Circle compactification, confining and conformal gauge dynamics

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with Mithat Ünsal — SLAC/Stanford

I won't say anything of immediate use to the LHC - while I'll talk about hadronic string tensions, these won't be the right ones to use in a hadronization algorithm...

This is a talk about nonperturbative gauge dynamics:

There are many things one would like to understand about any gauge theory:

- does it confine?
- does it break its (super) symmetries?
- is it conformal?
- what are the spectrum, interactions...?

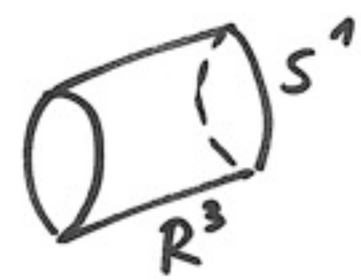
Tough to address, in almost all theories.

But interesting for:

satisfying theorist's curiosity
QCD
SUSY extensions of the Standard Model
non-SUSY extensions of the Standard Model

...what's new?

Will use older and more recent results to study a regime where the nonperturbative dynamics of 4-d gauge theories - SUSY or not, chiral or vectorlike - is analytically tractable: compactifying 4d gauge theories on a small circle is a "deformation" where nonperturbative dynamics is under theoretical control (as "friendly" as SUSY, e.g. Seiberg- Witten theory).

Studying gauge theories on 

or, say, 

of characteristic size "L" has a long history...

e.g., Bjorken's "femtouniverse" & Luescher (1983),
Eguchi-Kawai's "large-N reduction" (1982),
Gonzalez-Arroyo, Perez (1980's-90's)...

While our inspiration came from Seiberg-Witten (1997) and Aharony-Intriligator-Hanany-Seiberg-Strassler (1998) on SUSY circle compactifications (the most successful/celebrated examples) it turns out that there are connections to the earlier work.

punchline:

We will gain new, sometimes perhaps surprising, insight (both analytic and qualitative) into the physics of confinement and abelian or discrete chiral symmetry breaking gauge theories with massless fermions in vectorlike or chiral representations

- in a "locally 4d" setting.

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- in a "locally 4d" setting.

But - don't expect to analytically compute detailed properties of QCD, or other gauge theories, on R^4 !

Compactifying 4d theory on a small circle is no magic bullet.
Many aspects of the physics are not analytically accessible in the regime we will study - but the relative simplicity is what gives us theoretical control!

On the positive side, it appears that at least some aspects of the dynamics relevant for the transition to conformality (as the number of massless fermion species increases) are retained. This is especially interesting in theories with confinement without chiral symmetry breaking (as in SUSY or perhaps in some non-SUSY chiral theories).

The plan

of this talk is to tell you, largely in pictures, what the advertised insight amounts to, what its “abilities” and limitations are...

time

inf{rigour}

Conformality or confinement (II): One-flavor CFTs and mixed-representation QCD
JHEP 0912:011,2009; 0910.1245, 33pp

Conformality or confinement: (IR)relevance of topological excitations
JHEP 0909:050,2009; 0906.5156, 42pp

Chiral gauge dynamics and dynamical supersymmetry breaking
JHEP 0907:060,2009; 0905.0634, 31pp

Index theorem for topological excitations on R*3 x S*1 and Chern-Simons theory
JHEP 0903:027,2009; 0812.2085, 29pp

(all by M. Unsal and E.P.)

this is the only model above 2d where an explicit and analytic (& physicsal) understanding of confinement is available!
- see Banks's recent textbook where it (finally) made it...

3d Polyakov model & "monopole-instanton"-induced confinement

Polyakov, 1977

continuum picture: 3d Georgi-Glashow [on the lattice - compact U(1)]

$$L \sim \frac{1}{g_3^2}\left(F^a_{\mu\nu} F^a_{\mu\nu} + D_\mu \phi^a D^\mu \phi^a\right) \qquad \mu, \nu = 1, 2, 3$$

$$[A_\mu] = [\phi] = 1 \qquad [g_3^2] = 1$$

due to some Higgs potential $\langle \phi \rangle = (0, 0, v)$

$SU(2) \xrightarrow{v} U(1)$ at low energies, $E \ll m_W \sim v$

free U(1) theory $A^3_\mu \equiv A_\mu$

$$\mathcal{L}_{eff} = \frac{1}{g_3^2} F^2_{\mu\nu} + \ldots$$

"..." are perturbatively calculable
& not very interesting

$$B_\mu = \epsilon_{\mu\nu\lambda} F_{\nu\lambda}$$

"magnetic field"
topologically conserved current of **"emergent topological U(1) symmetry"** responsible for conservation of magnetic charge

$$B_\mu = g_3^2 \, \partial_\mu \sigma$$

3d photon dual to scalar (as one polarization only)

Abelian duality

$$\partial_\mu B_\mu = 0 \qquad \longleftrightarrow \qquad \partial_\mu^2 \sigma = 0$$

Bianchi identity ⟷ equation of motion

$$\mathcal{L}_{eff} = \frac{1}{g_3^2} F_{\mu\nu}^2 + \ldots \qquad \mathcal{L}_{eff} = g_3^2 (\partial_\mu \sigma)^2 + \ldots$$

topological U(1) symmetry = shift of "dual photon"

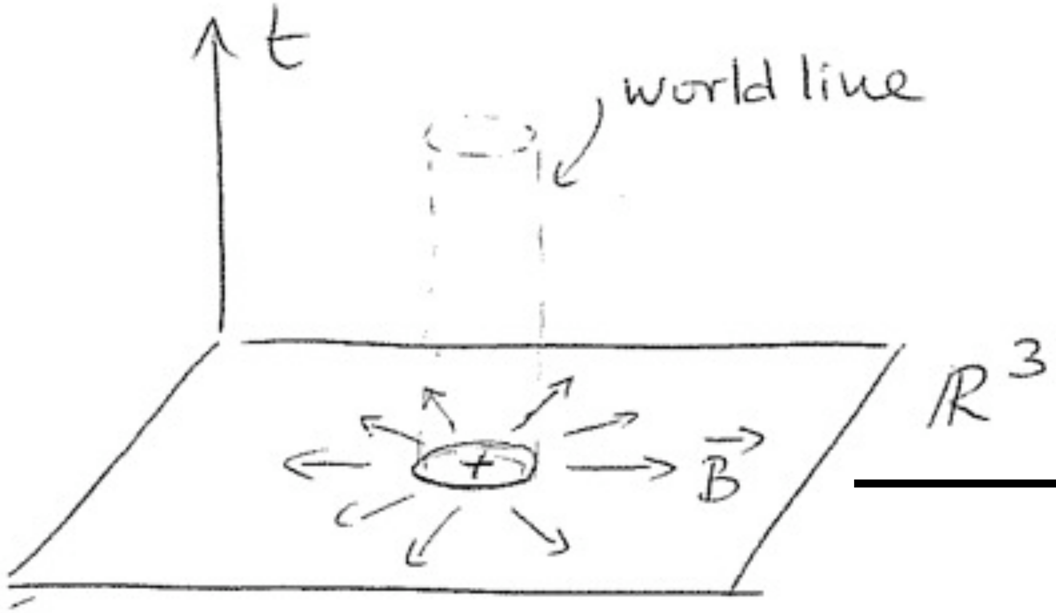
a rather "boring-boring" duality - if not for the existence of monopoles:

monopoles $\partial_\mu B_\mu$ = quantized magnetic charge - shift symmetry broken

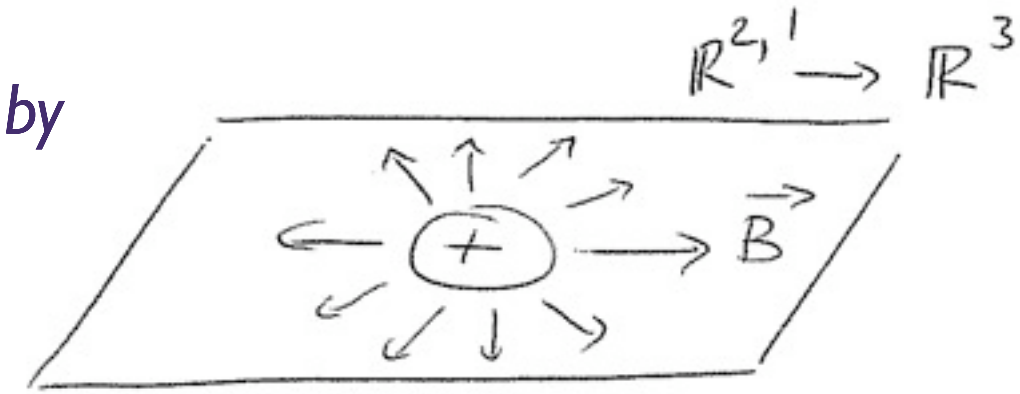
- **dual photon gains mass** & **electric charges confined**

how?
...in pictures:

“‘t Hooft-Polyakov monopole” - *static finite energy solution of Georgi-Glashow model in 4d*



get Euclidean 3d by “forgetting time”



solution of Euclidean eqns. of motion of finite action: a “monopole-instanton”

$$E_M = \frac{4\pi v}{g_4^2} \longrightarrow S_0 = \frac{4\pi v}{g_3^2}$$

$$e^{-S_0} \to 0 \qquad g_3^2/v \to 0$$

M-M* pairs give exponentially suppressed (at weak coupling) “semiclassical” contributions to the vacuum functional

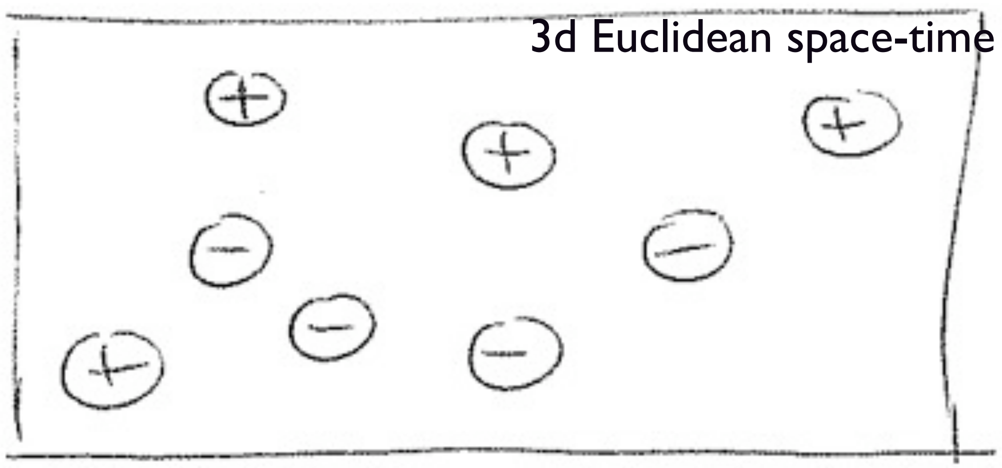
vacuum “is” a dilute monopole-antimonopole plasma

number of M’s per unit volume ~ $v^3 e^{-S_0}$

(analogous to B+L violation in electroweak model; at T=0 exponentially small)

vacuum is a dilute M-M* plasma -
but interacting, unlike instanton gas in 4d (in say, electroweak theory)

