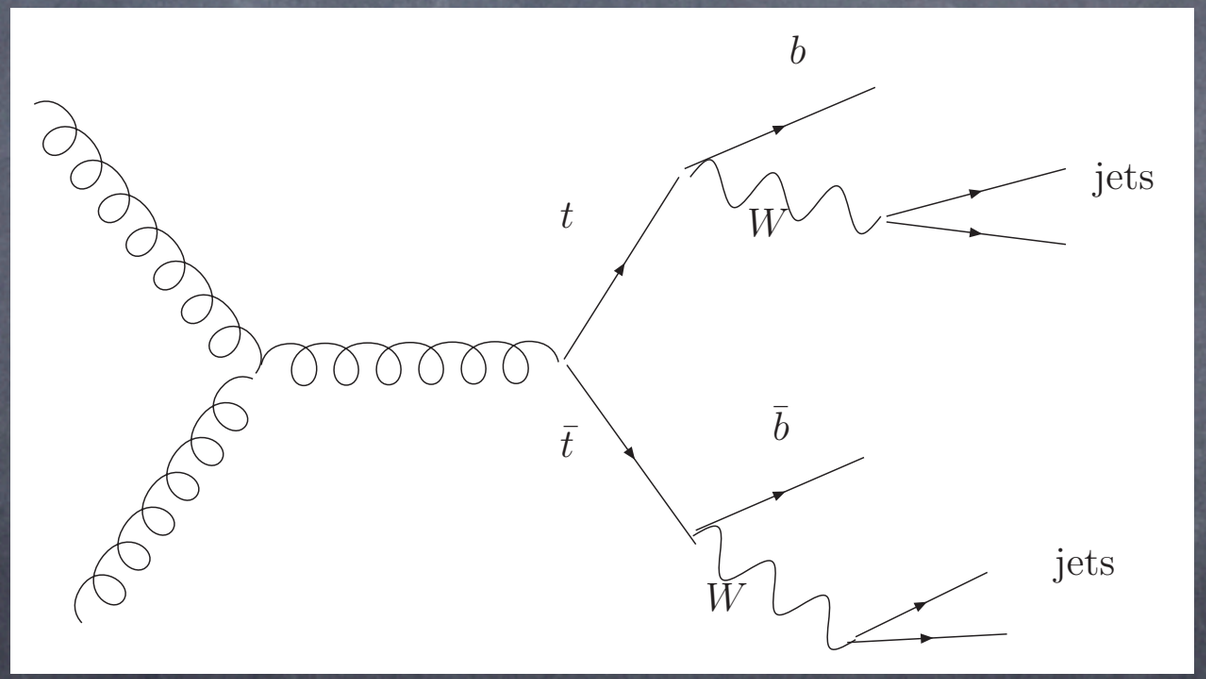




- have to be "lucky"!

Fully visible II (not so easy)

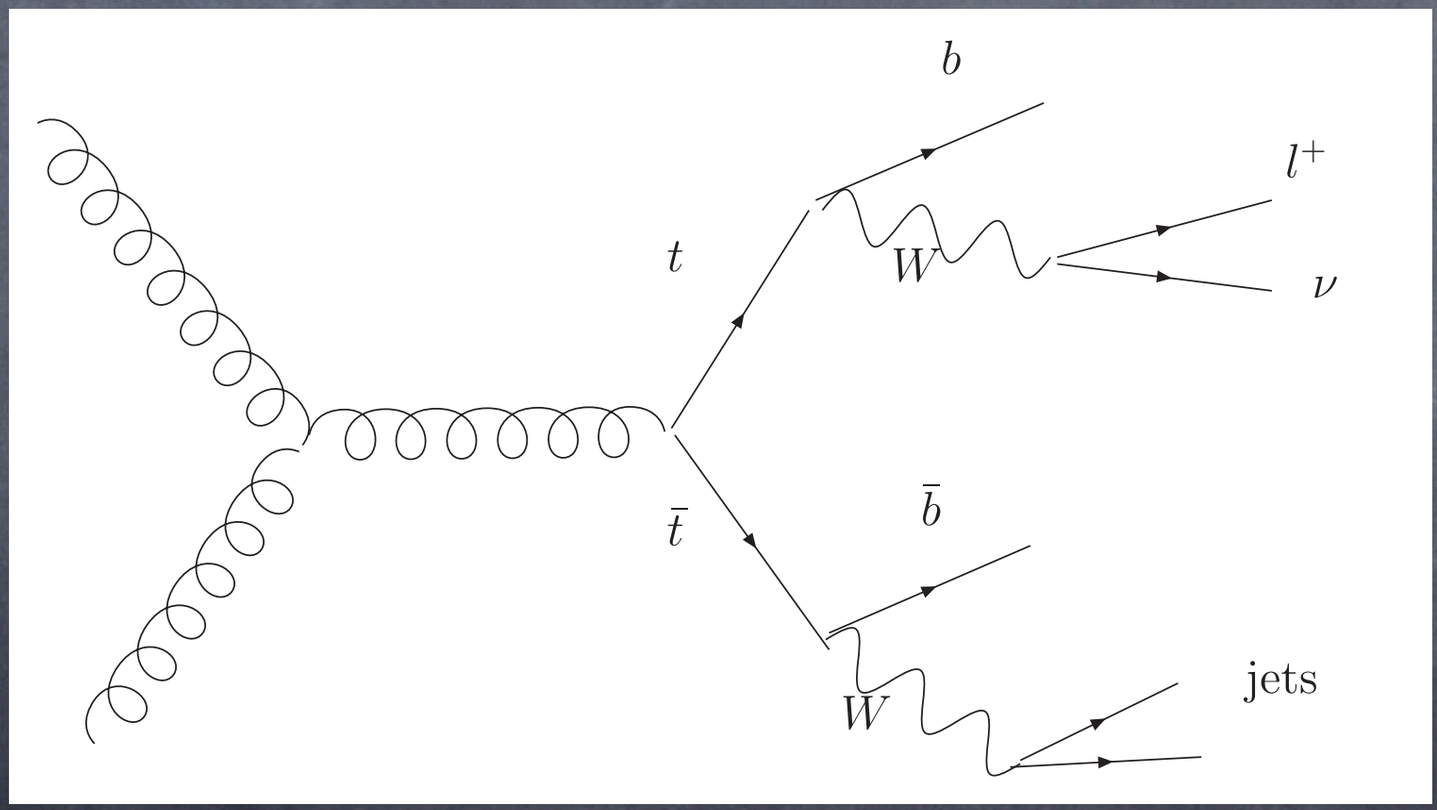
- fully hadronic top decay



- problem: all jetty + combinatorics (compounded by jets from initial/final state radiation)

"Partially" visible I (can be reconstructed)

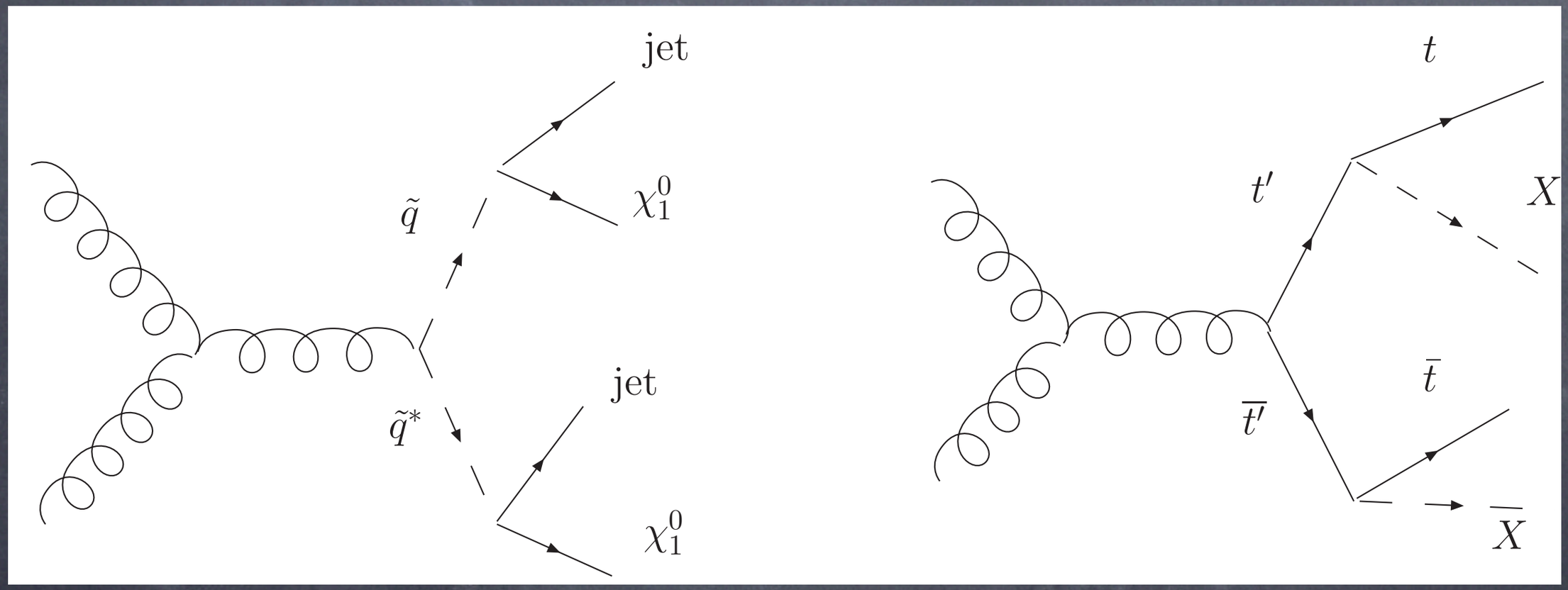
- 1 daughter fully visible, other partially
- semileptonic top decay ("less" jetty)



- still "issues": discrete ambiguity in reconstructing W; uses MET; still combinatorics (which W with which b)...

"Partially" visible II (cannot be reconstructed)

- 1 daughter fully visible, other fully invisible (maybe DM)
- R-parity conserving SUSY, top-partner in T-parity little Higgs models...



- (generalized) transverse mass (M_{T2}): uses MET
- razor: M_R based on (plausible) assumptions about boosts

Bottomline: (in my opinion)

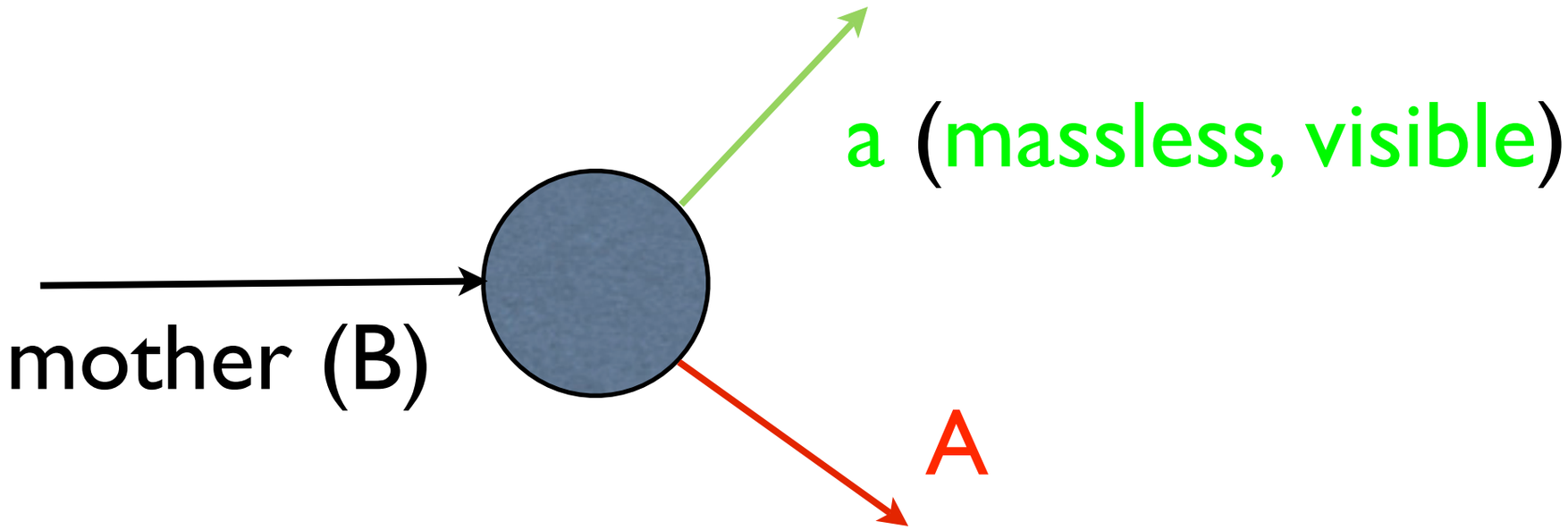
no slam dunk!

- useful to have more techniques, especially simpler; complementary (different systematics, e.g., avoid MET or combinatorics or assumptions about boosts)

NEW OBSERVATION TECHNIQUE

Basic assumptions

- 2-body decay: one daughter (fully) visible, massless:



- ...other (A) don't care (almost)!
- more assumptions later
- extensions/generalizations later

Energy (not invariant) of daughter

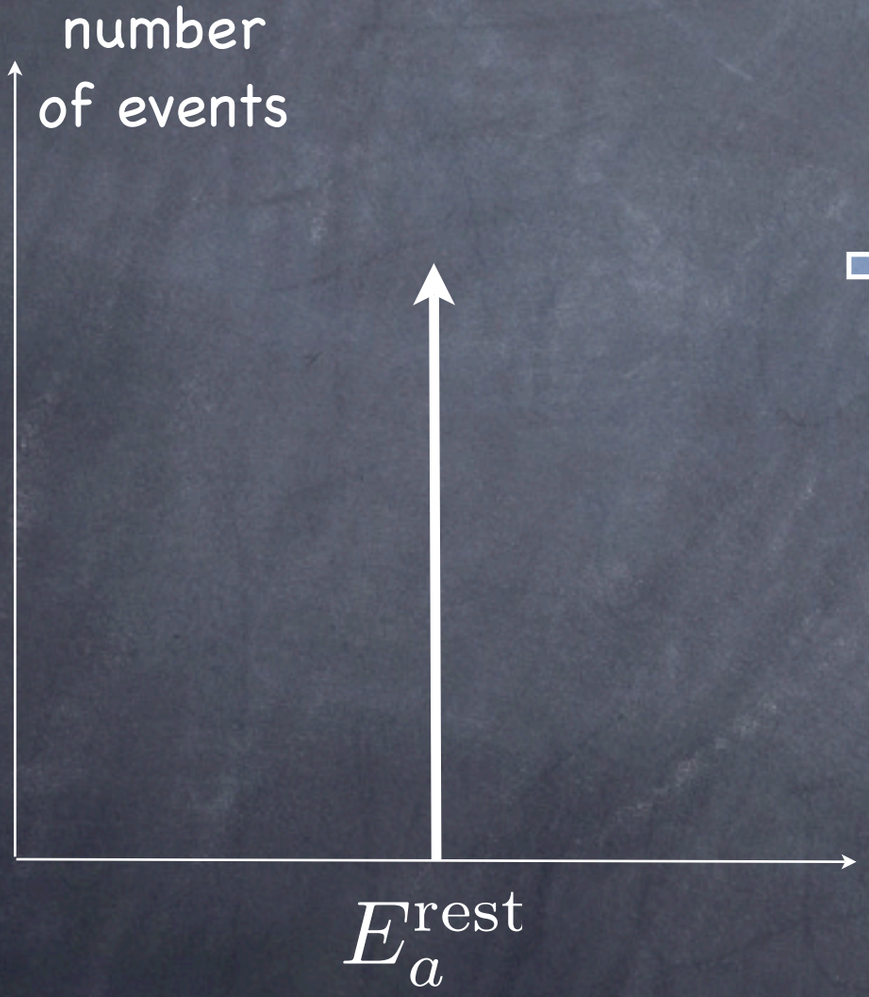
- mono-chromatic and simple function of masses in rest frame of mother:

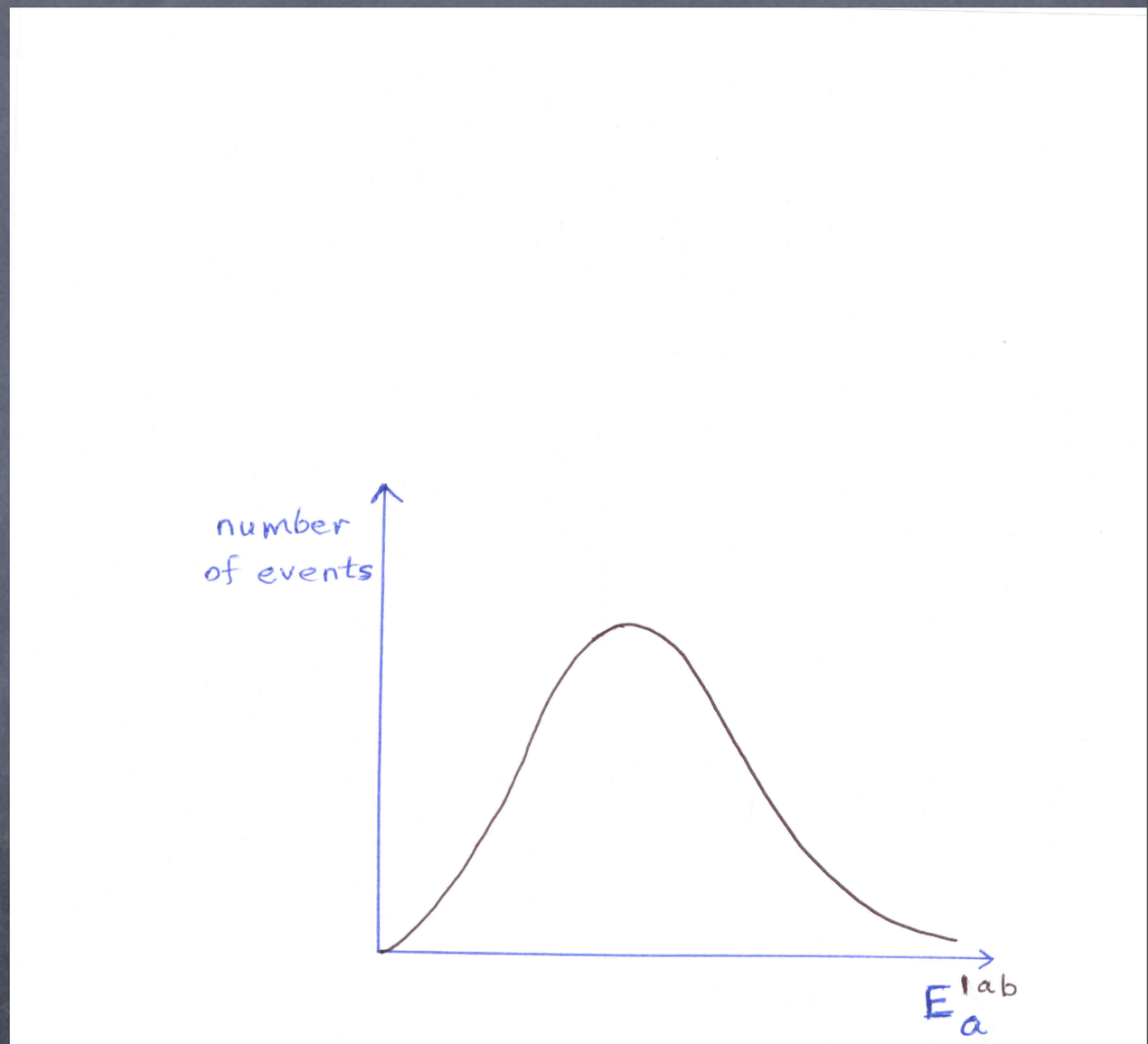
$$E_a^{\rm rest} = \frac{M_B^2 - M_A^2}{2M_B}$$

- determine M_B if M_A known and $E_a^{\rm rest}$ measured

...too simple to be practical/useful?!

- hadron collider: mother has unknown boost; varies event to event ⇒ distribution in $E_a^{\rm lab}$

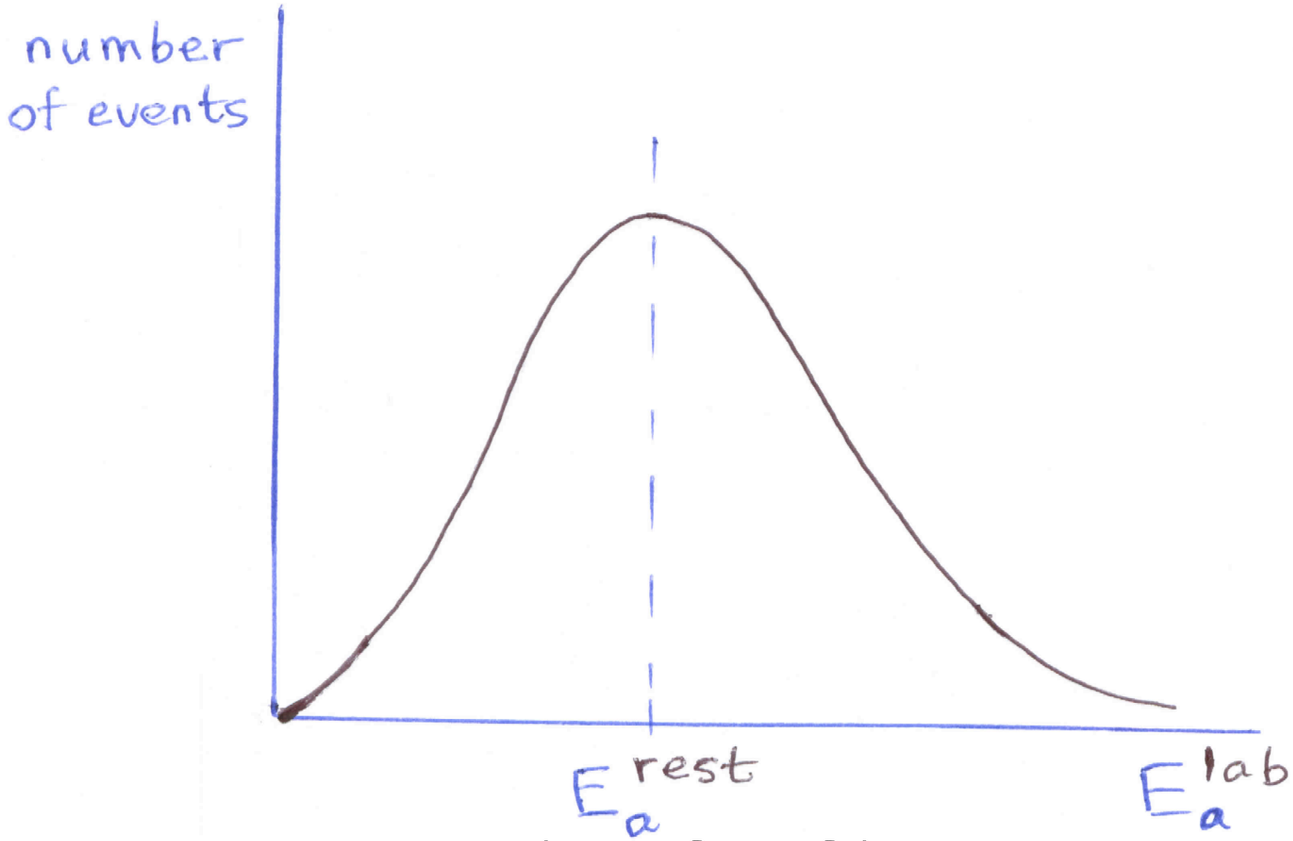




- lose rest-frame information?!

Outline

- Peak (of lab. distribution) still retains this information...as simply, precisely, robustly!
- ``Test'' application (top mass):
  obtain approximation to theory curve
  Fit it to (simulated) data for extracting peak
- New physics (Cascade decay):
  general idea
  SUSY example
- Three-body decay
- Conclusions

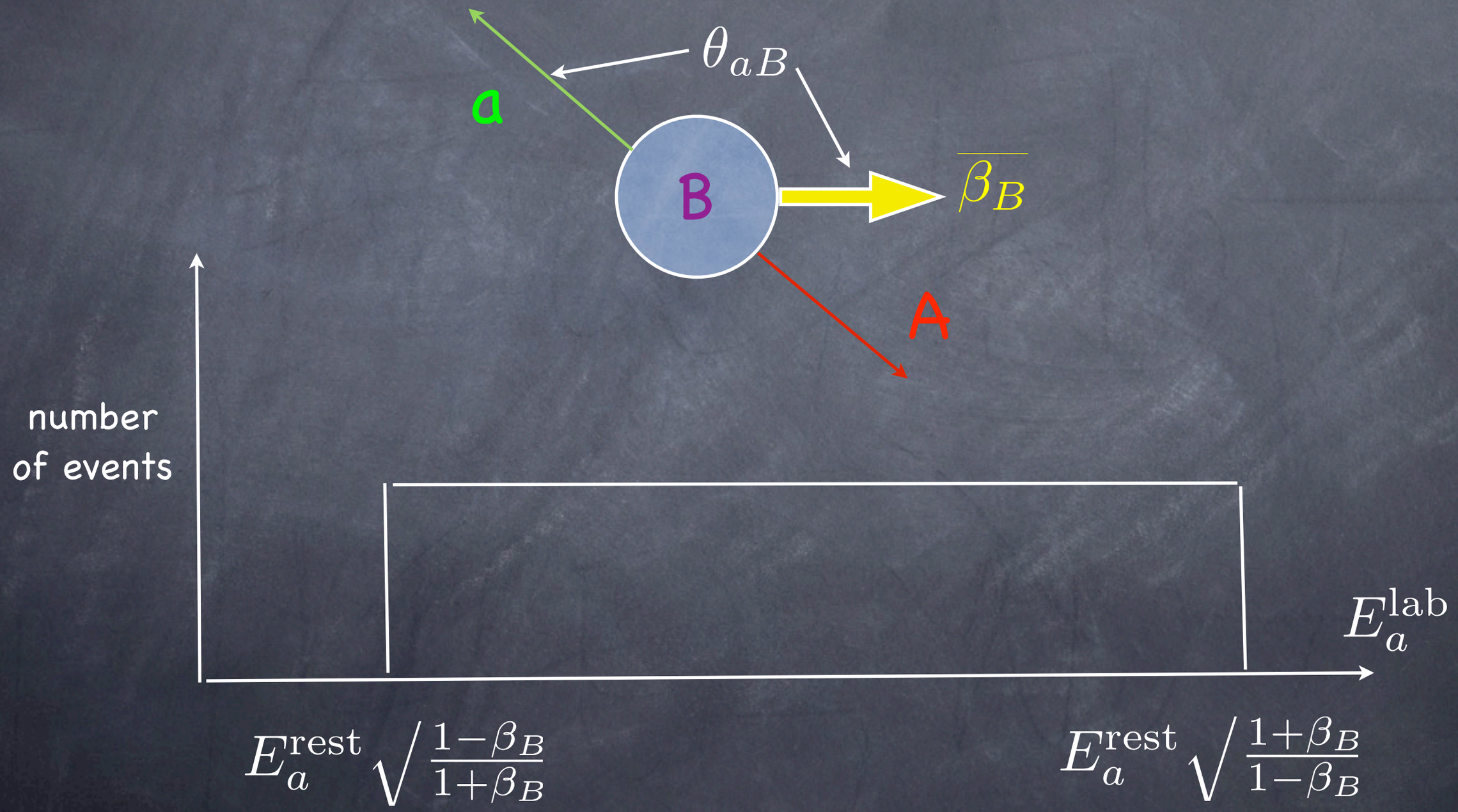


$$\left(= \frac{M_B^2 - M_A^2}{2M_B}\right)$$

``INVARIANCE'' OF TWO-BODY DECAY KINEMATICS

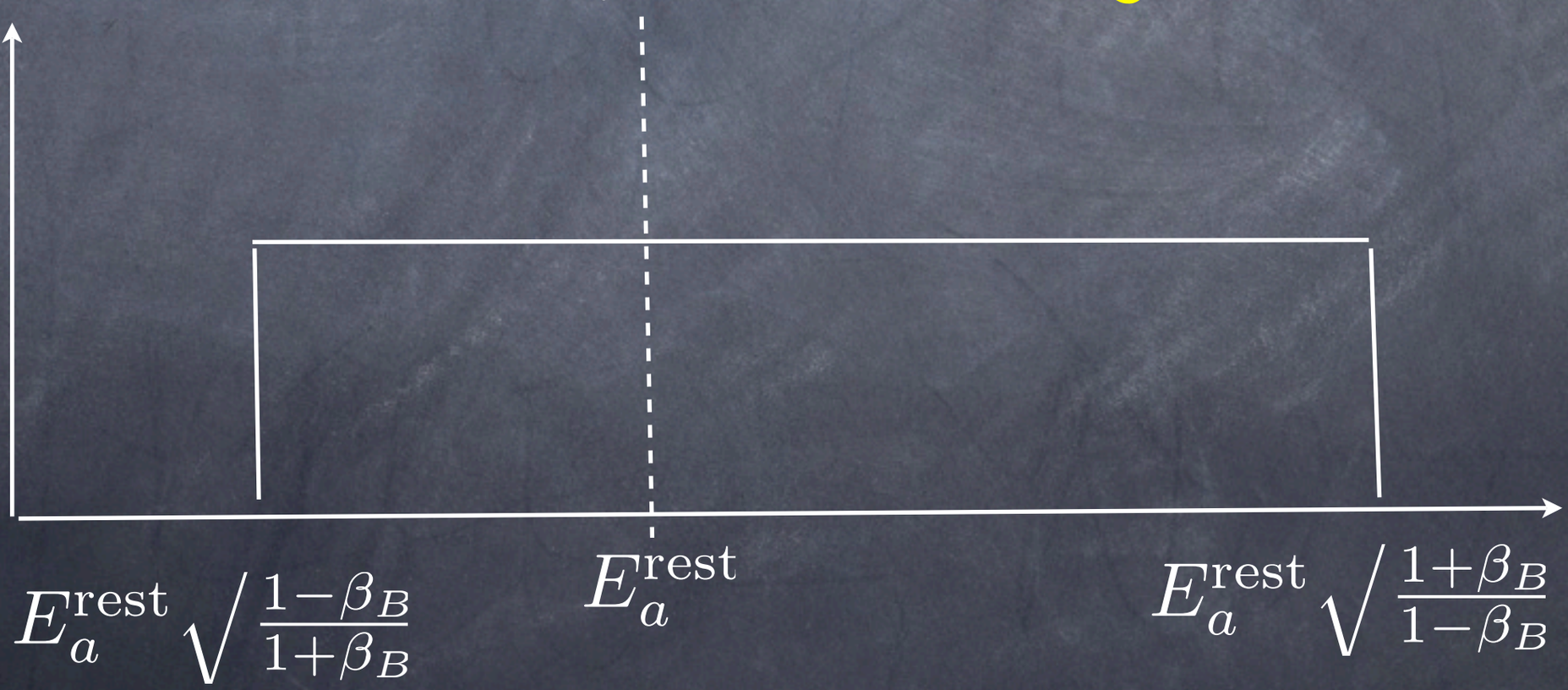
Rectangle for fixed, but arbitrary boost

- In general: $E_a^{\rm lab} = E_a^{\rm rest}\gamma_B\,(1+\beta_B\cos\theta_{aB})$
- Assume unpolarized mother: $\cos\theta_{aB}$ is flat



Rectangle vs. rest energy

- contains $E_a^{\rm rest}$ (for any boost)
- no other $E_a^{\rm lab}$ gets larger contribution from given boost than does $E_a^{\rm rest}$
- no other $E_a^{\rm lab}$ is contained in every rectangle (e.g., $\beta_B \to 0$)
- asymmetric on linear (symmetric on log...)