"picture" of vacuum



physics is that of Debye screening

by analogy:

electric fields are screened in a charged plasma ("Debye mass for photon"), so in the monopole-antimonopole plasma, the dual photon obtains mass from screening of magnetic field:

$$\mathcal{L}_{eff} = g_3^2 (\partial\sigma)^2 + (\#)\, v^3 e^{-S_0}\left(e^{i\sigma} + e^{-i\sigma}\right) + \ldots$$

"(anti-)monopole operators"

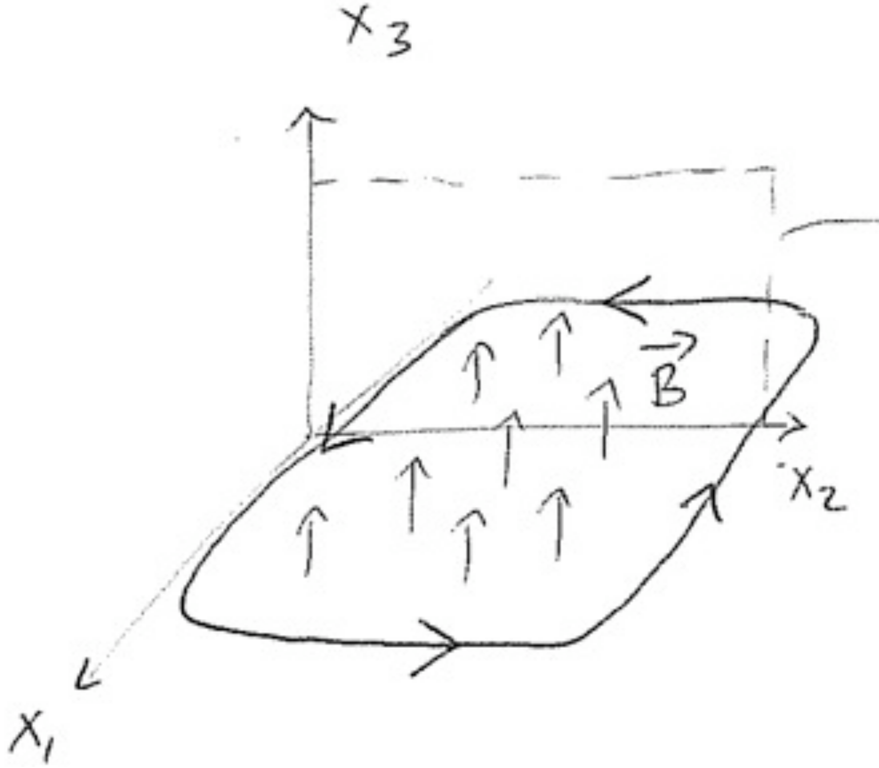
aka *"disorder operators"* because
not locally expressed in terms of gauge fields
(Kadanoff-Ceva; 't Hooft - 1970s)

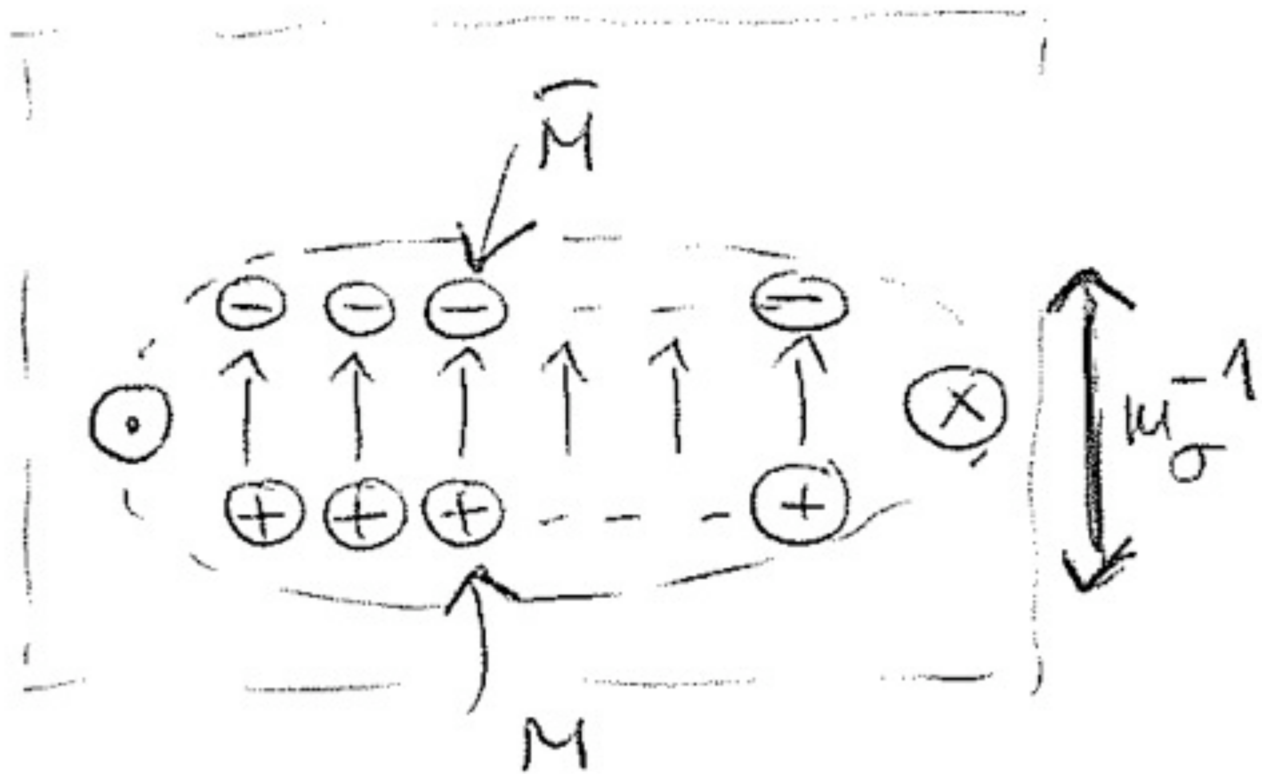
also by analogy with Debye mass:
dual photon mass^2 ~ M-M* plasma density

$$m_\sigma \sim v\, e^{-S_0/2} = v\, e^{-\frac{4\pi v}{2 g_3^2}}$$

next:
dual photon mass
~ confining string tension...

confining string tension:





screening of magnetic field in plasma
= Wilson loop area law:

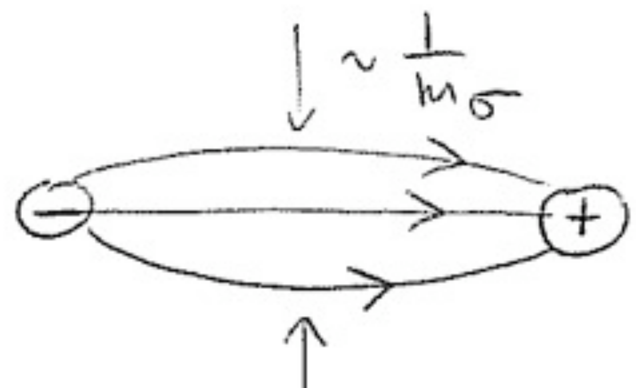
$$\langle e^{i\oint A\,dx}\rangle \sim e^{-(\mathrm{Area})\, m_\sigma g_3^2}$$

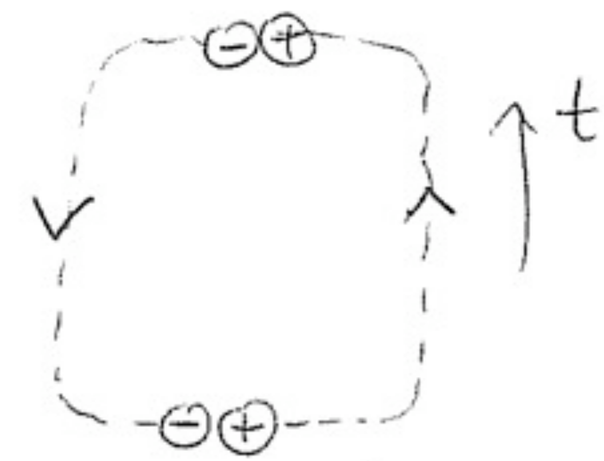
Minkowski space interpretation of Wilson loop: x_1 - time

$$0 \neq B^3 = \epsilon^{312} F^{12}$$

$$0 \neq \epsilon^{302} F^{02}$$

electric field





electric!

confining flux tube: **tension^{-1} ~ thickness ~ inverse dual photon mass**

3d Polyakov model & "monopole-instanton"-induced confinement

Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$

K. Lee, P.Yi, 1997

P. van Baal, 1998

we want to go to 4d - **by "growing" a compact dimension:**

$$S^1 : \quad x^4 \sim x^4 + L$$

"monopole-instantons" on $R^3 \times S^1$

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A_4 is now an adjoint 3d scalar Higgs field $\quad \partial_4 + A_4 \longrightarrow \frac{2\pi n}{L} + A_4$

but it is a bit unusual -
a compact Higgs field: $\quad \langle A_4 \rangle \sim \langle A_4 \rangle + \frac{2\pi}{L}$

such shifts of A_4 vev absorbed into shift of KK number "n"

thus, natural scale of "Higgs vev" is $\quad \langle A_4 \rangle \sim \frac{\pi}{L}$ leading to $\quad SU(2) \xrightarrow{\frac{1}{L}} U(1)$

(clearly, semiclassical and weakly coupled if L << 1/strong scale)

$\langle A_4 \rangle \sim \frac{\pi}{L}$ breaks SU(2) to U(1) so there are monopoles:

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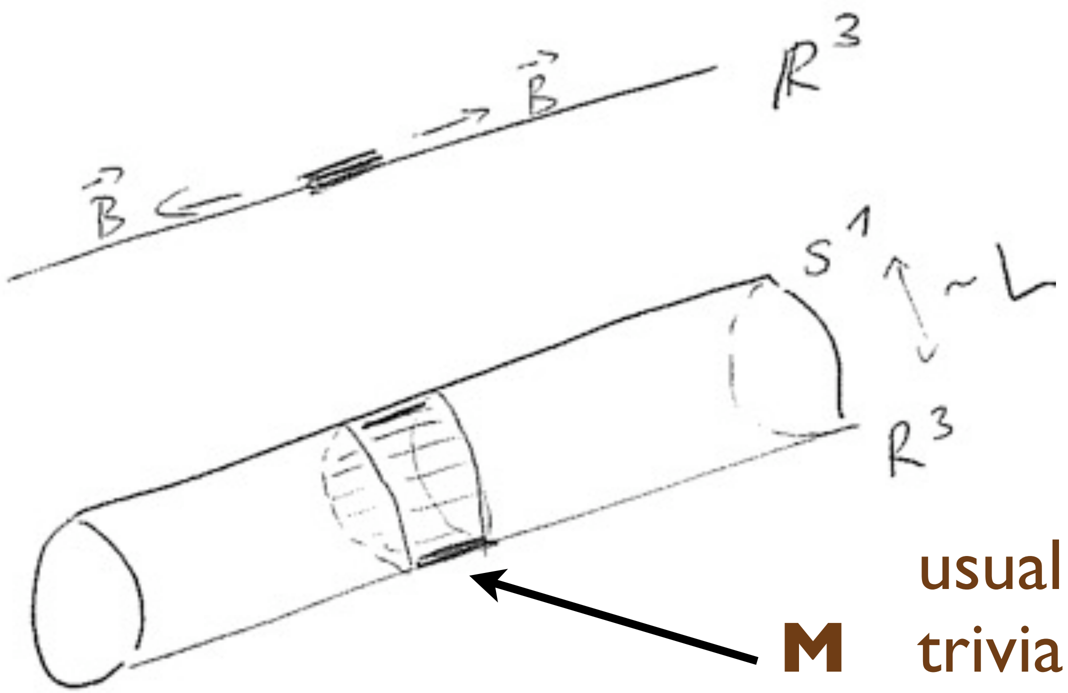

M

usual monopole
trivially
embedded in 4d

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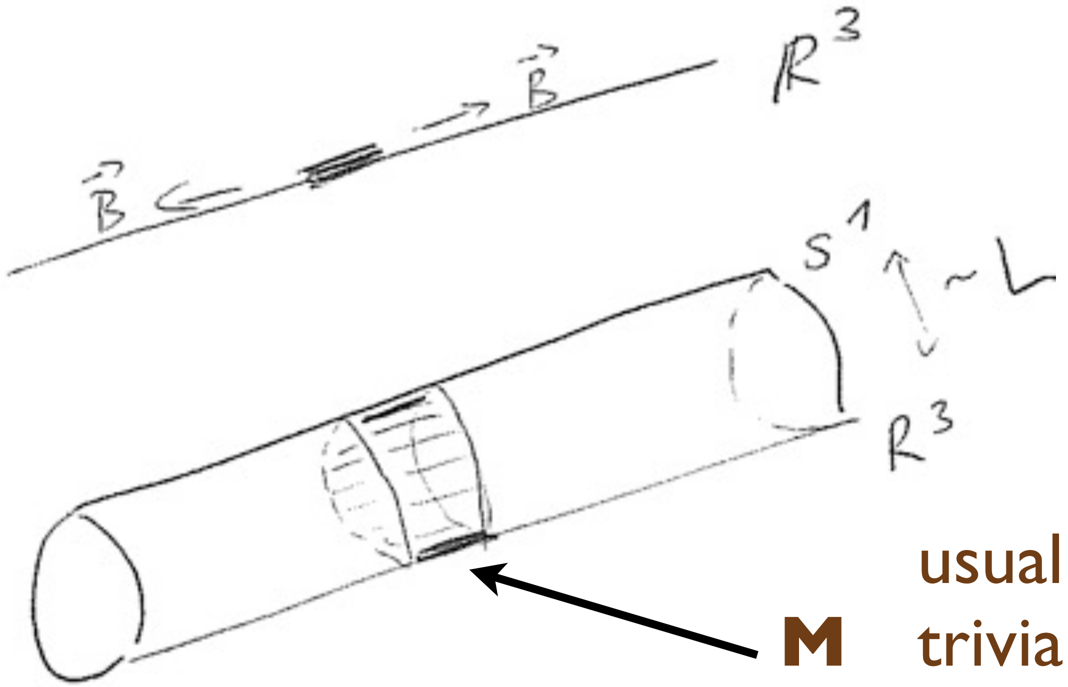
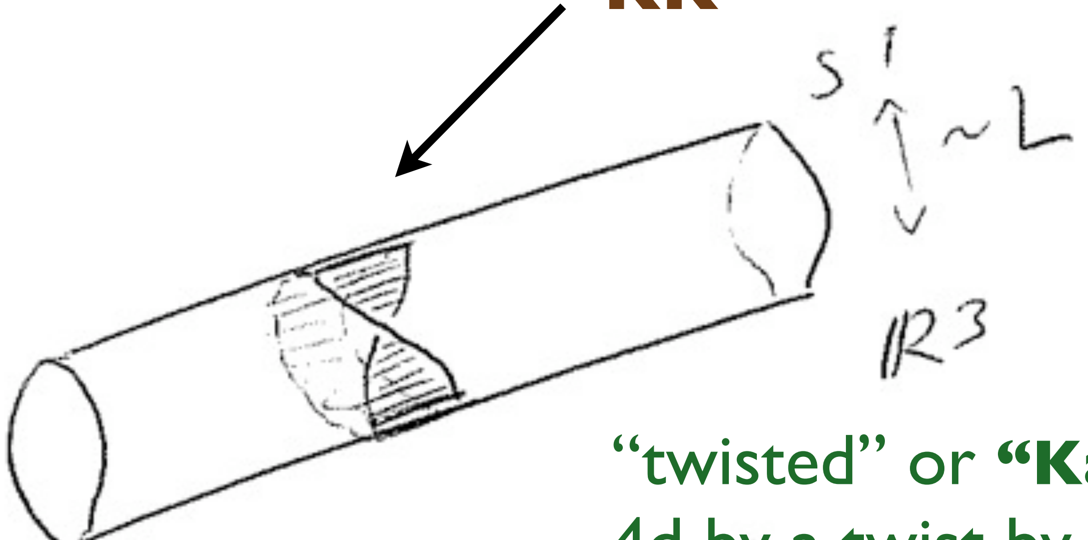


M usual monopole trivially embedded in 4d

KK



"twisted" or **"Kaluza-Klein":** monopole embedded in 4d by a twist by a "gauge transformation" periodic up to center - in 3d limit not there! (infinite action)

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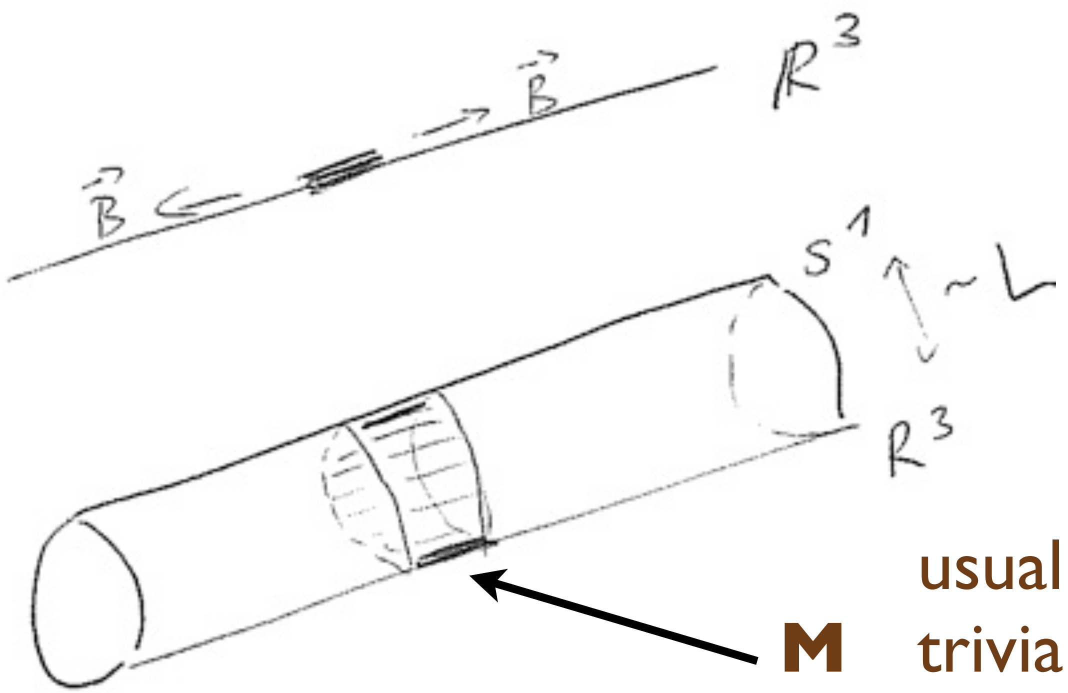
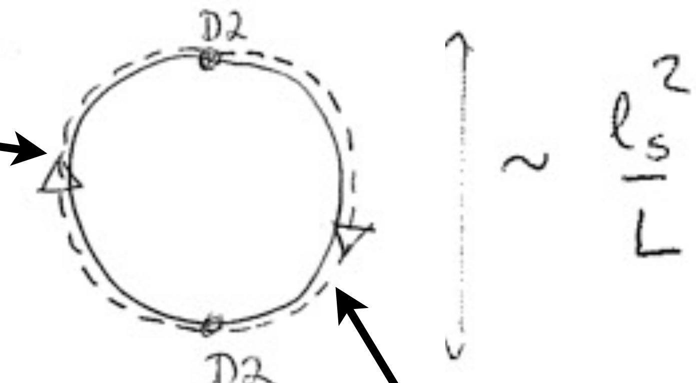


M usual monopole trivially embedded in 4d

KK discovered by K. Lee, P. Yi, 1997, as *"Instantons and monopoles on partially compactified D-branes"*