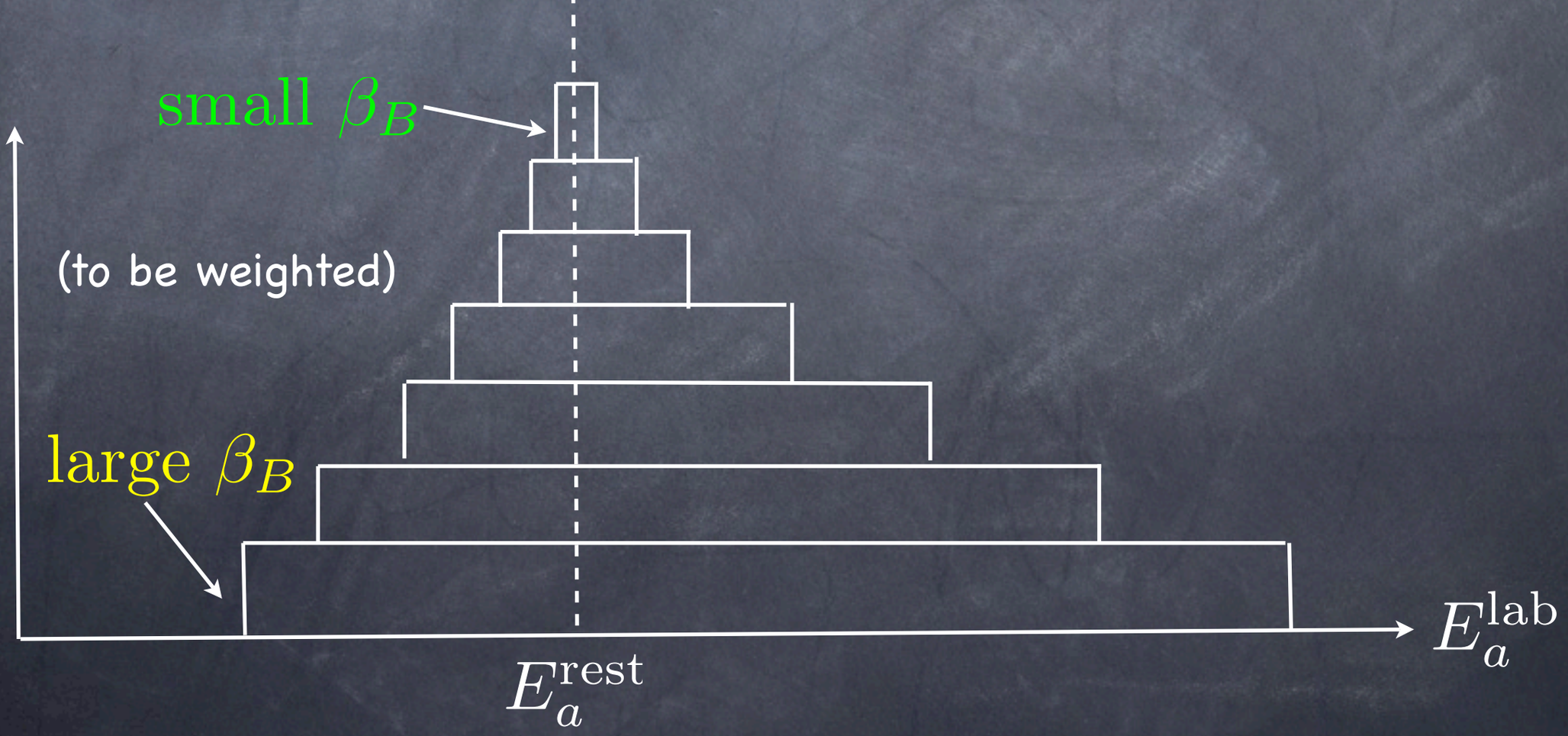


(Generic) Boost distribution: ``stacking'' up rectangles

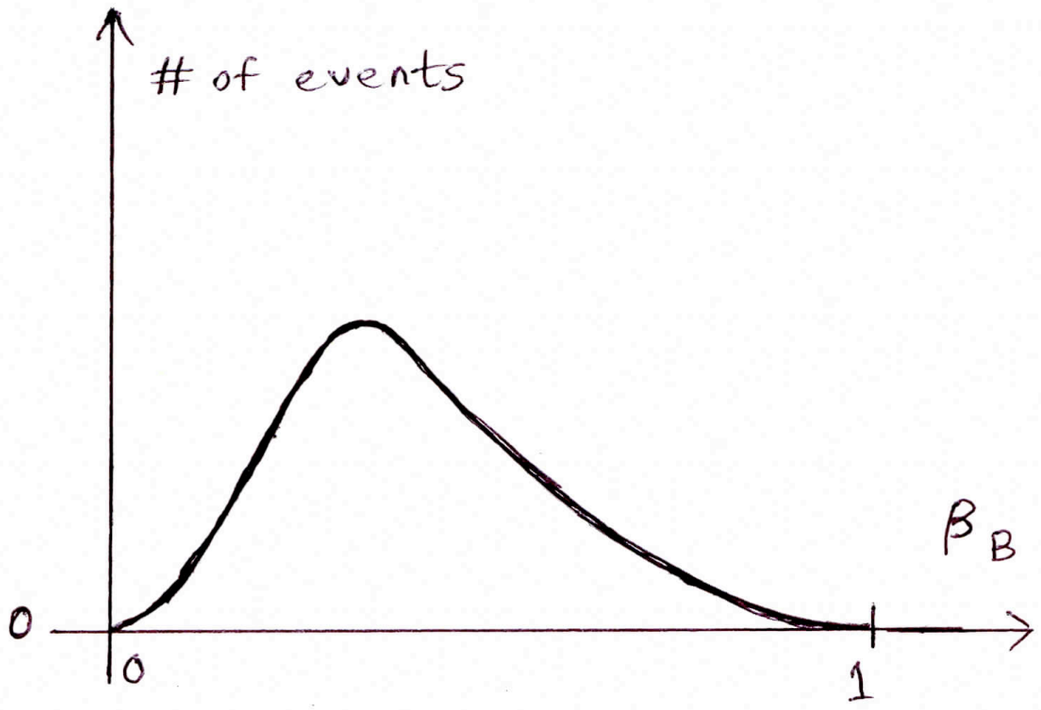
(KA, Franceschini, Kim: 1209.0772)

(see also Stecker: ``Cosmic gamma rays'')

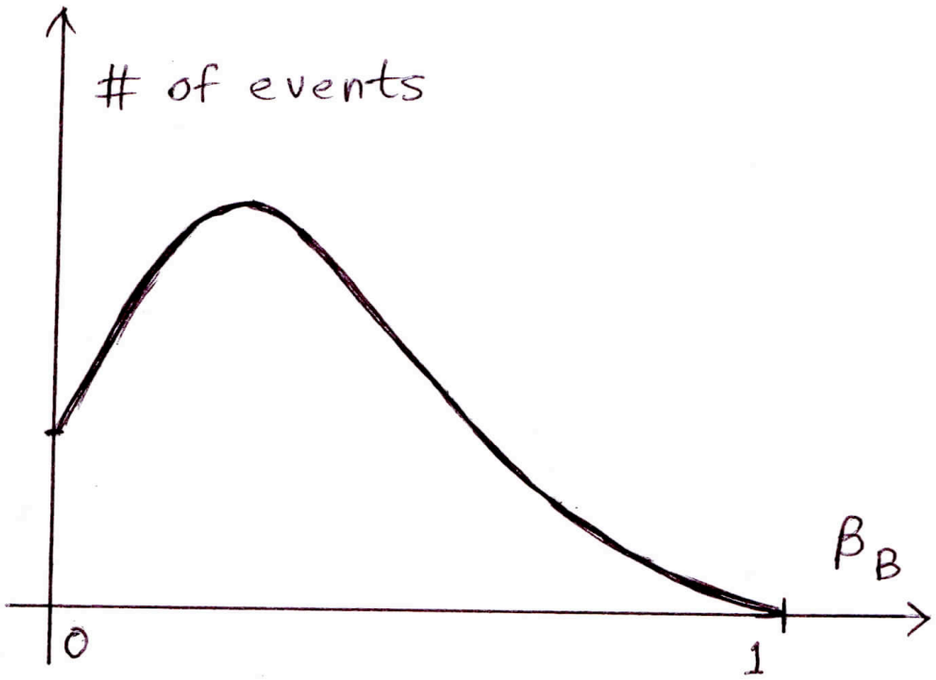
- distribution of $E_a^{\rm lab}$ has peak at $E_a^{\rm rest}$
- ....no matter what is the boost distribution!
- boost distribution depends on production mechanism, mother mass, PDF's...



Boost distributions: I & II



boost distribution for 2→2 (previous)

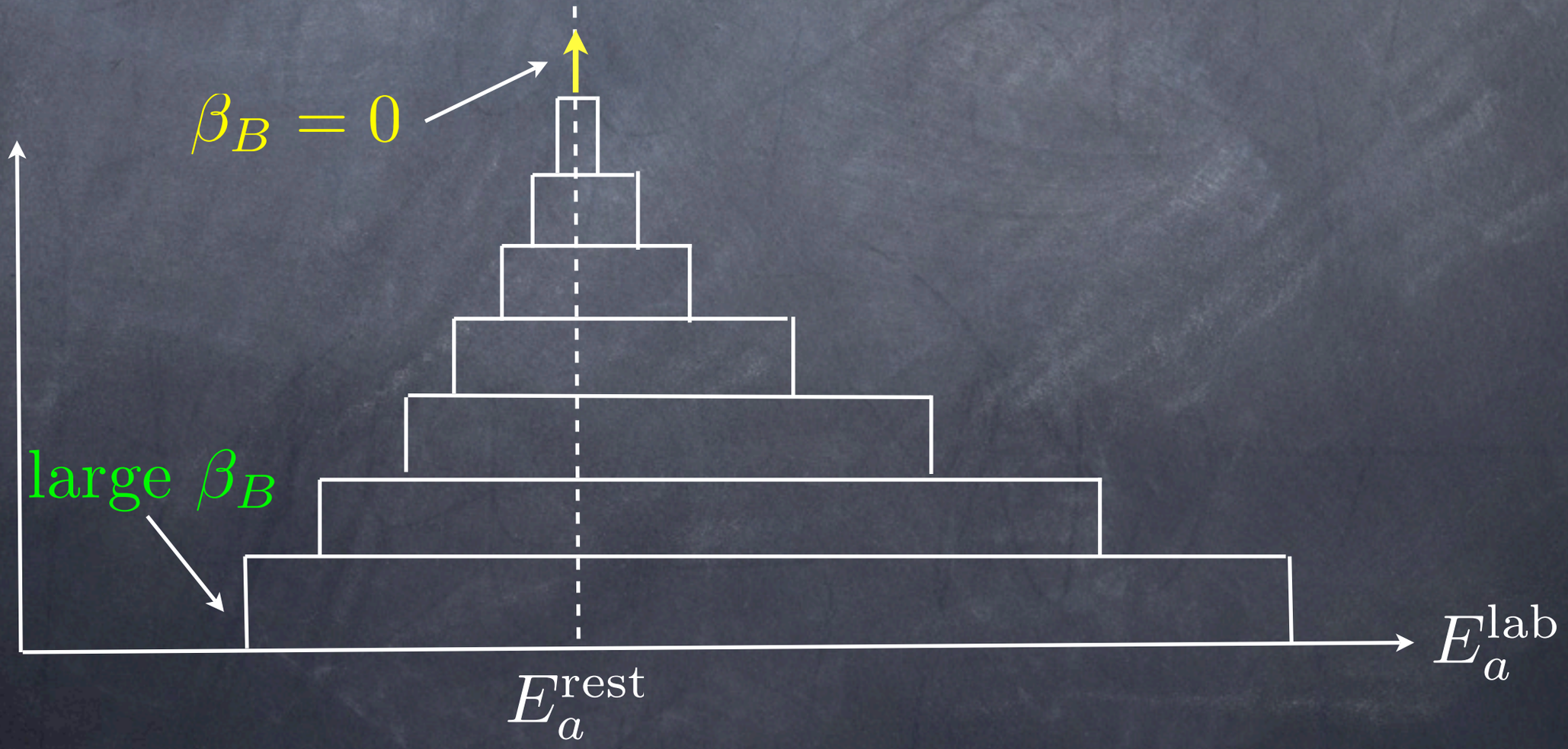


boost distribution for 2→1 (next)

Single mother production

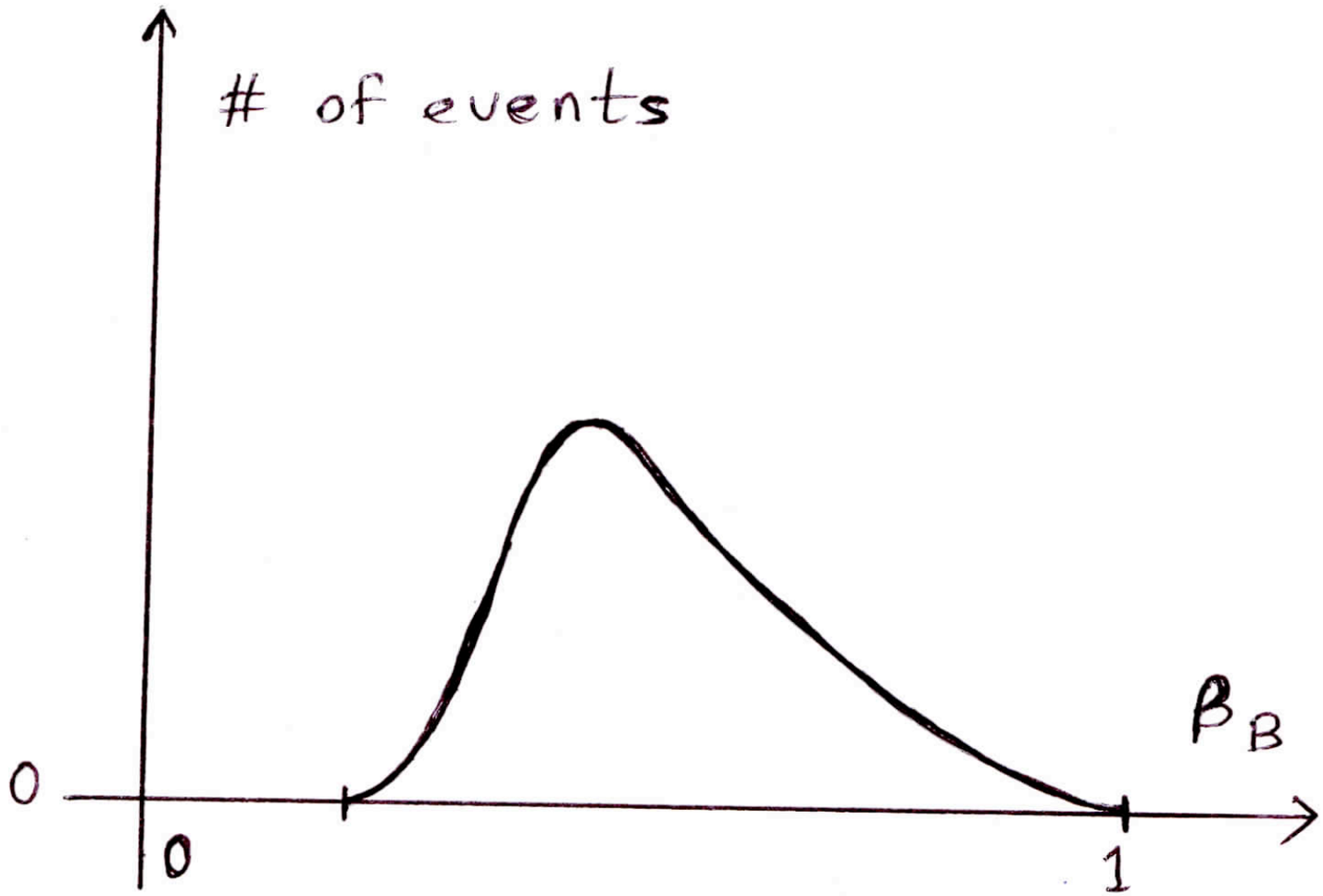
[e.g., gg→Higgs; KK graviton (?): Chen, Davoudiasl, Kim]

- distribution of $E_a^{\rm lab}$ has "kink" at $E_a^{\rm rest}$



Boost distributions: III

- Due to cuts (or highly boosted secondary mother), boost distribution vanishes close to $\beta_B = 0$



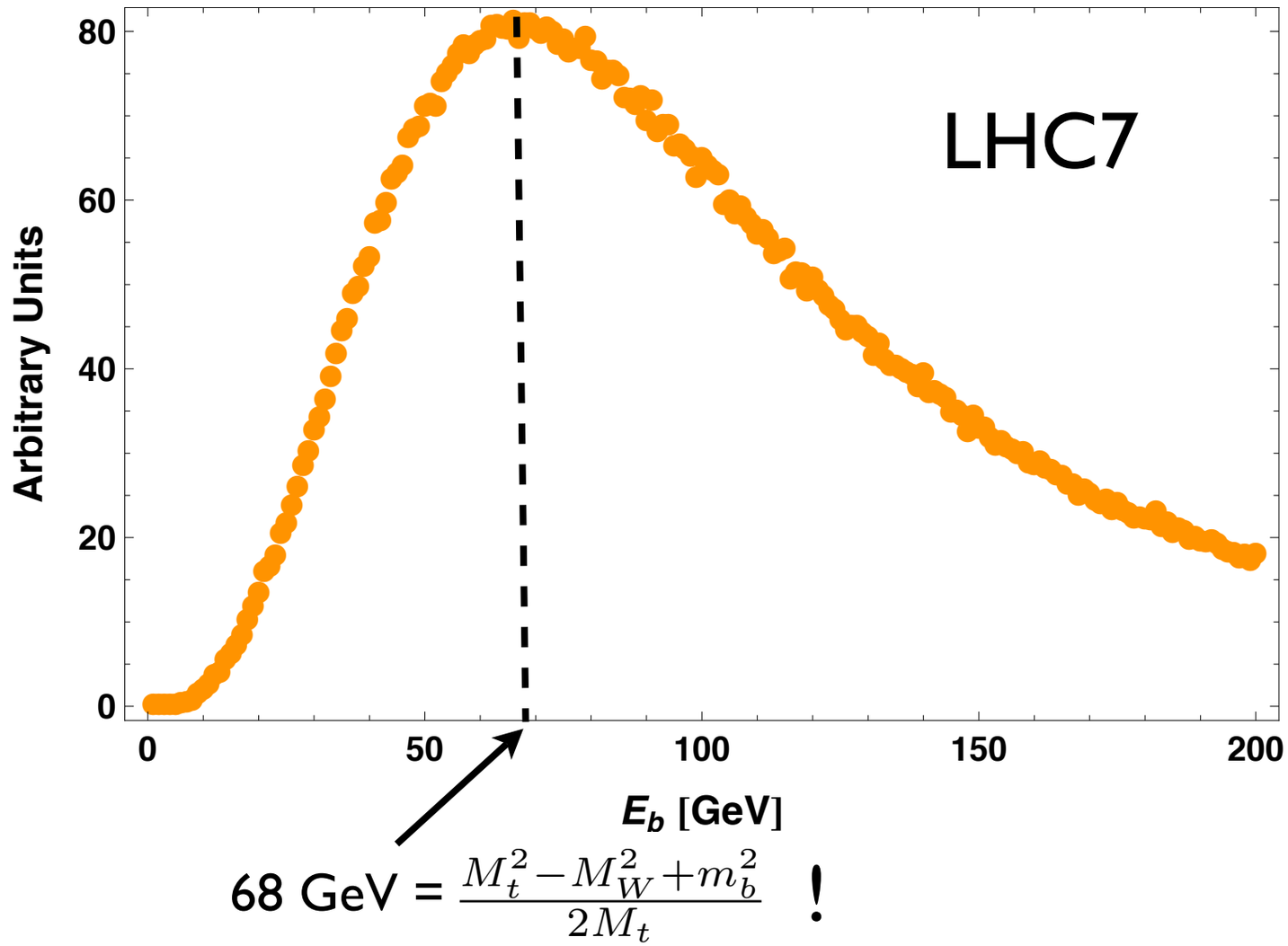
...plateau is not "generic"

small boost

No need really, but anyway, actual calculation...

- bottom from top quark decay as example:

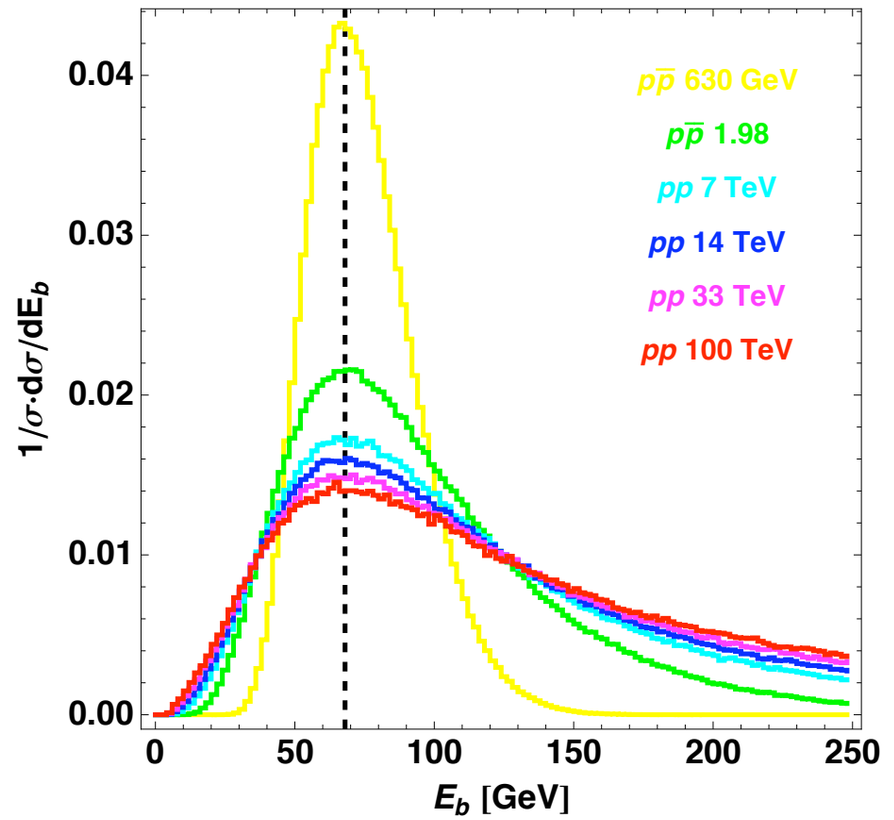
bottom mass negligible ⇒ peak is not expected to shift from $E_b^{\text{rest}} = \frac{M_t^2 - M_W^2 + m_b^2}{2M_t}$ (modified)

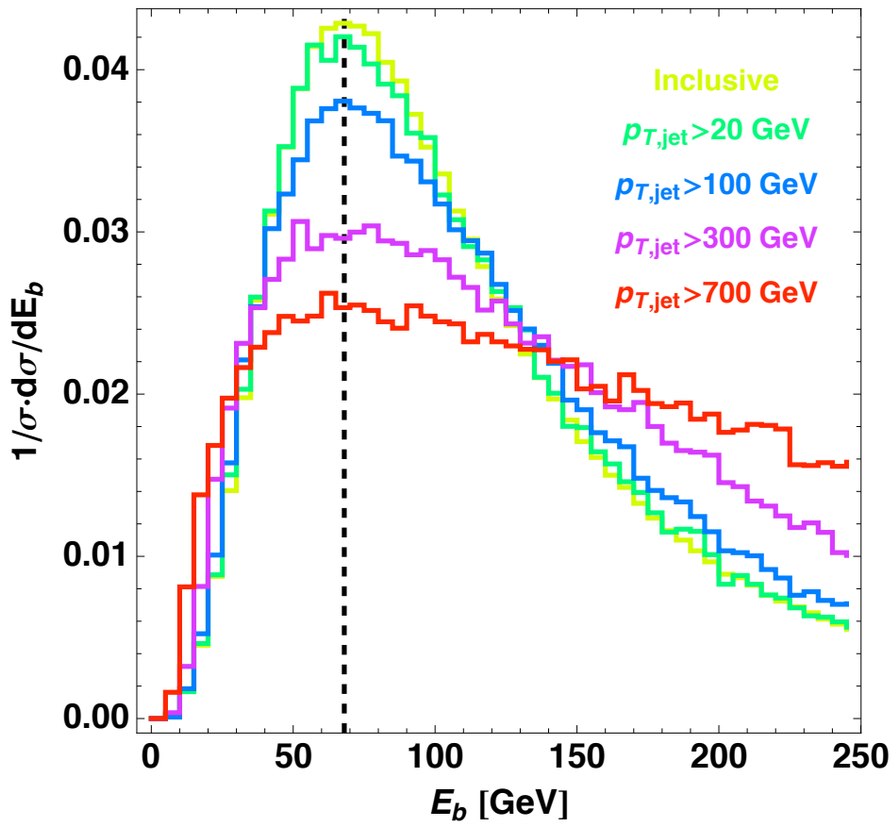


- ...maybe an "accident"?!

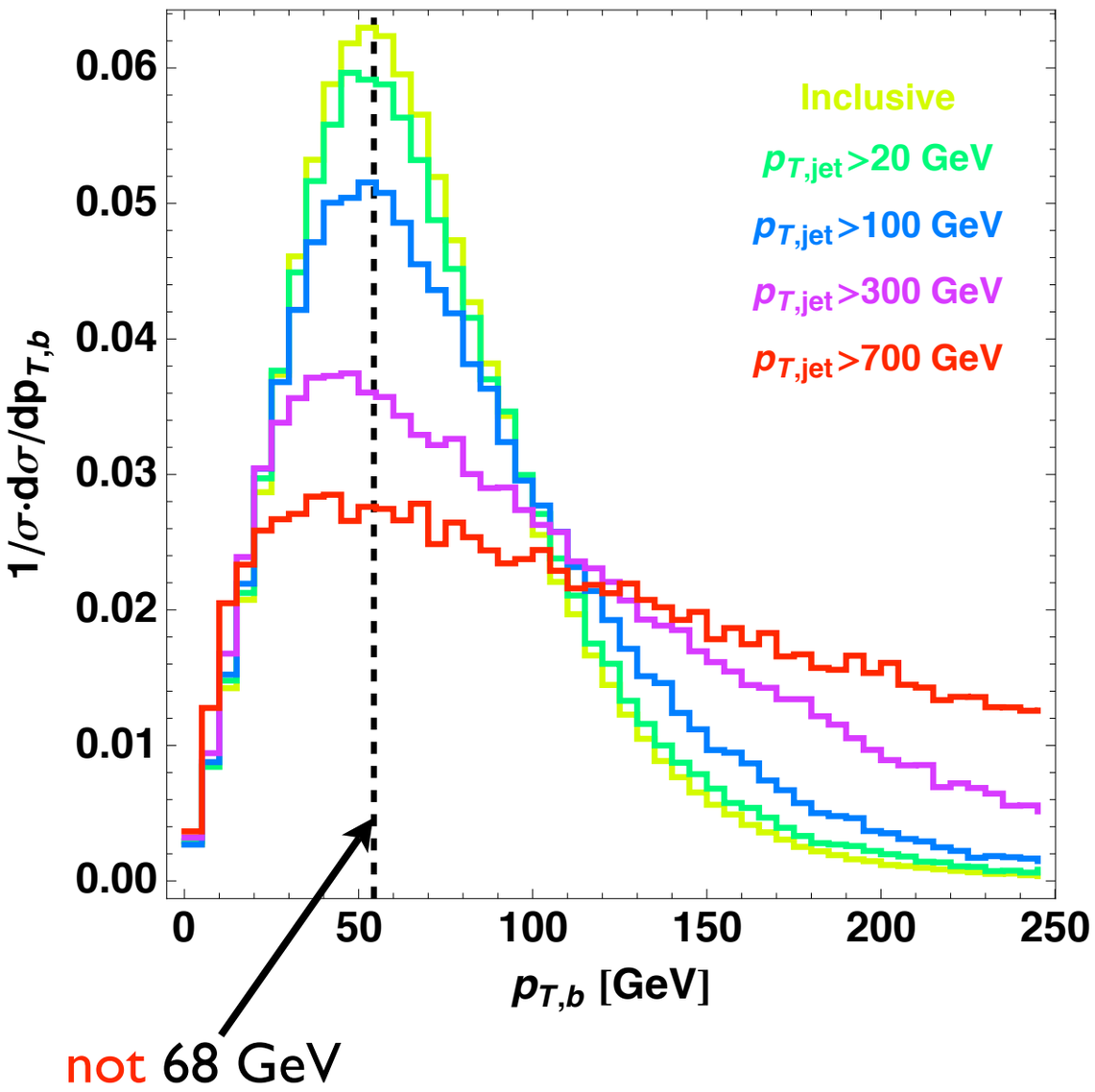
``Invariant" (under boost distributions) feature in non-invariant (energy)distribution: subtle!

- vary collider energy
- vary ISR
- ...but, peak stays put, even though shape changes (broadens for more boosted top)

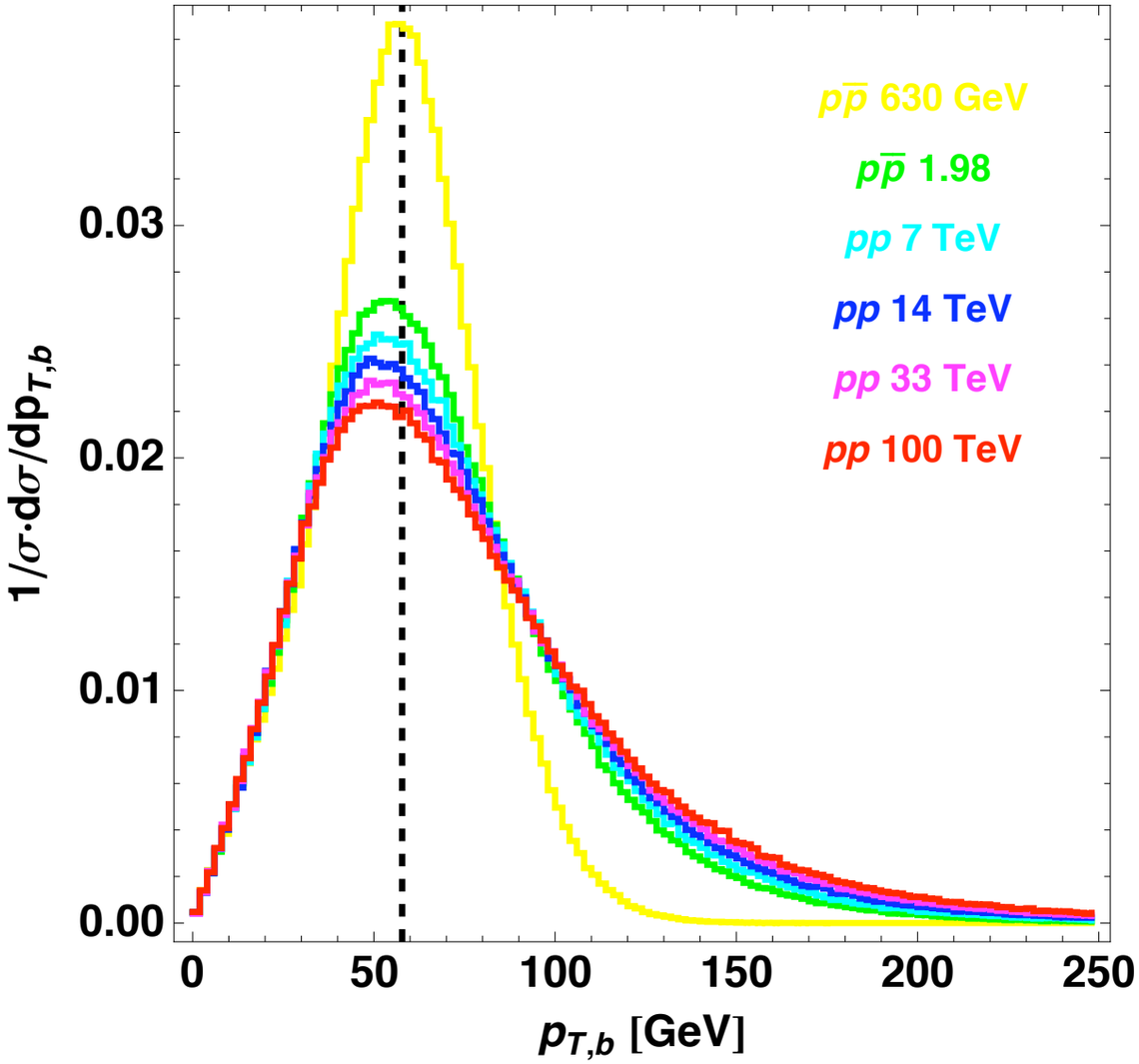




...accidents don't happen: no such invariance for p_T!



not 68 GeV

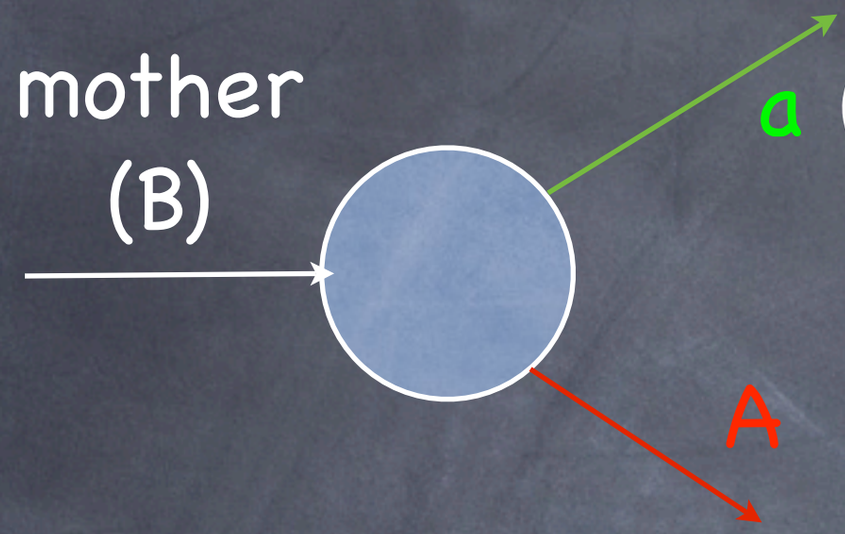


- peak (and shape) change...

(POSSIBLE) APPLICATIONS:

``BETTER'' ONES TO COME?!