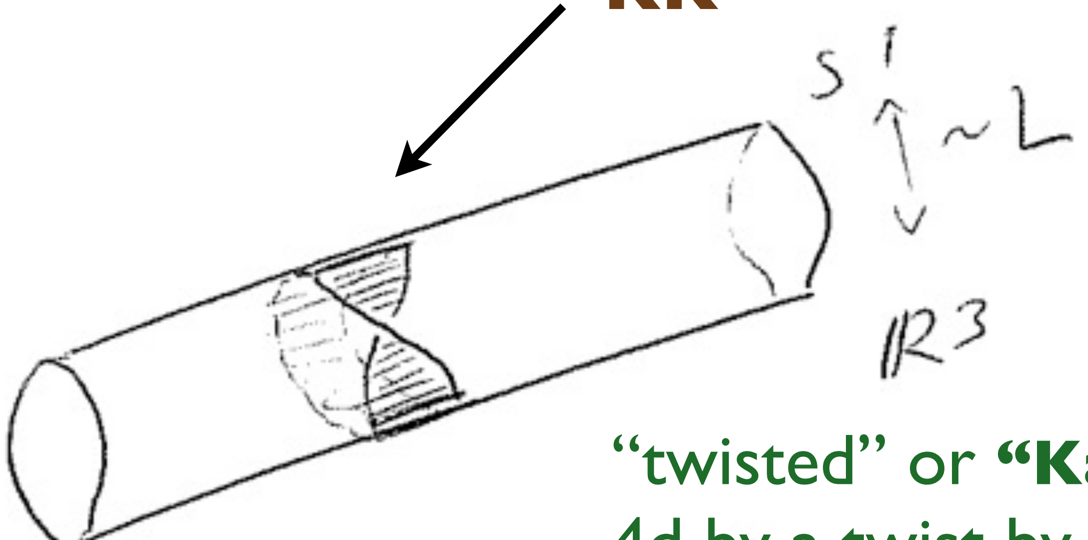
(but see earlier 1987 lattice work by Kronfeld, Schierholz&Wiese on "maximal abelian projection")

M Euclidean D0-brane



KK Euclidean D0-brane

KK



"twisted" or **"Kaluza-Klein":** monopole embedded in 4d by a twist by a "gauge transformation" periodic up to center - in 3d limit not there! (infinite action)

Summary: **“elementary” topological excitations on R3xS1**

M & KK both self-dual objects, of opposite magnetic charges

	magnetic charge	topological charge	semiclassical suppression
M	+1	1/2	e^{-S_0}
KK	-1	1/2	e^{-S_0}
BPST	0	1	e^{-2S_0}

+ their anti-”particles”

- thus, BPST instanton “ = M+KK ”

(also P. van Baal, 1998)

$$e^{-S_0} = e^{-\frac{4\pi v}{g_3^2}} = e^{-\frac{4\pi^2}{L g_3^2}} = e^{-\frac{4\pi^2}{g_4^2(L)}}$$ for SU(2)

$$SU(N): \quad e^{-S_0} = e^{-\frac{8\pi^2}{g_4^2(L)\,N}}$$

(large-N survive!)

M & KK have, in SU(N), 1/N-th of the ‘t Hooft suppression factor

3d Polyakov model & “monopole-instanton”-induced confinement

Polyakov, 1977

“monopole-instantons” on $R^3 \times S^1$

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the relevant index theorem

Nye, Singer, 2000
Unsal, EP, 2008

for more detail,
see M. Unsal, EP
0812.2085

like on R^3 Callias $\xleftarrow{\text{physicist derivation}}$ E. Weinberg, 1970s, but on $R^3 \times S^1$,
so must incorporate anomaly equation, some interesting effects

for this talk it is enough to consider 4d SU(2) theories with N_w adjoint Weyl fermions

"applications":

$N_w=1$ is N=1 SUSY YM

$N_w=4$

- "minimal walking technicolor"
- happens to be N=4 SYM without the scalars

M KK M* KK* each have $2N_w$ zero modes

disorder operators:

M:

$$e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_w}$$

KK:

$$e^{-S_0} e^{-i\sigma} (\lambda\lambda)^{N_w}$$

M*:

$$e^{-S_0} e^{-i\sigma} (\bar\lambda\bar\lambda)^{N_w}$$

KK*:

$$e^{-S_0} e^{i\sigma} (\bar\lambda\bar\lambda)^{N_w}$$

where:

$$(\lambda\lambda)^{N_w} = \det_{\underset{SU(N_w)}{I,J}} \overset{SU(2)}{\lambda^a_{\alpha I}} \lambda^a_{\underset{SL(2,\mathbb{C})}{\beta}J} \epsilon^{\alpha\beta}$$

remarks:

- operator due to M+KK = 't Hooft vertex; independent of dual photon
- "our" index theorem interpolates between 3d Callias and 4d APS index thms.

3d Polyakov model & "monopole-instanton"-induced confinement

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center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008

- Abelianization occurs only if there is a nontrivial holonomy (i.e., A_4 has vev)

- upon thermal circle compactifications, gauge theories with fermions do not Abelianize: center symmetry is broken at small circle size - transition to a deconfining phase - $A_4 = 0$, $\langle trW \rangle \neq 0$ - deconfinement - at high-T, 1-loop V_{eff} (Gross, Pisarski, Yaffe, early 1980s)

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008

without much unnecessary detail, the jargon I use is (for SU(2)) that if

$\langle A_4 \rangle \sim \frac{\pi}{L}$ then "center symmetry" is preserved to ensure calculability at small L and smooth connection to large L in the sense of center symmetry:

avoid phase transition in L and ensure that theory abelianizes at small L?

I. non-thermal compactifications - periodic fermions ("twisted partition function")

- with $N_W > 1$ adjoint fermions center symmetry preserved (Unsal, Yaffe 2007) as well as with other, "exotic" fermion reps (Unsal, EP 2009)
- in many supersymmetric theories, can simply choose center-symmetric vev

II. add double-trace deformations: force center symmetric vacuum at small L (Shifman, Unsal 2008; also Unsal, Yaffe 2007)

In what follows, I assume center-symmetric vacuum - due to either I. or II. - will explicitly discuss only theory where center symmetry is naturally preserved at small L (**I**.)

3d Polyakov model & "monopole-instanton"-induced confinement — Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$ — K. Lee, P. Yi, 1997; P. van Baal, 1998

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center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations — Shifman, Unsal, 2008; Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009 as an example, again consider 4d SU(2) theories with N_w adjoint Weyl fermions

classical global chiral symmetry is $SU(N_w) \times U(1)$

but 't Hooft vertex $(\lambda\lambda)^{2N_w} e^{-\frac{8\pi^2}{g_4^2}}$ only preserves $\mathbb{Z}_{4N_w}: \lambda \to e^{i\frac{2\pi}{4N_w}} \lambda$

so, quantum-mechanically we have only $SU(N_w) \times Z_{4N_w}$ exact chiral symmetry

now **M, KK**(+*) operators all look like: $e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_w}$

hence $(\lambda\lambda)^{N_w} \xrightarrow{\mathbb{Z}_{4N_w}} e^{i\pi} (\lambda\lambda)^{N_w}$

invariance of **M**, **KK**(+*) operators under exact chiral symmetry means that **dual photon must transform under the exact chiral symmetry** i.e., topological shift symmetry is intertwined with chiral symmetry:

$$\mathbb{Z}_{4N_w}: \sigma \to \sigma + \pi$$

$\sigma \to \sigma + \pi$ $\quad \cancel{\cos\sigma} \quad \cos(2\sigma)$ ✓

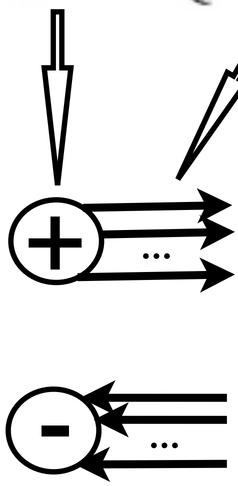
so the exact chiral symmetry allows a potential - **but what is it due to?**

$e^{-S_0}\, e^{i\sigma} (\lambda\lambda)^{N_w}$

to generate $\cos(2\sigma)$ must have

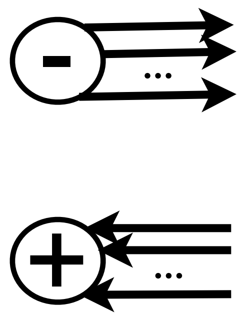
i. magnetic charge 2

ii. no zero modes



M:

M*:



KK:

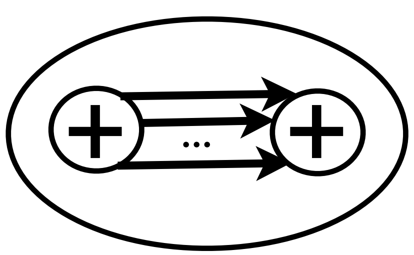
KK*:

M + KK* bound state? (Unsal, 2007)

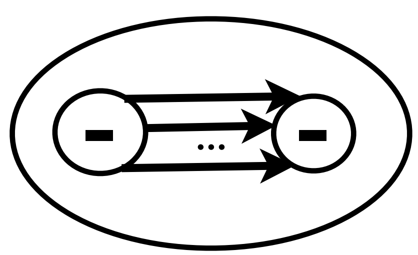
- same magnetic charge ~ 1/r-repulsion
- fermion exchange ~ log(r)-attraction

M + KK* = B - magnetic "bion" size of bound state $1/g_4(L)^2$ larger than UV cutoff length L

B:



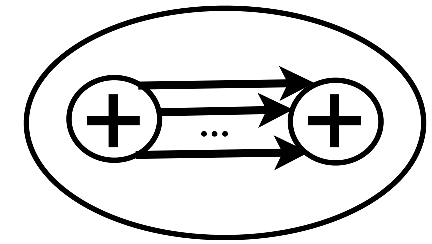
B*:



$$e^{-2S_0}\left(e^{2i\sigma} + e^{-2i\sigma}\right)$$

dual photon mass is induced by magnetic "bions"- the leading cause of confinement in SU(N) with adjoints at small L (incl. SYM) to summarize, in QCD(adj),

M + KK* = B - magnetic "bions" -



- carry magnetic charge
- no topological charge (non self-dual)

  (locally 4d nature crucial: no KK in 4d)
- generate "Debye" mass for dual photon

main tools used:

- intertwining of topological shift symmetry & chiral symmetry
- index theorem

thus, topological objects generating magnetic screening (and confinement) depend on massless fermion content - not usually thought that fermions relevant

using these tools, one can analyze any theory...

name codes:

U=Unsal
S=Shifman
Y=Yaffe
P=the speaker

vectorlike | chiral

Theory all SU(N)	Confinement mechanism on $\mathbb{R}^3 \times S^1$	Index for monopoles $[\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_N]$ Nye-M.Singer '00; PU '08	Index for instanton $I_{inst.} = \sum_{i=1}^{N} I_i$ Atiyah-Singer	$(\text{Mass Gap})^2$ units ~ $1/L^2$
YM Y,U '08	monopoles	$[0, \ldots, 0]$	0	e^{-S_0}
QCD(F) S,U '08	monopoles	$[2, 0, \ldots, 0]$	2	e^{-S_0}
SYM /QCD(Adj) U '07	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(BF) S,U '08	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(AS) S,U '08	bions and monopoles	$[2, 2, \ldots, 2, 0, 0]$	$2N - 4$	e^{-2S_0}, e^{-S_0}
QCD(S) P,U '09	bions and triplets	$[2, 2, \ldots, 2, 4, 4]$	$2N + 4$	e^{-2S_0}, e^{-3S_0}
SU(2)YM $I = \frac{3}{2}$ P,U '09	magnetic quintets	$[4, 6]$ SUSY version: ISS(henker) model of SUSY [non-]breaking	10	e^{-5S_0}
chiral $[\mathrm{SU}(N)]^K$ S,U '08	magnetic bions	$[2, 2, , \ldots, 2]$	$2N$	e^{-2S_0}
$AS + (N-4)\overline{\mathrm{F}}$ S,U '08	bions and a monopole	$[1, 1, , \ldots, 1, 0, 0]$ + $[0, 0, \ldots, 0, N-4, 0]$	$(N-2)\mathrm{AS} + (N-4)\overline{\mathrm{F}}$	e^{-2S_0}, e^{-S_0},
$S + (N+4)\overline{\mathrm{F}}$ P,U '09	bions and triplets	$[1, 1, , \ldots, 1, 2, 2]$ + $[0, 0, \ldots, 0, N+4, 0]$	$(N+2)\mathrm{S} + (N+4)\overline{\mathrm{F}}$	e^{-2S_0}, e^{-3S_0},

Table 1. Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbf{R}^3 \times \mathbf{S}^1$. Unless indicated otherwise,

+ SO(N),SP(N) - S. Golkar 0909.2838; for mixed-representation/higher-index reps. SU(N) - P,U 0910.1245

So, I have now introduced all the key players:

3d Polyakov model & "monopole-instanton"-induced confinement	Polyakov, 1977
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center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations	Shifman, Unsal, 2008 Unsal, Yaffe, 2008
"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement	Unsal, 2007 Unsal, EP, 2009

The upshot:

the dual lagrangian of QCD(adj) on a circle of size L

$$\frac{g^2(L)}{2L}(\partial\sigma)^2 - \frac{b}{L^3}\, e^{-2S_0} \cos 2\sigma + i\bar{\lambda}^I \gamma_\mu \partial_\mu \lambda_I + \frac{c}{L^{3-2N_f}}\, e^{-S_0} \cos\sigma (\det_{I,J} \lambda^I \lambda^J + \text{c.c.})$$

B, B*

M, KK+*

leading-order perturbation theory (A_4 is massive, except in SUSY when must be added)

perturbative corrections ~ $g_4(L)^2$ omitted

$$m_\sigma \sim \frac{1}{L} e^{-S_0} = \frac{1}{L} e^{-\frac{8\pi^2}{N_c g_4^2(L)}}$$

$$(\Lambda L)^{\beta_0} = e^{-\frac{8\pi^2}{g_4^2(L)}}$$

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_W N_c$$

$$m_\sigma = \frac{1}{L} (\Lambda L)^{\frac{\beta_0}{N_c}} = \Lambda (\Lambda L)^{\frac{\beta_0}{N_c} - 1} = \Lambda (\Lambda L)^{\frac{8-2N_W}{3}}$$

mass gap ~ string tension behaves in an interesting way

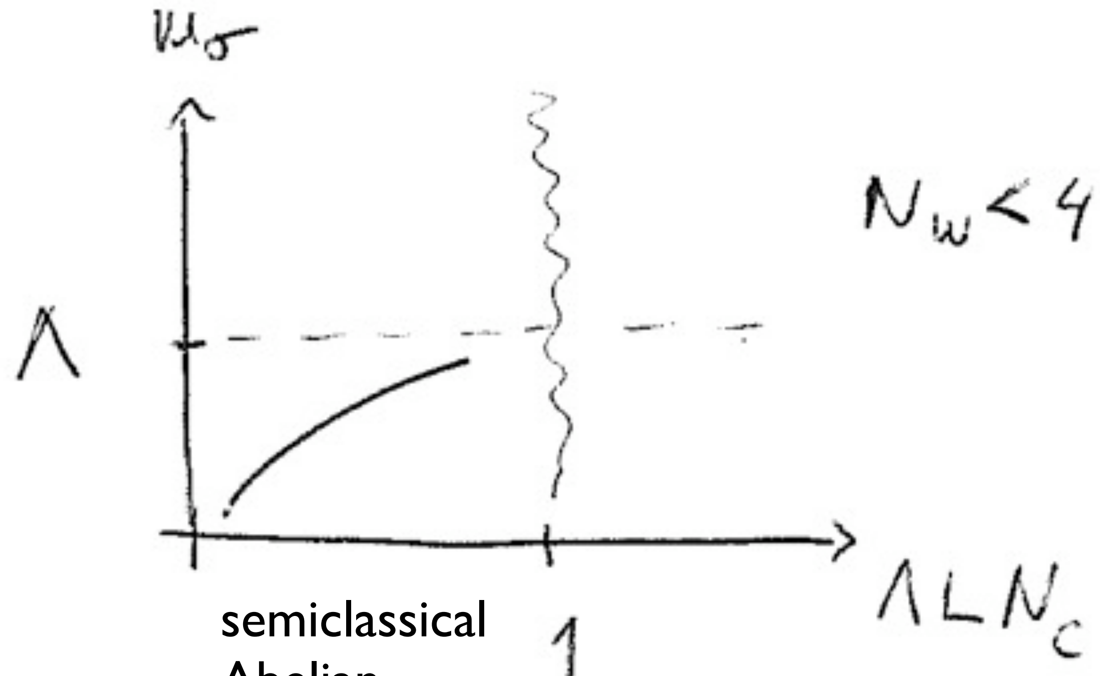
as L changes at fixed Λ ... $N_W^* = 4$?

recall, however, region of validity of semiclassical analysis:

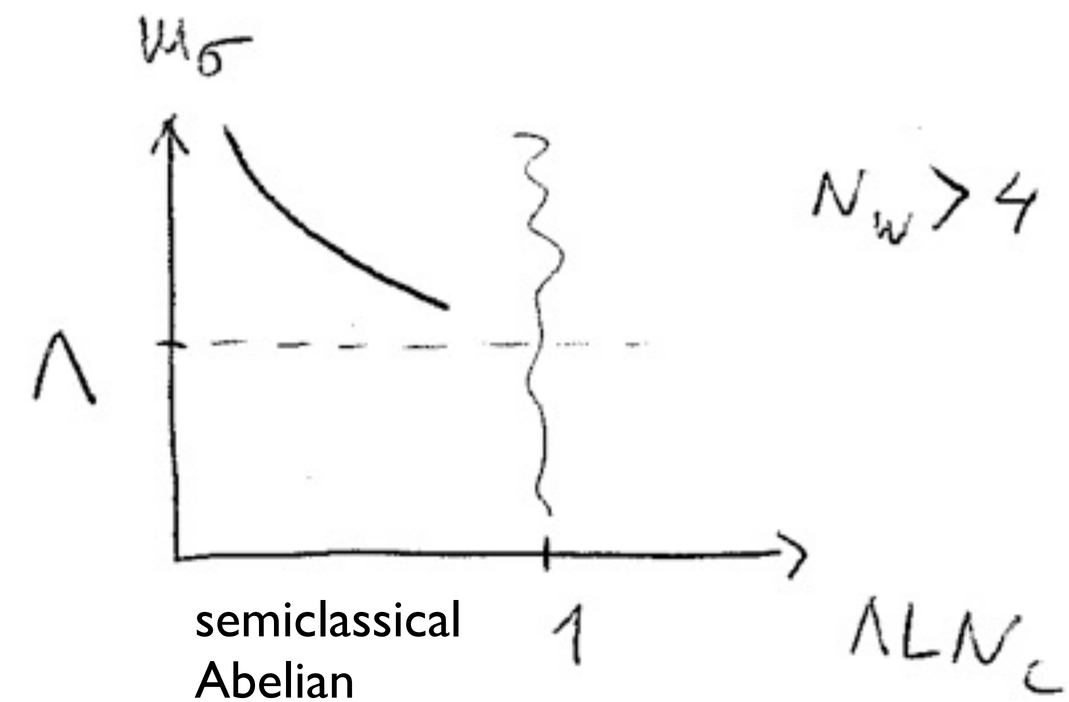
$$\Lambda L \ll 1 \qquad (N_c \Lambda L \ll 1 \text{, really})$$

as mass of W ~ 1/(NL)

$$m_\sigma \sim \Lambda (\Lambda L)^{(8-2N_w)/3}$$



semiclassical Abelian confinement



semiclassical Abelian confinement

analysis shows that this switch of behavior as number of fermion species is increased occurs in all theories - vectorlike or chiral alike

in each case we obtain a value for the critical number of "flavors" or "generations"... N_f^*

like $N_w^* = 4$ for QCD(adj)

does it tell us anything about R^4?

I know I am in danger of being arrested...

... how **dare** you study non-protected quantities?

even worse, in the end, I'll obtain "predictions" by pretending that "all functions are monotonic"... some circumstantial evidence:

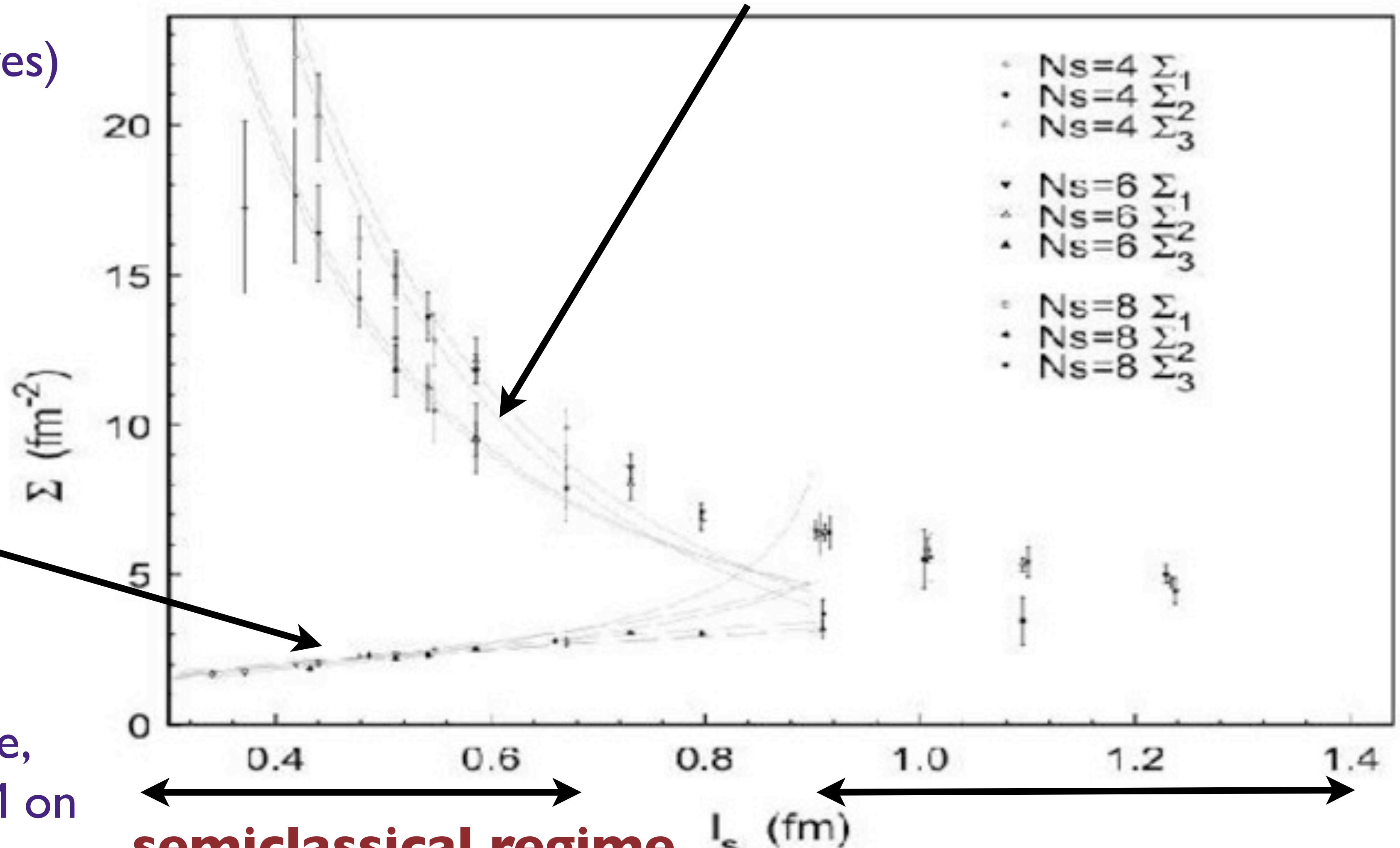
M. Perez, A. Gonzalez-Arroyo '93

pure YM - no fermions - on (small) T^3, twisted b.c. (center-symmetric!)

W-mass^2 ~ $1/L^2$

semiclassical calculation (curves)
vs
lattice Monte Carlo (points)

string tension
~$(\Lambda L)^{5/6}$



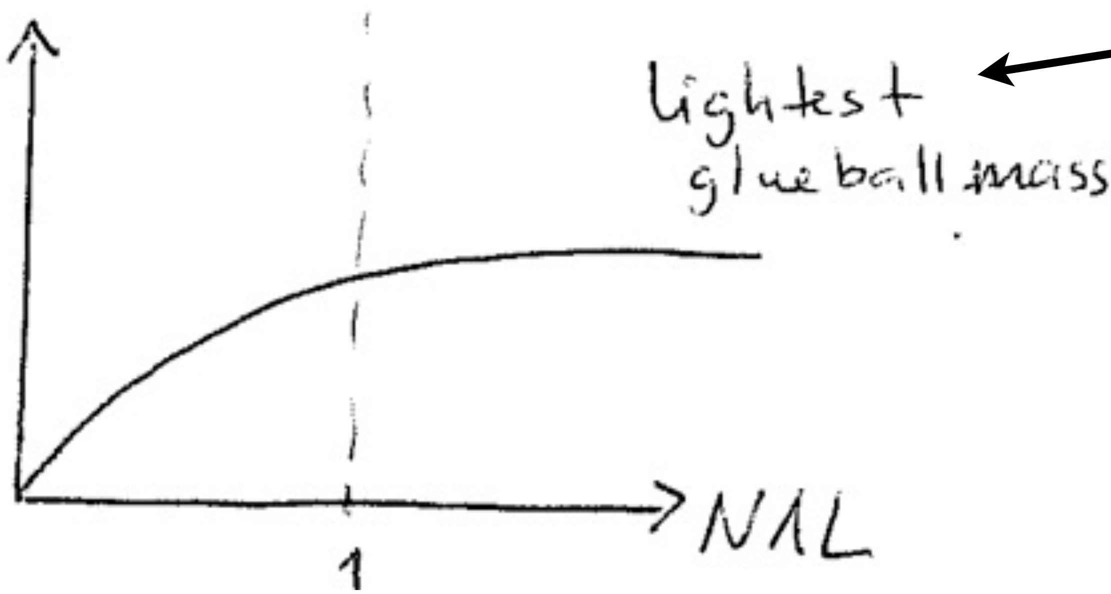
same L-scaling as our (U.+Yaffe, actually) prediction for pure YM on a circle - since also due to fractional instantons, but now on T^3!

semiclassical regime (calculable)

L << 1/strong scale

strong-coupling regime

now, a reasonable expectation of what happens at very small or very large number of "flavors" is this:



topological excitations become non-dilute with increase of L, cause confinement, M, KK+* operators

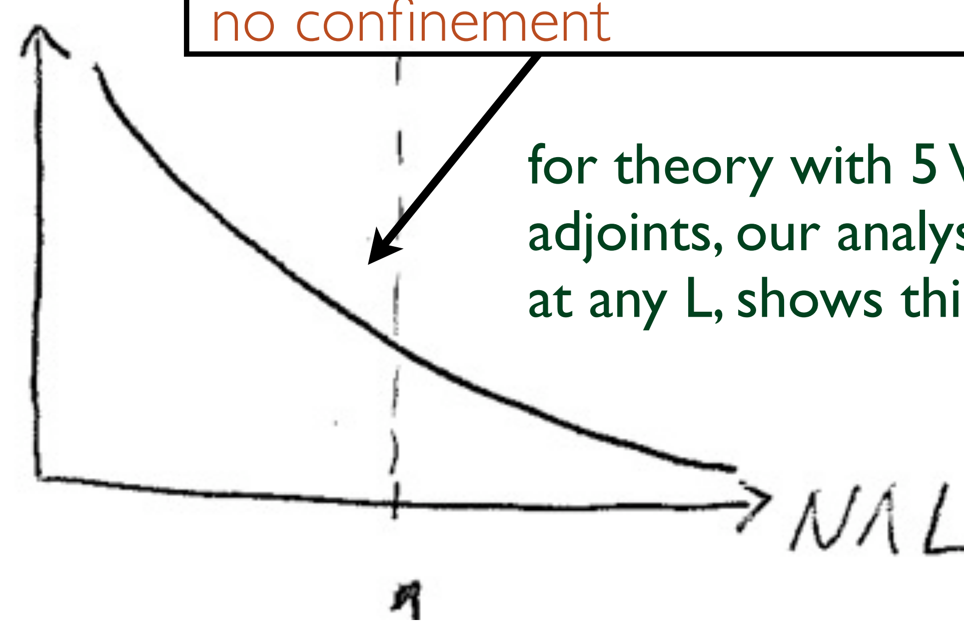
$$e^{-S_0} \cos\sigma(\det_{I,J} \lambda^I \lambda^J + \text{c.c.})$$

become strong, can cause chiral symmetry breaking (whenever the confining theories break their nonabelian chiral symmetries)

sufficiently small # fermion species
confining theories

sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

topological excitations that cause confinement dilute with increase of L, no confinement



for theory with 5 Weyl adjoints, our analysis is valid at any L, shows this behavior now, a reasonable expectation of what happens at very small or very large number of "flavors" is this:

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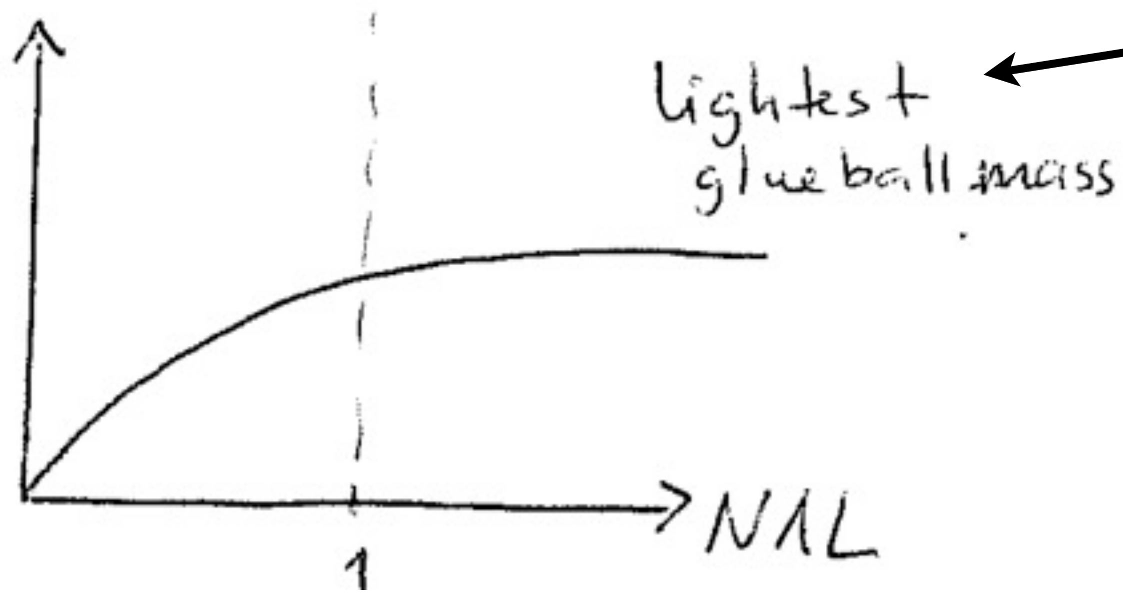