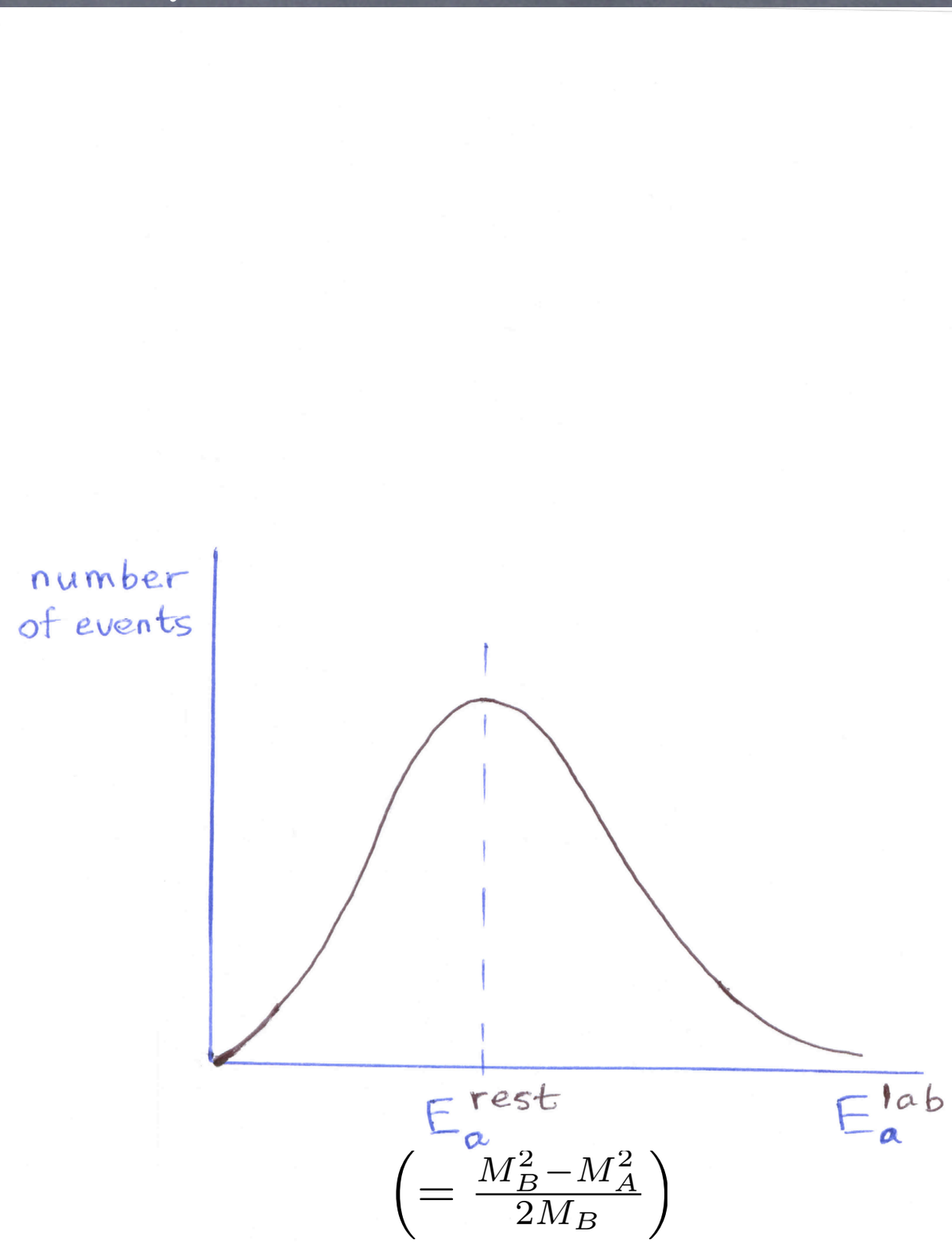
General, simple Idea



- determine M_B (if M_A known) using $E_a^{\rm rest}$ (measured from peak in $E_a^{\rm lab}$)

$$E_a^{\rm rest} = \frac{M_B^2 - M_A^2}{2M_B}$$

(independent of production mechanism of unpolarized mother)



Measuring the peak

- peak can be wide (difficult to read-off value ``by eye'')
- extract peak by fitting to ``theory curve'':
  a la Breit-Wigner [simple (2-parameter), analytic, model-independent function]
- ...but exact, analytic formula difficult to obtain here (depends on boost distribution, thus PDF's...)

APPROXIMATION TO THEORY CURVE

Do know (analytically) properties of distribution, f

- value of $f(x)$ remains the same under $x \leftrightarrow \frac{1}{x} \left(x \equiv \frac{E_a^{\rm lab}}{E_a^{\rm rest}}\right)$
- f is maximized at $x = 1$
- f vanishes as x approaches 0 or ∞
- f becomes a δ-function in some limit of its parameters

Ansatz (based on properties)

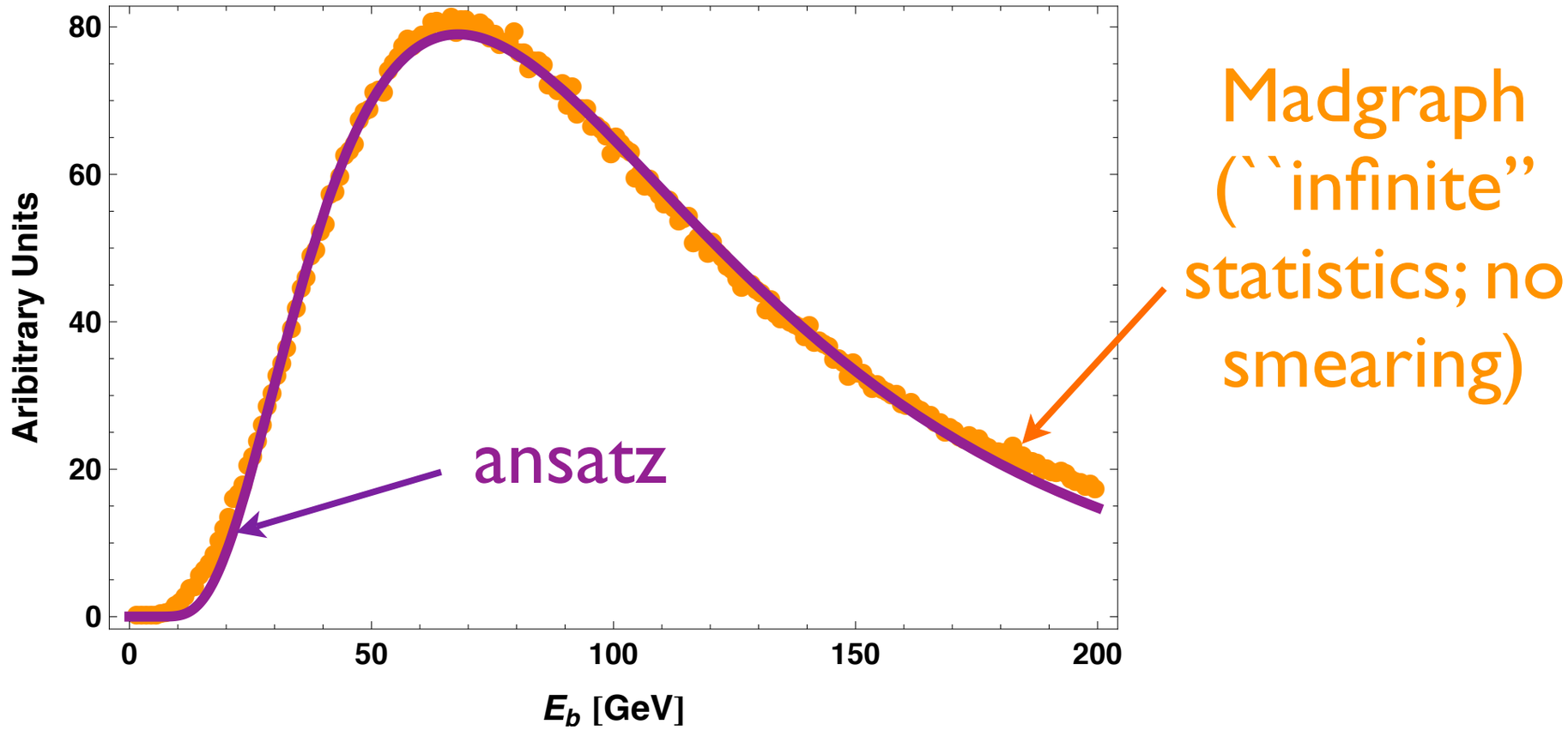
width parameter

$$f(x) = K_1^{-1}(p) \exp\left[-\frac{p}{2}\left(x + \frac{1}{x}\right)\right]$$

Bessel function (normalization)

- simple, but not unique "peak finder"...

``Test'' on b-jet energy from top quark decay (production unpolarized...)



- bottom (almost)``massless'': peak does not shift, shape property negligibly violated
- good fit for heavier ``top'' quark as well: different PDF's, boost distribution (width parameter encompasses this variation)

"New" Breit-Wigner

- Based on theory fits, assume

$$f(x) = K_1^{-1}(p) \exp\left[-\frac{p}{2}\left(x + \frac{1}{x}\right)\right]$$

FURTHER TEST:FIT TO (SIMULATED)DATA

(Again) Top quark decay: basic idea

neglect m_b in $E_b^{\rm rest}$

- Peak in measured b-jet energy distribution $\approx \frac{M_t^2 - M_W^2}{2M_t}$
- Assuming M_W (but no need to detect it at all!), get M_t

Top mass measurement: details

- Fully leptonic (opposite flavor) and 2 b-tags,
  with 5/fb at LHC7: expect 4000 S vs. 200 B

- Madgraph ⟶ Pythia ⟶ Delphes/Fastjet

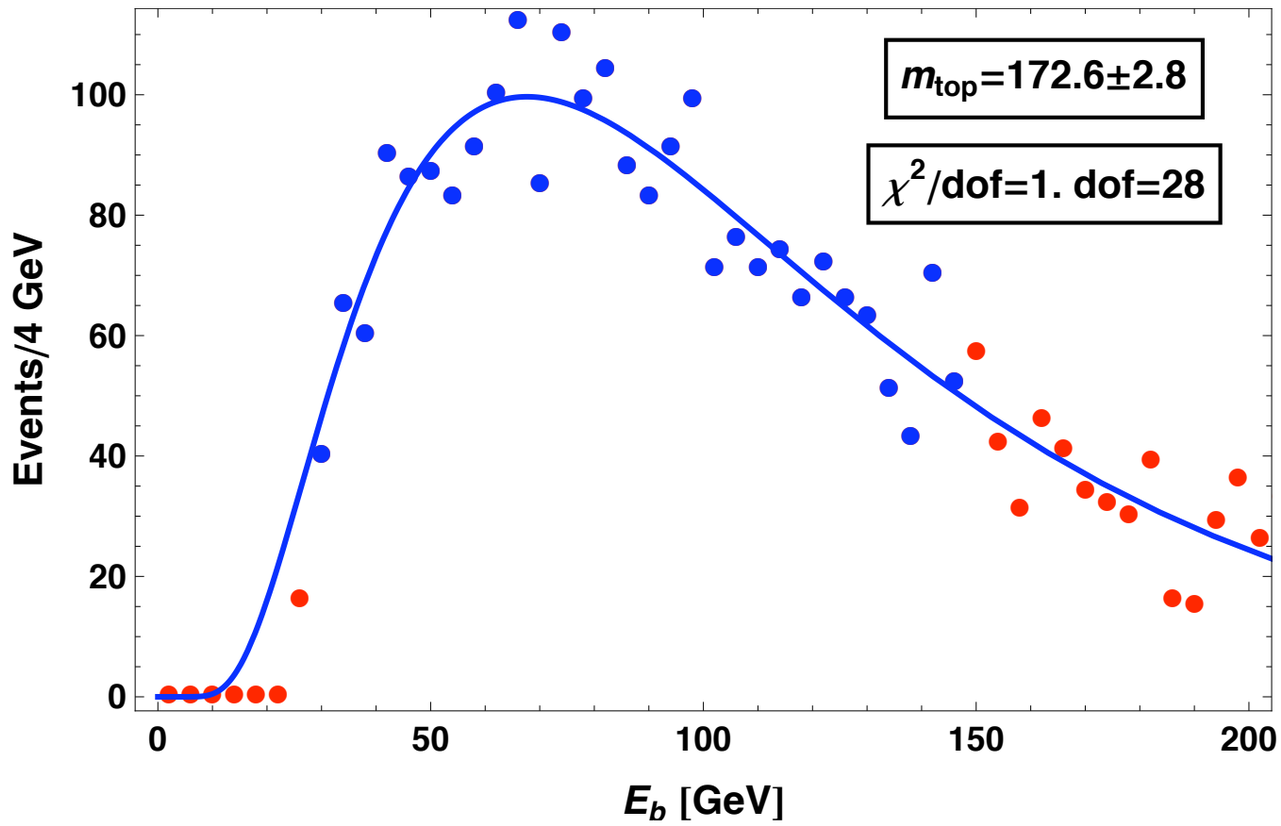
- 100 pseudo-experiments

- ATLAS/CMS choice of (mild) cuts:
  1209.2393; ATLAS-CONF-2012-097

- neglected background

Result

(1 pseudo-experiment shown)

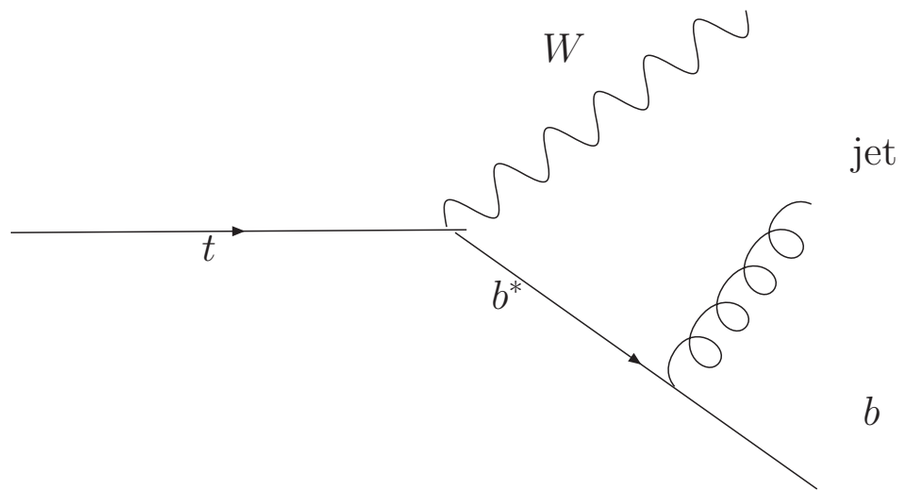


(use only blue dots)

- consistent with input value
- fitting not spoiled by cuts or detector effects

Discussion

- neglected hard radiation from bottom (3-body): suppressed by α_s/π + jet-veto (calculable in QCD)



- safe from soft radiation off of bottom
- safe from initial state radiation
- no combinatorics (include both b's)
- independent of production mechanism (single or pair; uncertainty in PDF's; new physics or SM) as long as unpolarized

Comparison (simplified!) of methods for top mass

- complementary/cross-check: different systematics, e.g., use of MET in some earlier methods vs. not here

- theory systematics, based on (small) parameters:

$$\delta_{\rm prod}\ (\text{PDF's, new physics});\ \epsilon_{\rm FSR}\ (\text{NLO, jet-veto});\ f_{\rm pol.}\ (\text{new physics})$$

- error in top mass with
  - matrix element; full reconstruction (combinatorics): $\delta_{\rm prod}$
  - b-jet energy-peak: $\epsilon_{\rm FSR} \times \delta_{\rm prod.} + f_{\rm pol.}$

$\epsilon_{\rm FSR}$ for QCD production is calculable

- "test" for applications to new physics

What about real data?

- CMS email to us in July, 2013:

``...I guess you will be pleased to hear that we have now someone within CMS who is planning to try an mt extraction with the 8 TeV data following your Ansatz....

However, since that group is only starting now, we can't expect to see results too soon..."

....October, 2013: contact with M. Irfan Asghar from CMS about actual implementation!

(Preliminary results with semileptonic and dileptonic)

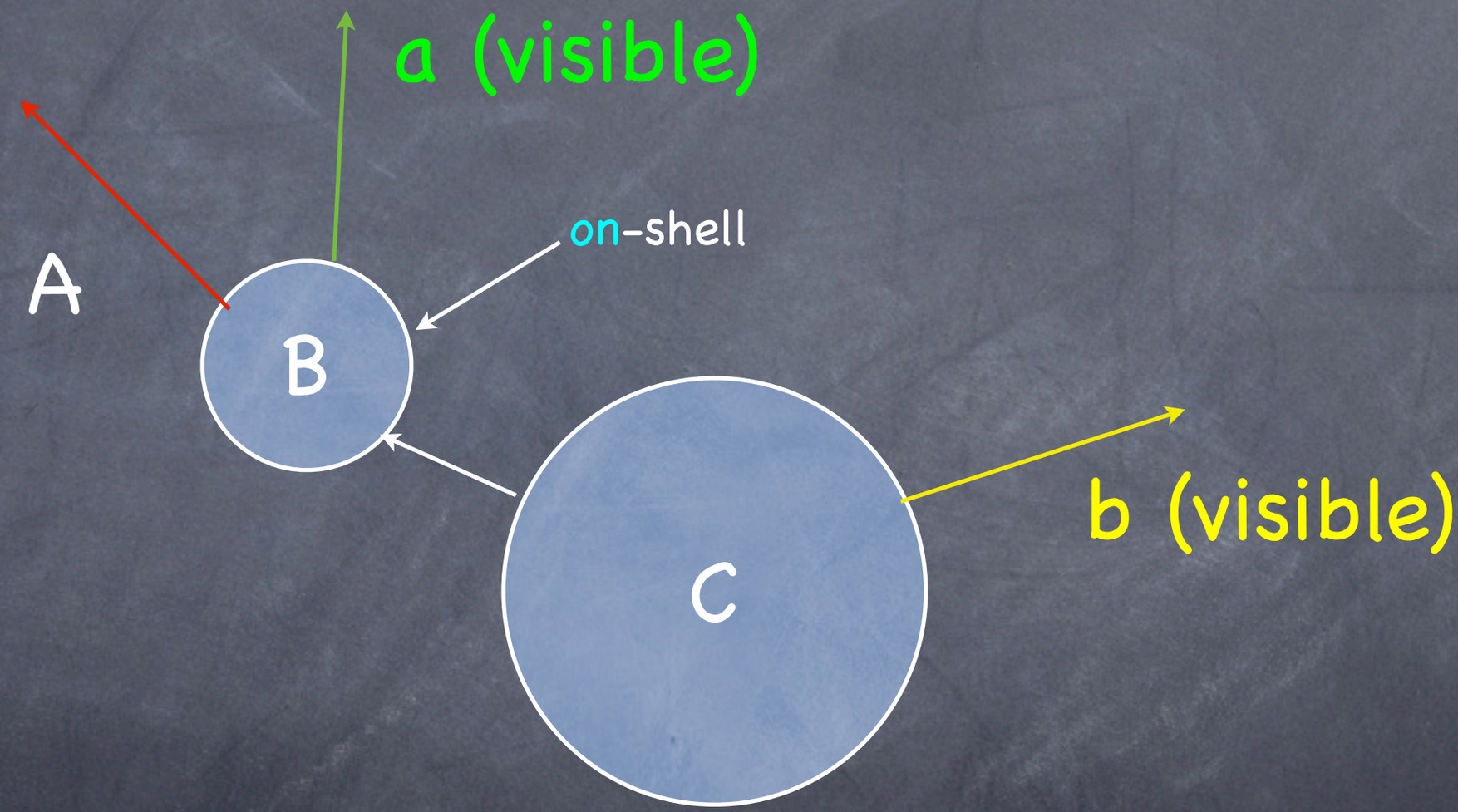
- Can ATLAS be far behind?!