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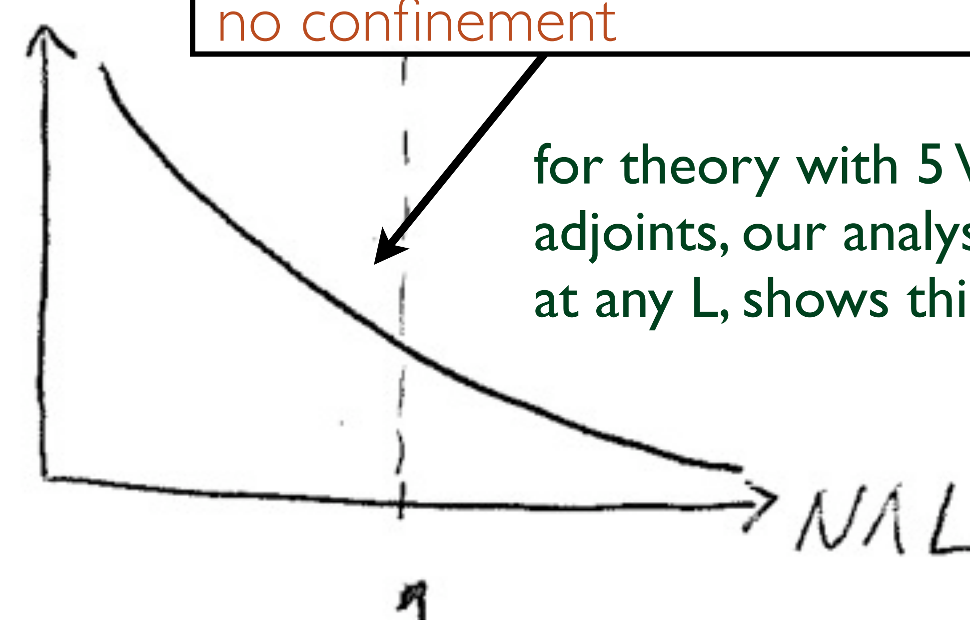


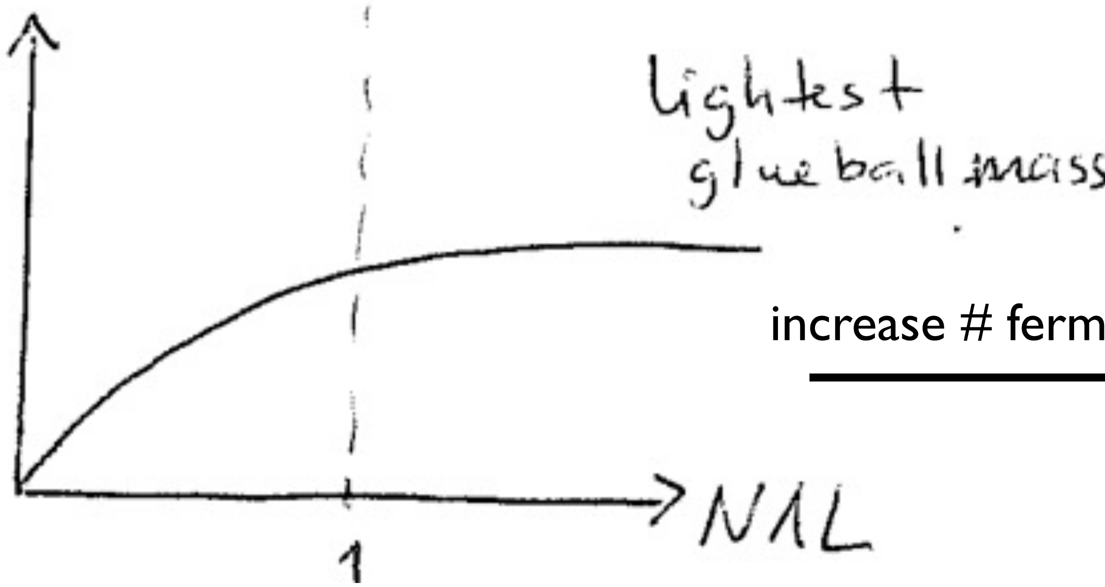
for theory with 5 Weyl adjoints, our analysis is valid at any L, shows this behavior

but where does the transition **really** occur?
is it at our value N_f*?

there appear to be three possibilities
(in any given class of theories, only one is realized)

A.) our N_f* is the true critical value N_{crit} [theory that may be in this class: QCD(adj), experiment (lattice)]

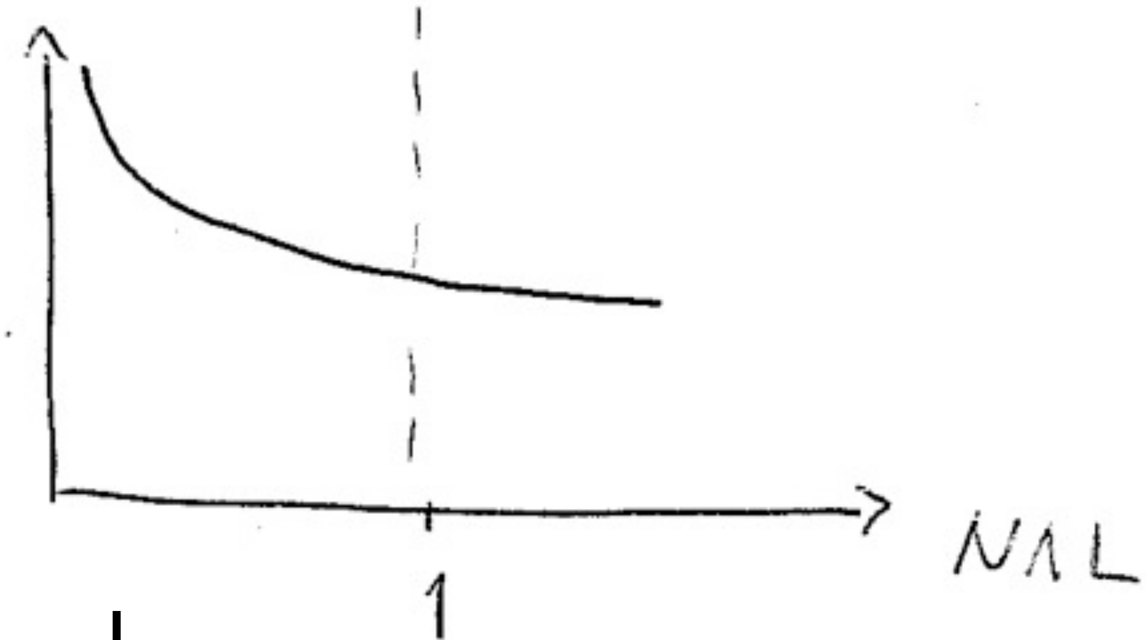
B.) if, as # species is increased above N_f^*



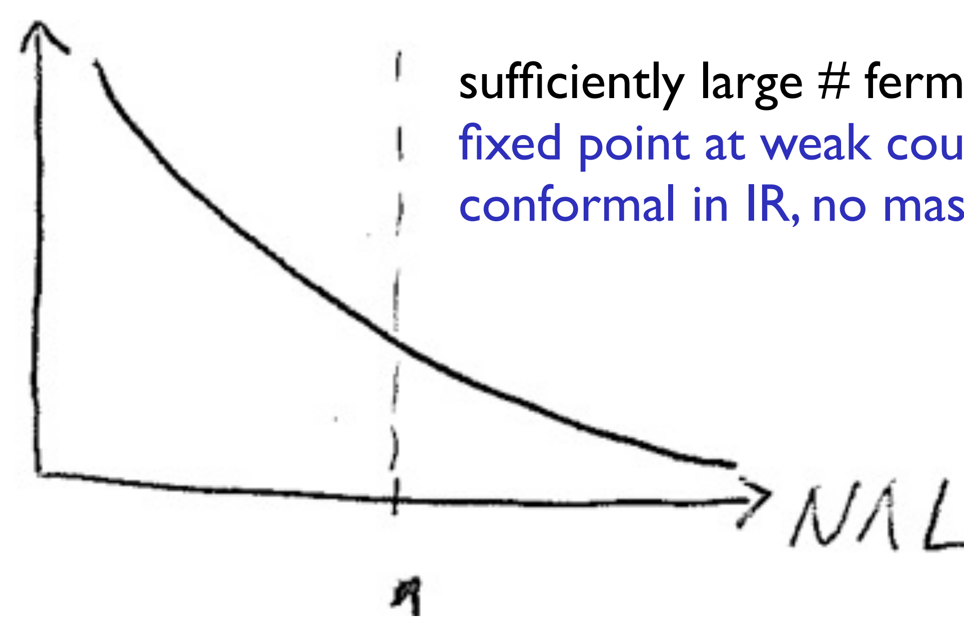
sufficiently small # fermion species

confining theories

increase # fermions →



↓ increase # fermions



sufficiently large # fermion species

fixed point at weak coupling

conformal in IR, no mass gap

then, $N_{crit} > N_f^*$

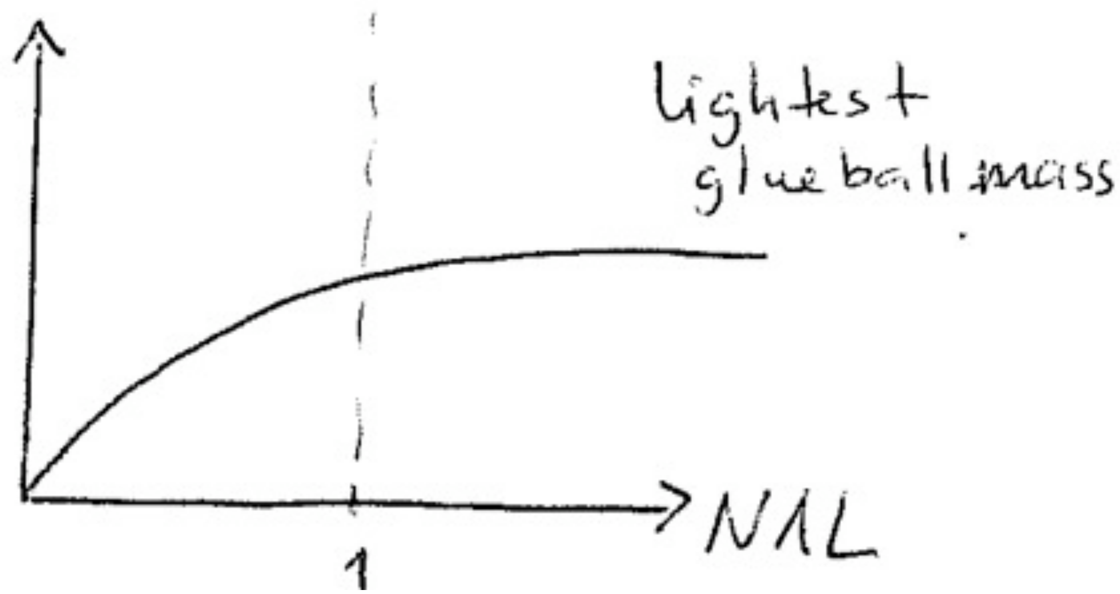
↑

true value of critical # "flavors"

thus, for such theories N_f^* is a lower bound thereof

[theory believed to be in this class: QCD(F) - arguments using mixed reps., experiment (lattice)]