A NEW PHYSICS APPLICATION (METHOD "TESTED" ON TOP MASS): CASCADE DECAY

(KA, Franceschini, Kim: 1309.4776)

In General: Topology

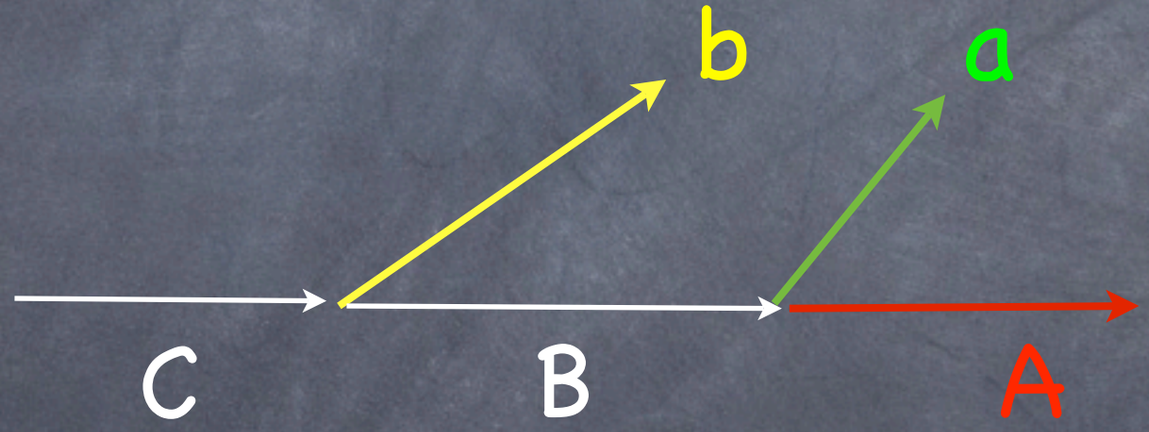
- Two 2-body decays: primary (C) and secondary (B) mothers)



Two energy peaks

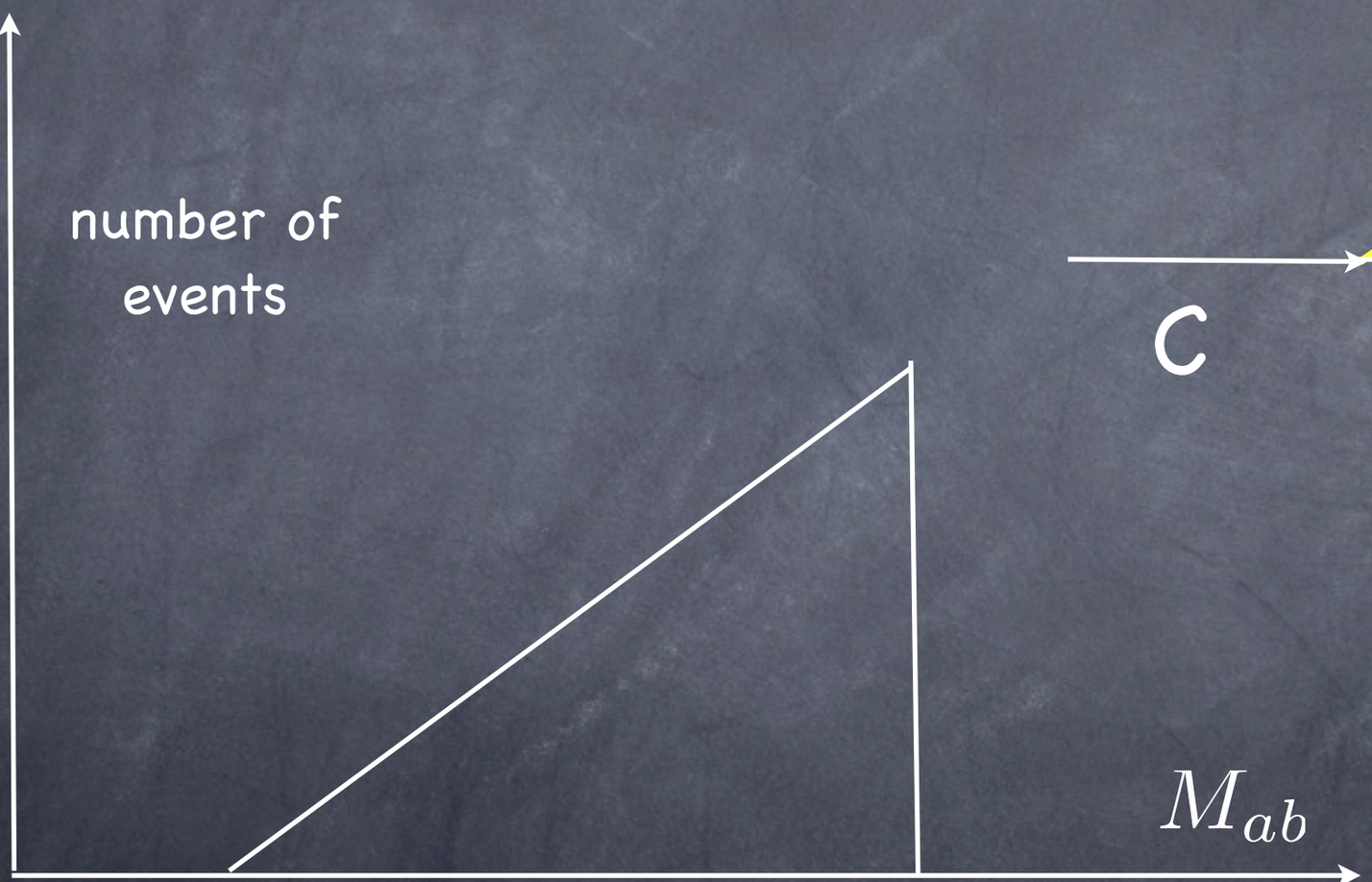
- Based on new observation:

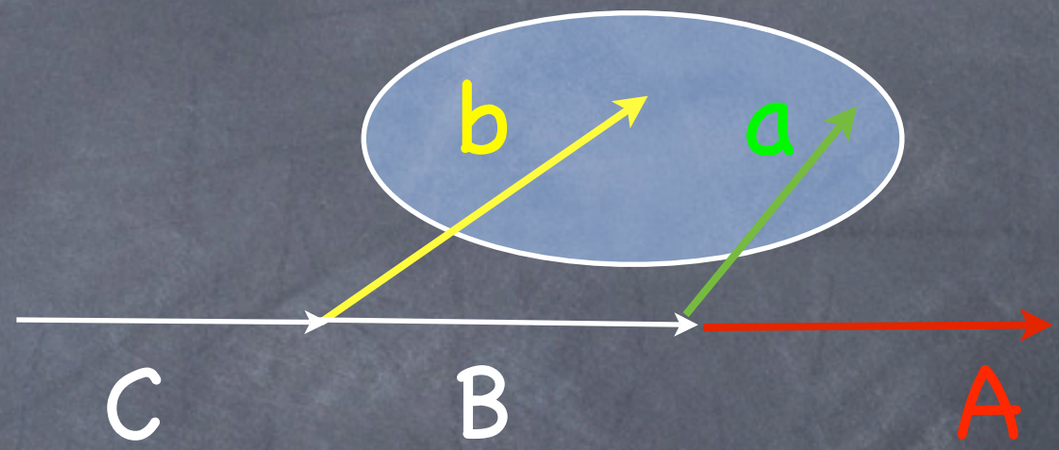
$$E_b^{\text{peak}} = \frac{M_C^2 - M_B^2}{2M_C} \text{ and } E_a^{\text{peak}} = \frac{M_B^2 - M_A^2}{2M_B}$$



Edge in invariant mass (old)

- On-shell intermediate particle ⟹ (sharp) edge





$$\sqrt{\frac{\left(M_C^2-M_B^2\right)\left(M_B^2-M_A^2\right)}{M_B}}$$

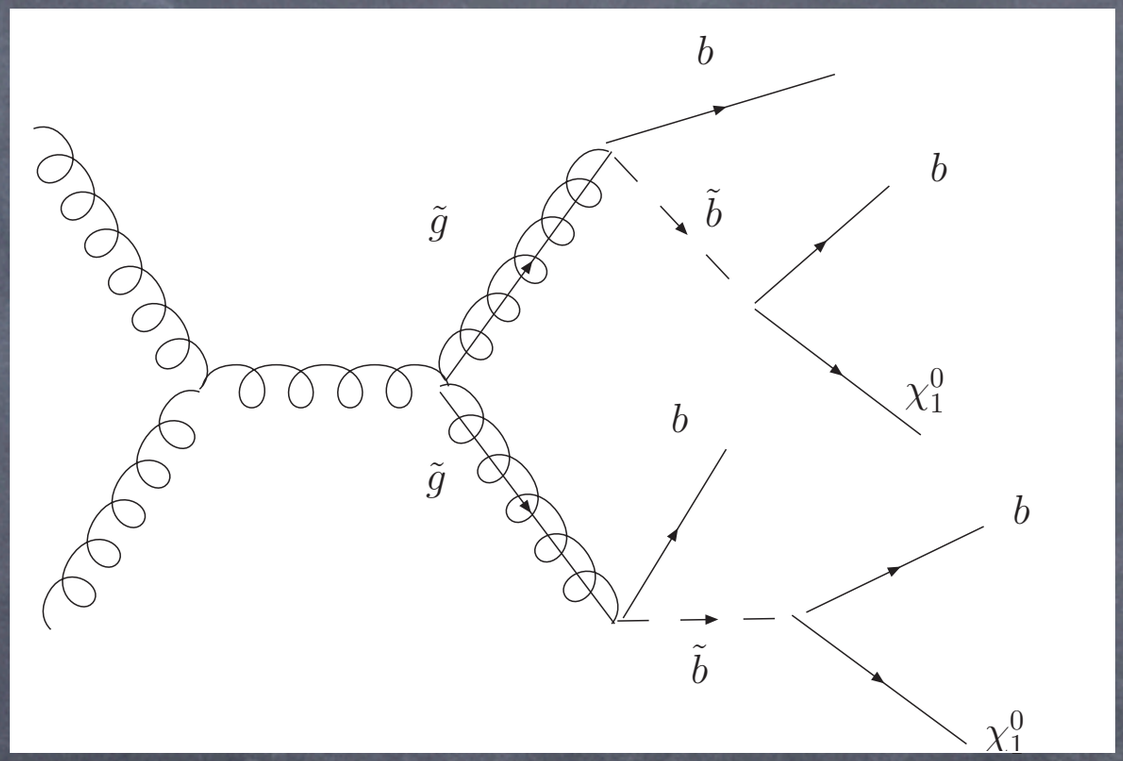
= 3 (independent)
observables for
determining 3 masses!

...(in principle) determine invisible particle
mass without measuring MET!

CASCADE DECAY IN SUSY

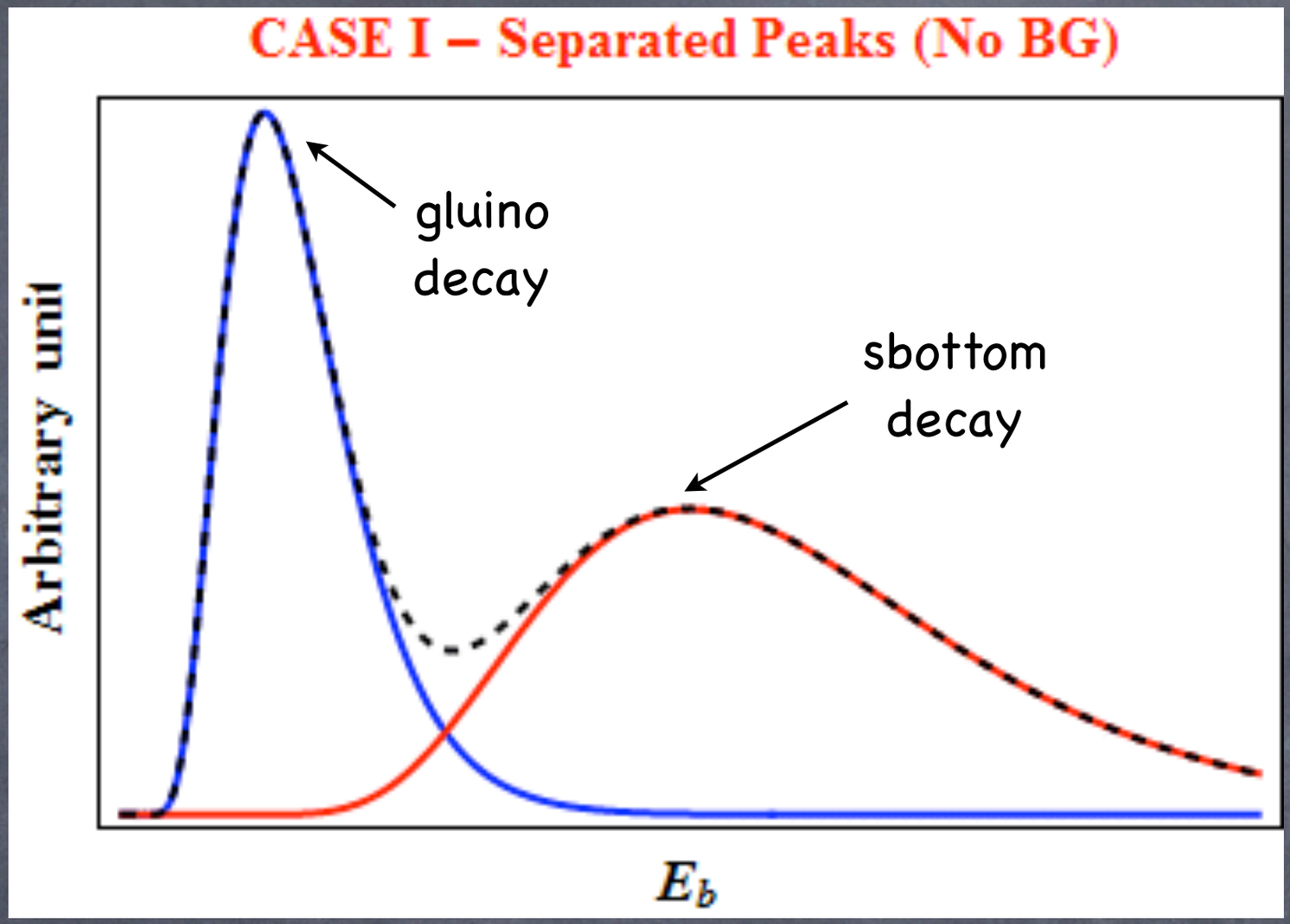
(AN EXAMPLE)

Gluino, sbottom, neutralino



- natural SUSY: 1st/2nd generation squarks heavy, stop/sbottom and gluino, Higgsino light

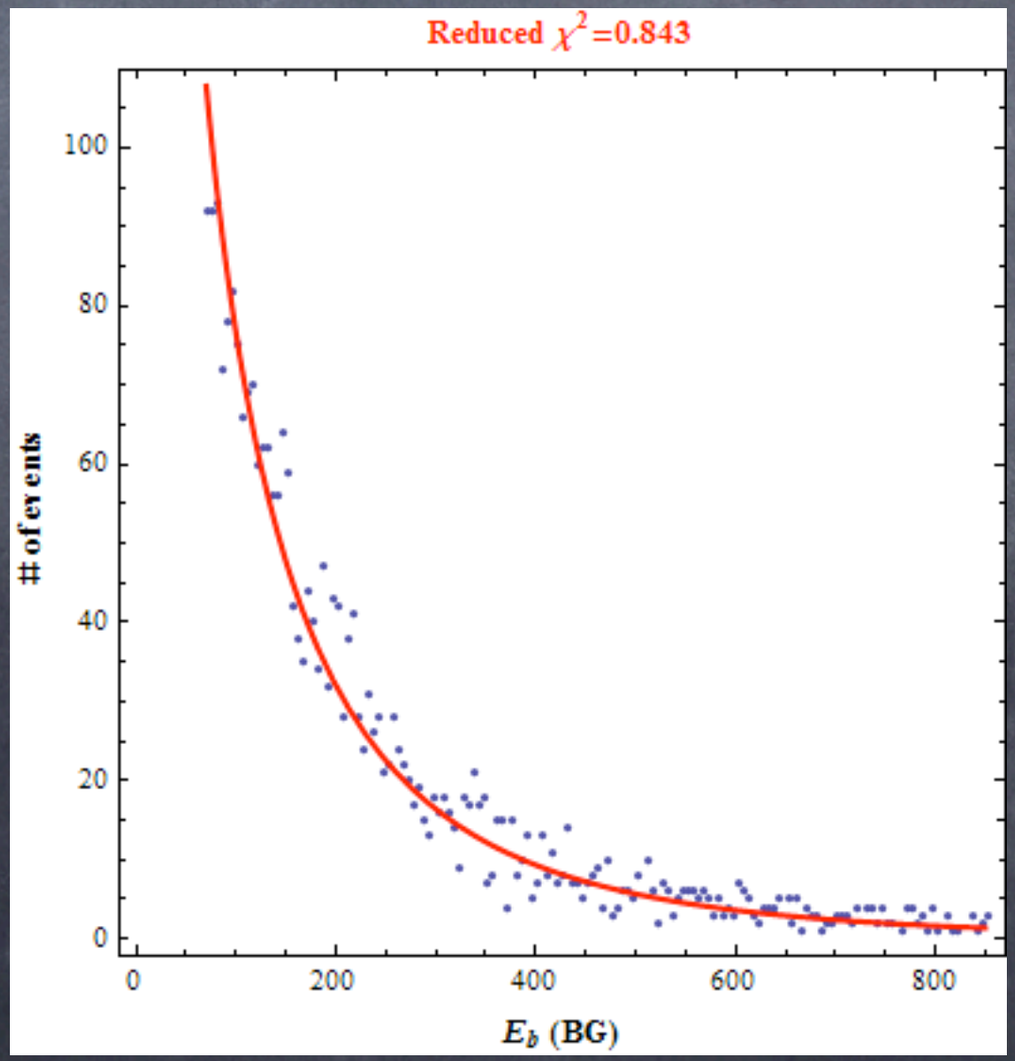
Double (b-jet energy) peak



- mass hierarchy: $M_{\tilde{g}} \approx M_{\tilde{b}} \gg M_{\chi_1^0}$ ⇒ "soft" & hard b-jets

Background

- $\bar{t}t\bar{b}b$ reducible and $Z + 4b$ irreducible
- template for background: $N_{p'} \exp\left(-p'\sqrt{E}\right)$

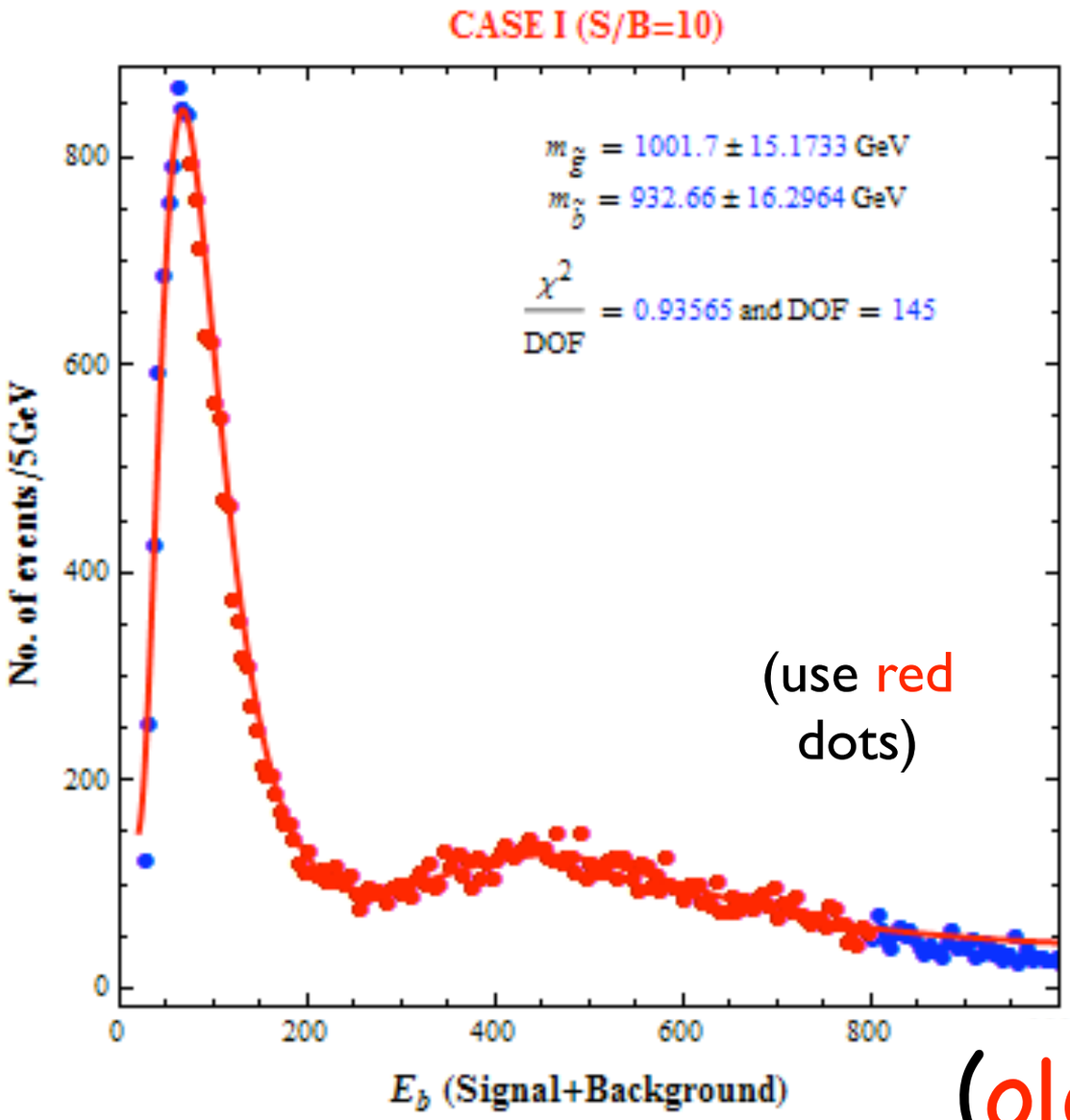


$Z + 4b$

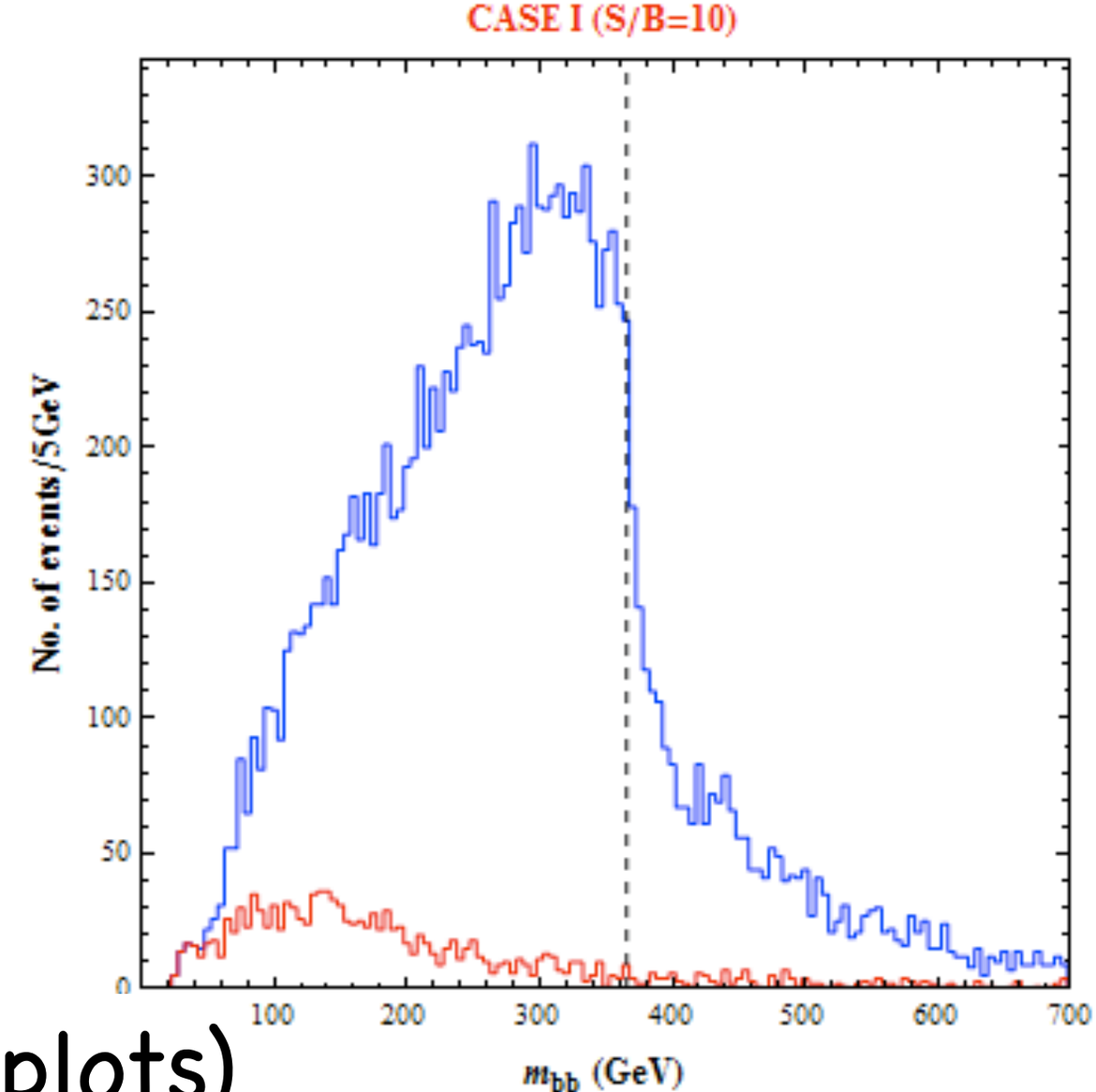
(old plot)

Results

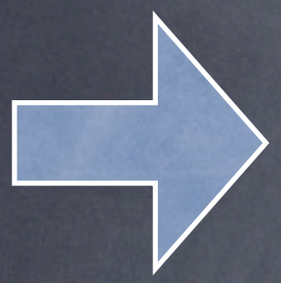
- $M_{\tilde{g}} = 1000$ GeV; $M_{\tilde{b}} = 930$ GeV and $M_{\chi_1^0} = 100$ GeV with 300 / fb at LHC14
- 3 (2 signal + 1 background) template fit (assume this model)
- little sensitivity to $M_{\chi_1^0}$: $2\sqrt{E_b^{\text{peak 1}} E_b^{\text{peak 2}}} \approx M_{bb}^{\max}$



(use red dots)



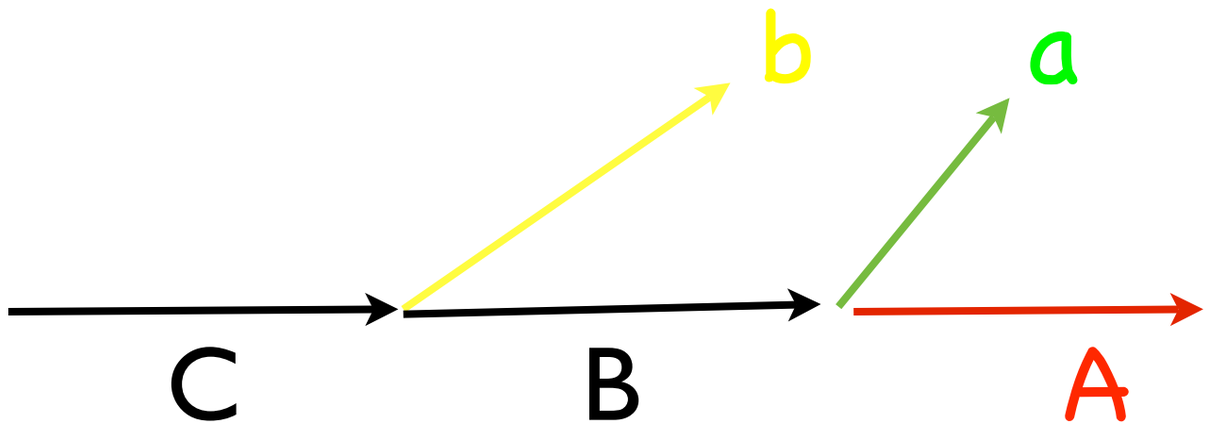
(old plots)



ansatz/fitting function **works** for (boost distribution of) a ``secondary'' mother **as well**!

Other/cleaner possibilities

- $a \neq b$: peaks in different distributions (no ``pollution'' between peaks)



- lepton instead of jet

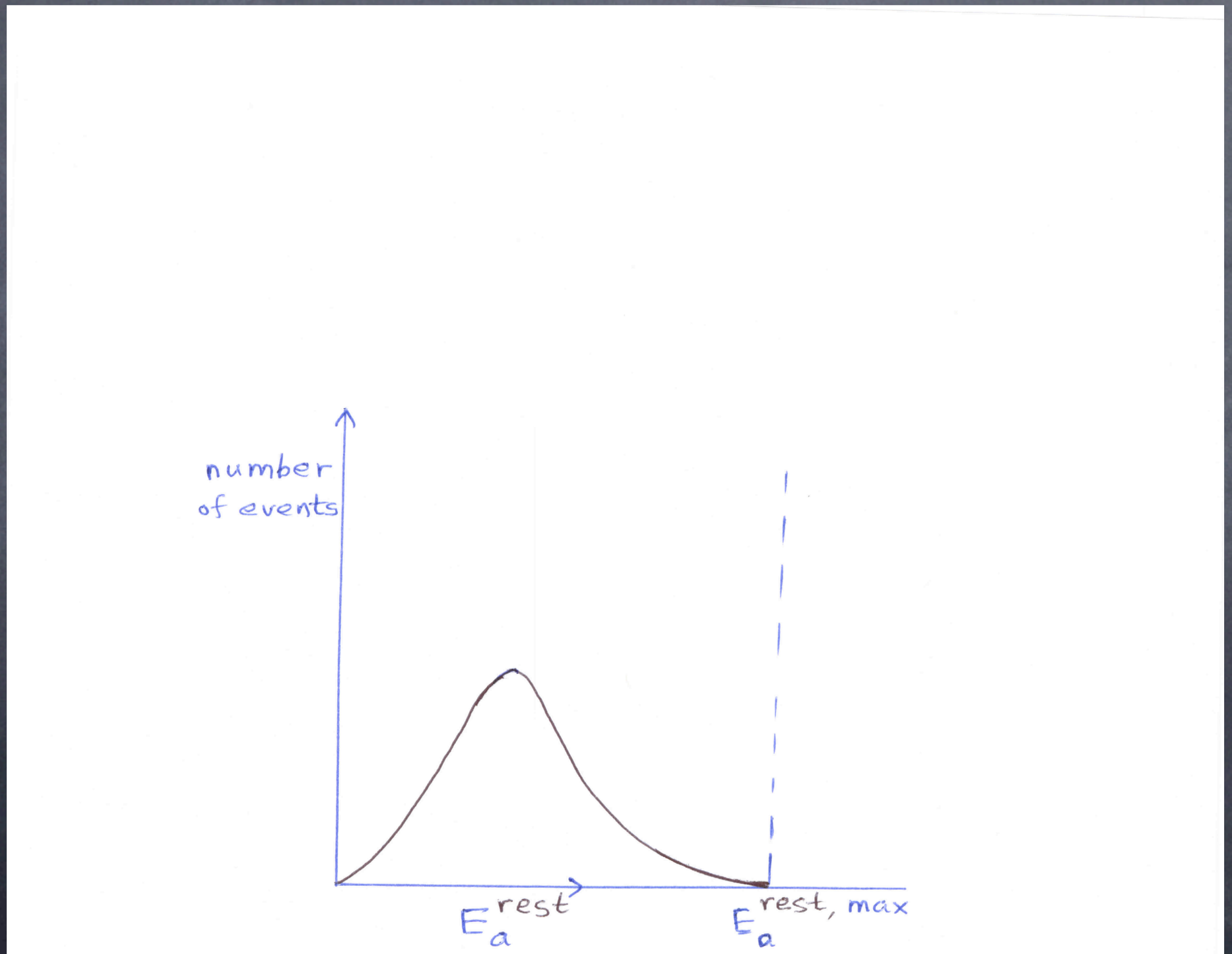
Generalizations

- Massive daughter
- Three-body decay with 2 visible (e.g., off-shell sbottom): for fixed invariant mass of 2 visible, apply 2-body result