c.) if, as # species has not yet reached N_f^*



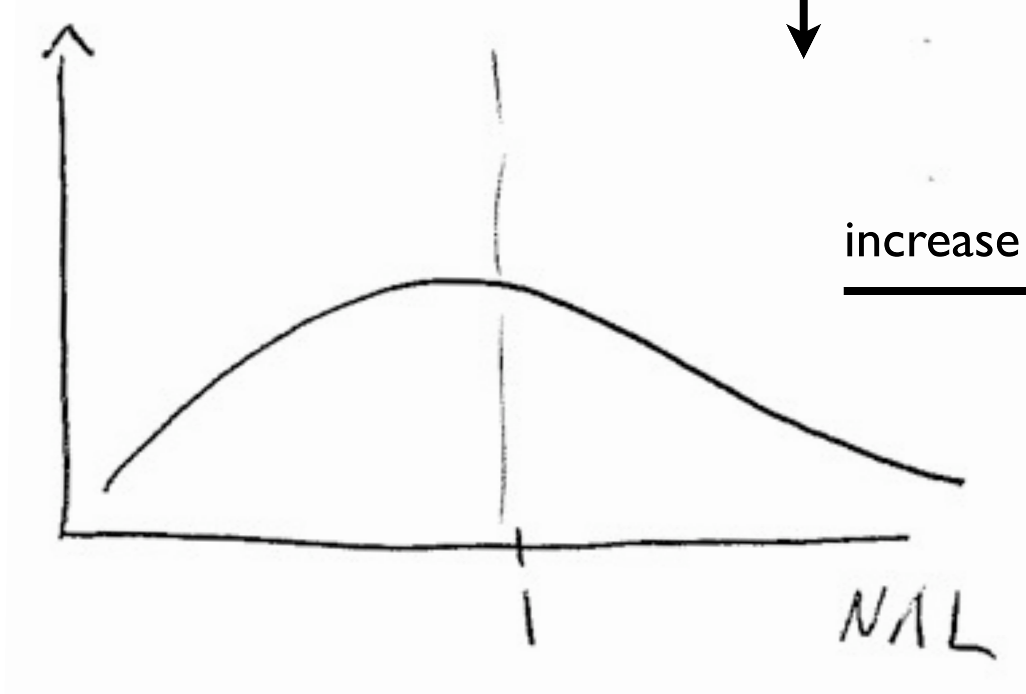
sufficiently small # fermion species
confining theories

then,

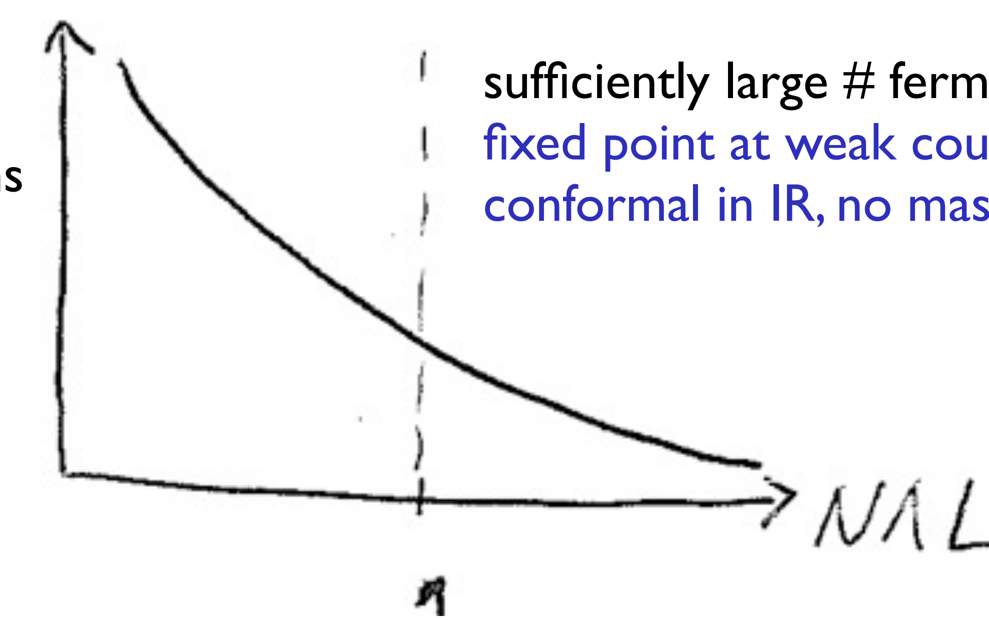
$N_{crit} < N_f^*$

thus, for this class of theories N_f^* is an upper bound on critical # “flavors”

increase # fermions



increase # fermions



sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

[only one theory we know is believed to be in this class: SU(2) 4-index symmetric tensor Weyl, theory arguments]

comparing theory estimates of critical number of fermions for SU(N)

"experiment"

Weyl adjoints [no deformation needed]

	our estimate	gap eqn	beta function gamma=2/1	AF lost
any N	4	4.15	2.75/3.66	5.5

4 ? e.g.: *Catterall et al; del Debbio, Patella,Pica; Hietanen et al.*

Dirac 2-index (anti)symmetric tensor [deformation needed]

N	our estimate	gap eqn	beta function gamma=2/1	AF lost
3	2.40	2.50	1.65/2.2	3.30
4	2.66	2.78	1.83/2.44	3.66
5	2.85	2.97	1.96/2.62	3.92
10	3.33	3.47	2.29/3.05	4.58
∞	4	4.15	2.75/3.66	5.5

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but large-N orbifold/orientifold equivalence to adjoint!

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Dirac fundamentals [deformation needed]

N	our estimate (a/c)	gap eqn	functional RG	beta function gamma=2/1	AF lost
2	5/8	7.85	8.25	5.5/7.33	11
3	7.5/12	11.91	10	8.25/11	16.5
4	10/16	15.93	13.5	11/14.66	22
5	12.5/20	19.95	16.25	13.75/18.33	27.5
10	25/40	39.97	n/a	27.5/36.66	55
∞	$2.5N/4N$	$4N$	$\sim(2.75-3.25)N$	$2.75N/3.66N$	$5.5N$

12 **?** e.g.: *Appelquist,Fleming, Neal; Deuzemann, Lombardo,Pallante; Iwasaki et al; Fodor et al; Jin, Mahwinney; A. Hasenfratz*

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- gap equation and lattice - only vectorlike theories
- in chiral gauge theories with multiple "generations" our estimates were the only known ones until Sannino's recent 0911.0931 via the proposed exact beta function

this - largely (given the absence of credible error bars) - agreement is, to us, somewhat amusing/satisfying...

compare the tools used:

gap equation

Appelquist et al; Miransky et al; Ryttov, Sannino (FRG: Gies, Jaeckel)

conformality tied to absence of chiral symmetry breaking

compares fixed-point coupling to critical gauge coupling for chiral symmetry breaking

- *ladder diagram "approximation" of truncated Schwinger-Dyson eqns. for fermion propagator in Landau gauge*

beta function

Ryttov, Sannino
Dietrich, Sannino

postulate exact beta function; loss of conformality tied to anomalous dimension of fermion bilinear at IR fixed point violating unitarity bound (or close to it)

our "estimate"

conformality tied to absence of mass gap/string tension

see also Armoni, 2009 (worldine approach; very similar numbers)

semiclassical analysis on a non-thermal circle
dilution vs. non-dilution of topological excitations with L

lattice

in principle, a first-principle determination
but for (a, V, m, $)

Conclusions 1:

Compactifying 4d gauge theories on a small circle is a "deformation" where nonperturbative dynamics is under control - dynamics as "friendly" as in SUSY, e.g. Seiberg-Witten.

Confinement is due to various "oddball" topological excitations, in most theories non-self-dual.

Polyakov's "Debye screening" mechanism works on $R^3 \times S^1$ also with massless fermions, contrary to what many thought
- KK monopoles and index theorem-crucial ingredients of analysis.

Precise nature - monopoles, bions, triplets, or quintets - depends on the light fermion content of the theory.

U,P; 0812.2085, 0906.5156

Conclusions II:

Didn't have time for these:

Found chiral symmetry breaking (Abelian) due to expectation values of topological "disorder" operators: occurs in mixed-rep. theories with anomaly-free chiral U(1), broken at any radius

U,P; 0910.1245

Circle compactification gives another calculable deformation of SUSY theories - well known, yet not fully explored -

in I=3/2 SU(2) Intriligator-Seiberg-Shenker model we argued that theory conformal, rather than SUSY-breaking.

U,P; 0905.0634
agreement with
different arguments of
Shifman, Vainshtein `98
Intriligator `05

Conclusions III:

Gave “estimates” of conformal window boundary in vectorlike and chiral gauge theories (OK with “experiment” when available).

Conformality tied to dilution vs non-dilution of topological excitations with L as a function of # of fermion species.

U,P; 0906.5156

Clearly, on R^3x S^1 we only see the shadow of the “real” thing...

...wait for lattice people or
perhaps go back to SUSY?

- theorists’ “safe haven”:

Conclusions IV:

We argued that "bions" are responsible for confinement in N=1 SYM at small L - a particular case of our Weyl adjoint theory. This remains true if N=1 obtained from N=2 by soft breaking.

On the other hand, massless monopole or dyon condensation is responsible for confinement in N=2 softly broken to N=1 at large L (Seiberg, Witten `94).

Thus, in different regimes we have different descriptions of confinement in N=1 SYM.

Do they connect in an interesting way?

e.g., recent works Gaiotto, Moore, Neitzke (GMN `08-09); Chen, Dorey, Petunin `10 on "wall-crossing" at finite L ...

a "picture" appears to emerge ... details to be worked out:

Conclusions V:

N=2 softly broken to N=1 on a size-L circle : $m \ll \Lambda$

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N=2 softly broken to N=1 on a size-L circle : $m << \Lambda$

massless monopole/dyon condensation
gives mass to $U(1)_{magnetic}$ "photon"

"QCD string": Nielsen-Olesen vortex (electric flux tube) of dual Higgs model describing condensation

4d magnetically charged massless particles condense;
relevant singularity of (hyper)Kaehler metric persists for large-L (GMN `08-09)

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$1/\Lambda$ ——— $1/m$ ——— L

description entirely
in terms of dual photon
+superpartners; nothing else
becomes massless

"QCD string": dual photon
configurations with
nontrivial monodromy
"cut and paste" domain wall

3(+1)d magnetically charged
instantons (KK+M*=B)
generate dual photon mass

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Small-L "bion" mechanism applies to non-SUSY theories as well, as shown.
Large-L massless monopole/dyon condensation only in softly broken N=2.

Appear (smoothly?) **connected when we understand both large and small L.**