THREE-BODY DECAY: ONE VISIBLE

Endpoint of distribution in rest frame

- Endpoint related simply to masses



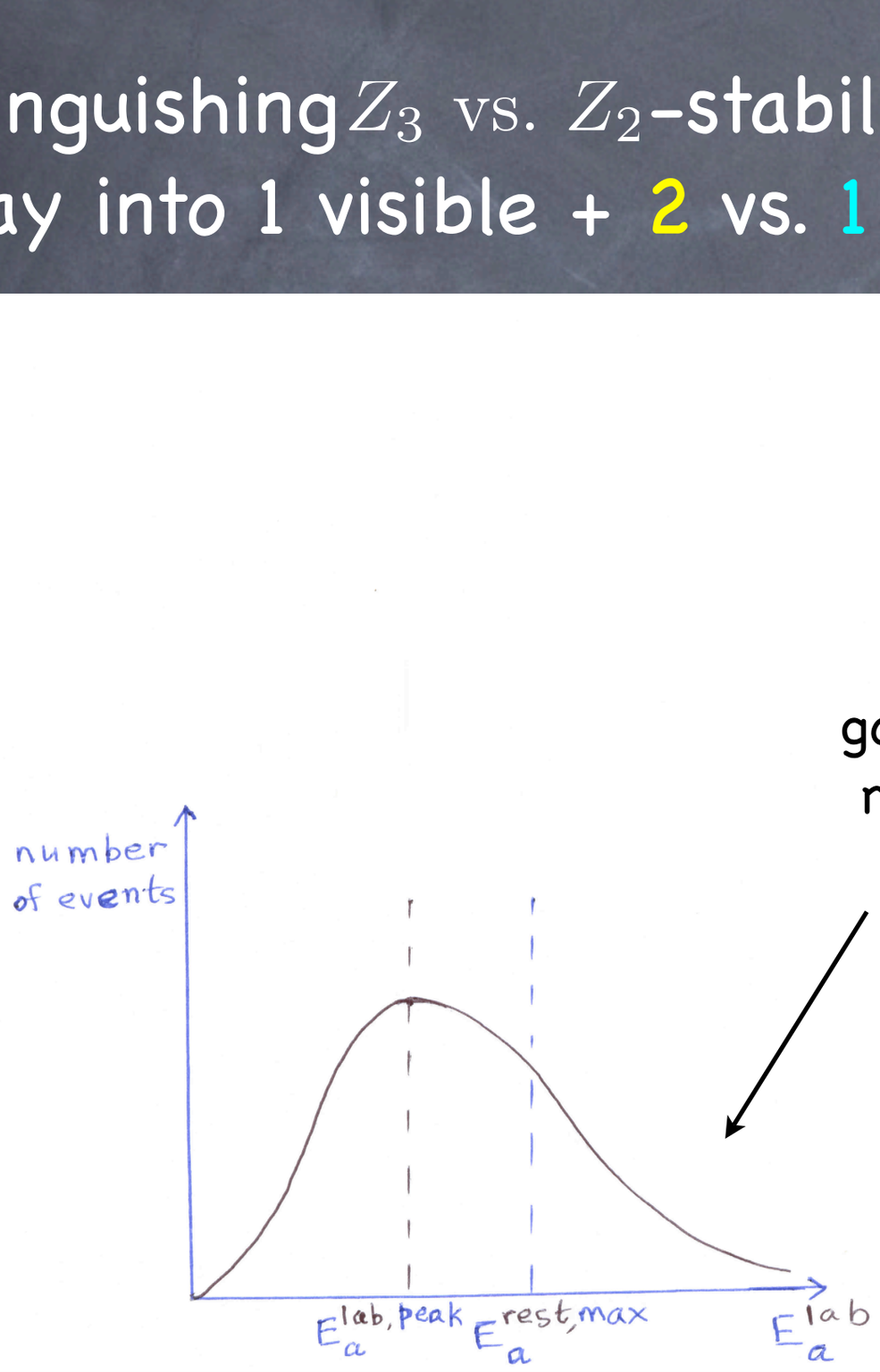
Peak of distribution in lab frame

$$E_a^{\text{lab,peak}} < E_a^{\text{rest,max}}$$

- Obtain inequality for masses

- distinguishing Z_3 vs. Z_2-stabilized dark matter (DM): decay into 1 visible + 2 vs. 1 DM ("same" final state!)

(KA, Franceschini, Kim, Wardlow: 1212.5230)



Conclusions

- Two body decay of unpolarized mother at hadron colliders:
  peak in energy distribution of massless daughter same as rest frame energy (simple function of masses)
- Obtain approximation to theory curve (for fitting to data to extract peak)
- Application(s):
  top quark mass (test)
  new particles decaying semi-invisibly: extract all masses from cascade decay (e.g., gluino to sbottom...)

BACK-UP

Formal proof

- Single Rectangle ($x = \frac{E_a^{\rm lab}}{E_a^{\rm rest}}$) :

$$\frac{1}{\Gamma}\frac{d\Gamma}{dx}\Big|_{\text{fixed } \gamma_B} = \frac{\Theta\left(x-\gamma_B+\sqrt{\gamma_B^2-1}\right)\Theta\left(-x+\gamma_B+\sqrt{\gamma_B^2-1}\right)}{2\sqrt{\gamma_B^2-1}}$$

- Stacking up rectangles:

$$f(x) \equiv \frac{1}{\Gamma}\frac{d\Gamma}{dx} = \int_{\frac{1}{2}\left(x+\frac{1}{x}\right)}^{\infty} d\gamma_B \frac{g(\gamma_B)}{2\sqrt{\gamma_B^2-1}}$$

- Slope:

$$f'(x) = \frac{\text{sgn}(1-x)}{2x} g\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$$

- Behavior at x = 1:

$f'(x=1) \propto g(1) = 0 \Rightarrow$ extremum or
$f'(x)$ flips its sign at $x = 1 \Rightarrow$ a cusp
$f(x)$ is positive and vanishes for both $x \to 0$ and $x \to \infty$
$\Rightarrow$ peak at $E_a^{\rm rest}$

``Massive'' daughter

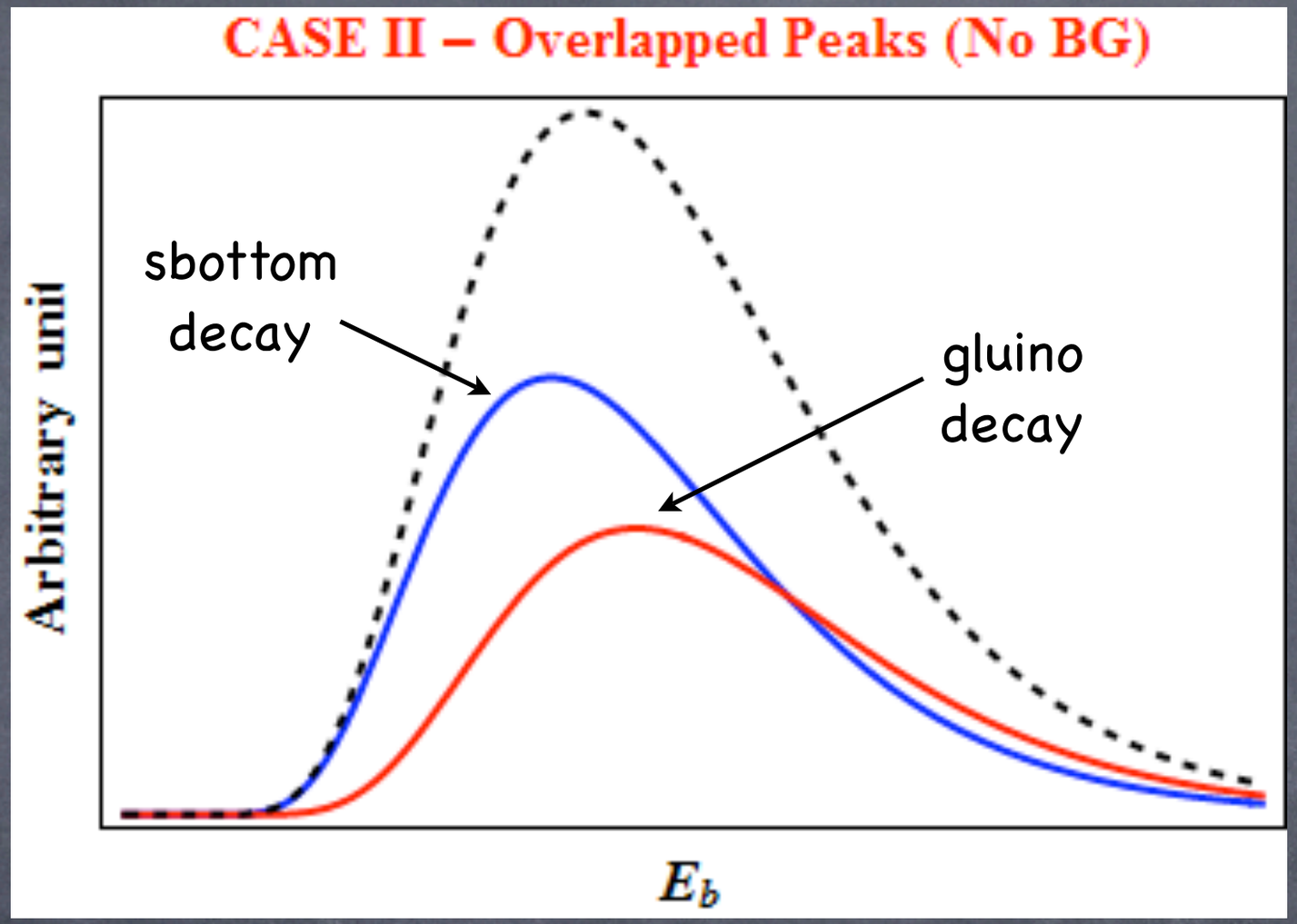
- argument goes thru' (rectangle contains $E_a^{\rm rest}$...) even for massive daughter if boost distribution restricted to $\gamma_B < \left[2\left(\gamma_a^{\rm rest}\right)^2 - 1\right]$

- This critical boost is typically large value for massive, but ``light'' daughter: e.g., for bottom from top quark decay ($\gamma_b^{\rm rest} \approx 15 \Rightarrow \gamma_{\rm top} \lesssim 500$ suffices)

Another SUSY spectrum: sensitivity to neutralino mass

- mass hierarchy: $M_{\tilde{g}} \gg M_{\tilde{b}} \gtrsim M_{\chi_1^0}$ → both b-jets hard

Overlapping peaks



- Ansatz can extract 2 peaks separately (assuming this model)

Other (possibly) related work

- razor (pair-produced polarized/unpolarized mothers):

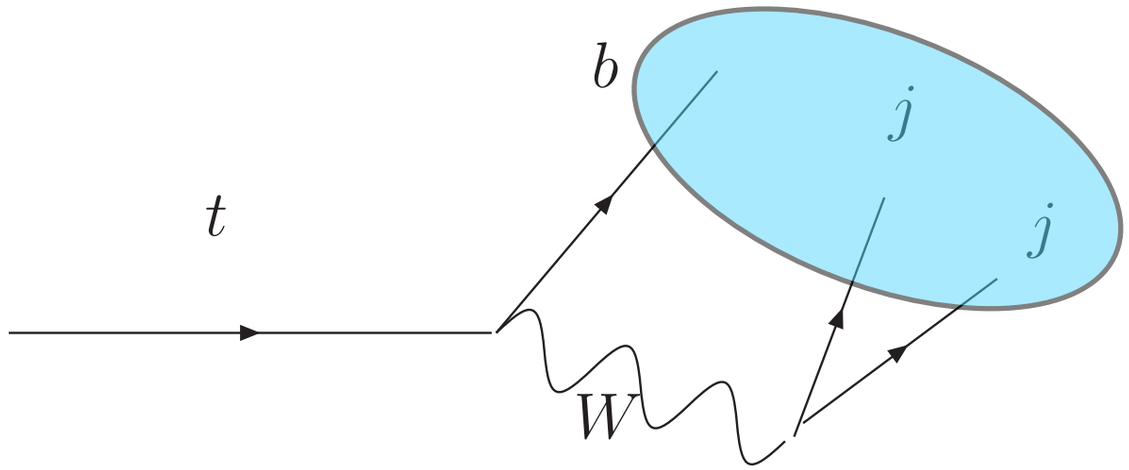
$M_R = 2E_a^{\rm rest}$, but *assuming* (a) mother at rest in COM of two mothers *and* (b) no transverse boost of this COM in lab frame

- 1107.4460, 1305.6150: use entire energy distribution, no (explicit) mention of location of peak (local feature)
- "Jacobian" peak at M_W in $2 \times p_T^{\rm lepton}$ and M_T (only) for single W production (+ events at this peak different than at energy-peak)

(Motivation for top mass: fundamental parameter of SM;enters calculation of other observables)

Conventional methods

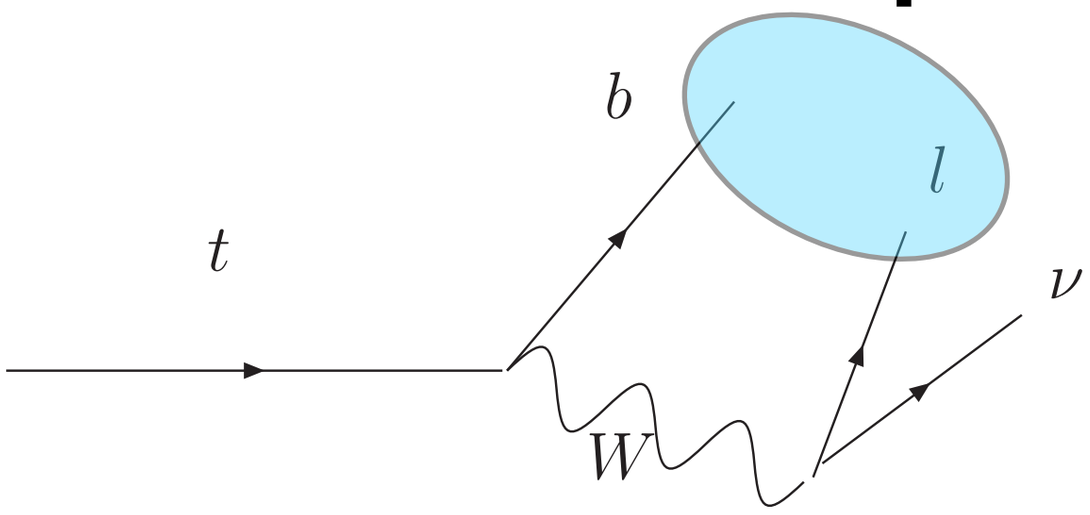
- Basic idea: reconstruct (full) decay of top

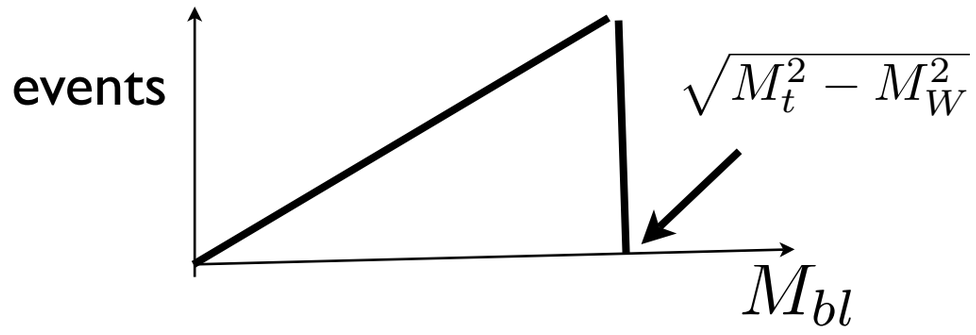


- can achieve O(0.6 GeV) uncertainty at LHC14, with 300/fb
- further gain may be possible with 3000/fb by using a more extended approach to constraining uncertainties using data

- Simulation (using SM matrix element in production) is used to handle combinatorics

Latest: endpoint of M_{bl}





- more cleanly interpreted as measurements of the pole quark mass

- combinatorics resolved with*out* assuming SM matrix element in production
  resulting top quark mass immune to possible contaminations from New Physics in production of top quarks

- can provide precision competitive with more conventional methods, especially using 3000/fb at LHC14

Comparison between M_{bl} and energy-peak

- energy-peak has larger statistics
- M_{bl} more robust (against hard radiation from bottom; polarized top quarks)

Using energy-peak for searches

- if background is flat or peaks elsewhere from signal
- Stops (Low: 1304.0491):

for $\tilde{t} \to b\tilde{\chi}_1^+$, peak in $E_b^{\rm lab}$ at $\left(M_{\tilde{t}}^2 - M_{\tilde{\chi}_1^+}^2\right) / (2M_{\tilde{t}})$...
can be $\gg \left(M_t^2 - M_W^2\right) / (2M_t)$ from $t\bar{t}$ background (from SM or from $\tilde{t} \to t\tilde{\chi}_1^0$)