Superconformal Flavor Simplified

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Flavor Hierarchies in SUSY

- Typical approach...
  - Hierarchies in superpotential:

$$W = y_u^{ij} Q_i U_j H_u + y_d^{ij} Q_i D_j H_d + y_l^{ij} L_i E_j H_d$$

$$y_a^{11} << y_a^{22} << y_a^{33}$$

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  - Horizontal Symmetries
  - Compositeness
  - RGE running? What about SUSY non-renormalization theorems?

Flavor Hierarchies in SUSY

Alternatively..

- Hierarchies in Kähler potential:

$$\mathcal{L} = \int d^4\theta \sum_i Z_i \Phi_i^\dagger \Phi_i \qquad y^{ij}_{phys} = \frac{1}{\sqrt{Z_i Z_j}} y^{ij}$$

$$Z_1 \gg Z_2 \gg Z_3$$

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- Allows for anarchical superpotential couplings $y^{ij} \sim O(1)$
- Flavor may have a *dynamical* origin!

Flavor Hierarchies in SUSY

- Taking $\epsilon_i \equiv Z_i^{-1/2}$, this structure gives:

$$\begin{array}{rcl} (m_t,\ m_c,\ m_u) & \approx & \langle H_u \rangle \left(\epsilon_{Q_3}\epsilon_{U_3}\epsilon_{H_u},\ \epsilon_{Q_2}\epsilon_{U_2}\epsilon_{H_u},\ \epsilon_{Q_1}\epsilon_{U_1}\epsilon_{H_u} \right) \\ (m_b,\ m_s,\ m_d) & \approx & \langle H_d \rangle \left(\epsilon_{Q_3}\epsilon_{D_3}\epsilon_{H_d},\ \epsilon_{Q_2}\epsilon_{D_2}\epsilon_{H_d},\ \epsilon_{Q_1}\epsilon_{D_1}\epsilon_{H_d} \right) \\ (m_\tau,\ m_\mu,\ m_e) & \approx & \langle H_d \rangle \left(\epsilon_{L_3}\epsilon_{E_3}\epsilon_{H_d},\ \epsilon_{L_2}\epsilon_{E_2}\epsilon_{H_d},\ \epsilon_{L_1}\epsilon_{E_1}\epsilon_{H_d} \right) \end{array}$$

$$|V_{\mathrm{CKM}}| \approx \begin{pmatrix} 1 & \epsilon_{Q_1}/\epsilon_{Q_2} & \epsilon_{Q_1}/\epsilon_{Q_3} \\ \epsilon_{Q_1}/\epsilon_{Q_2} & 1 & \epsilon_{Q_2}/\epsilon_{Q_3} \\ \epsilon_{Q_1}/\epsilon_{Q_3} & \epsilon_{Q_2}/\epsilon_{Q_3} & 1 \end{pmatrix}$$

Flavor Hierarchies in SUSY

- Works pretty well for mixing angles!

$$|V_{\rm CKM}| \approx \begin{pmatrix} 1 & \epsilon_{Q_1}/\epsilon_{Q_2} & \epsilon_{Q_1}/\epsilon_{Q_3} \\ \epsilon_{Q_1}/\epsilon_{Q_2} & 1 & \epsilon_{Q_2}/\epsilon_{Q_3} \\ \epsilon_{Q_1}/\epsilon_{Q_3} & \epsilon_{Q_2}/\epsilon_{Q_3} & 1 \end{pmatrix}$$

$$|V_{\rm CKM}|_{expt} \simeq \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.009 & 0.04 & 0.99 \end{pmatrix}$$

Flavor Hierarchies in SUSY

- What if we impose SU(5) GUT relations?

$$\epsilon_{Q_i} = \epsilon_{U_i} = \epsilon_{E_i} \equiv \epsilon_{T_i} \quad \text{and} \quad \epsilon_{D_i} = \epsilon_{L_i} \equiv \epsilon_{\overline{F}_i}$$

Flavor Hierarchies in SUSY

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Up-quarks: $\epsilon_{T_i}\sqrt{\epsilon_H} \approx (.001 - .002, .03 - .04, .7 - .9)$

Down-quarks: $\epsilon_{\overline{F}_i}\epsilon_{\overline{H}} \approx \tan\beta \times (.002 - .01, .002 - .01, .008 - .02)$

Leptons: $\epsilon_{\overline{F}_i}\epsilon_{\overline{H}} \approx \tan\beta \times (.001 - .002, .01 - .02, .01 - .03)$

[Extracted from Antusch&Spinrath '08]

Flavor Hierarchies in SUSY

- Simplest structure: '10-centered' model
  - Get within a factor of ~3 from:

$$\epsilon_{T_1} \simeq .003 \qquad \text{and} \qquad \epsilon_{T_2} \simeq .04$$

  - Prefers large $\tan\beta$

Flavor Hierarchies in SUSY

- Simplest structure: '10-centered' model
  - Get within a factor of ~3 from:

$$\epsilon_{T_1} \simeq .003 \qquad \text{and} \qquad \epsilon_{T_2} \simeq .04$$

  - Prefers large $\tan\beta$
- At smaller $\tan\beta$, could also generate suppressions in $\epsilon_{\overline{F}_i}$ or $\epsilon_{\overline{H}}$
- How do we do this with a model?

Nelson-Strassler Models

- SCFT dynamics generates hierarchy!
  - E.g., give $T_{1,2}$ large anomalous dimensions through couplings:

$$W_{int} = T_1 \mathcal{O}_1 + T_2 \mathcal{O}_2 + W_{CFT}$$

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  - E.g., give $T_{1,2}$ large anomalous dimensions through couplings:

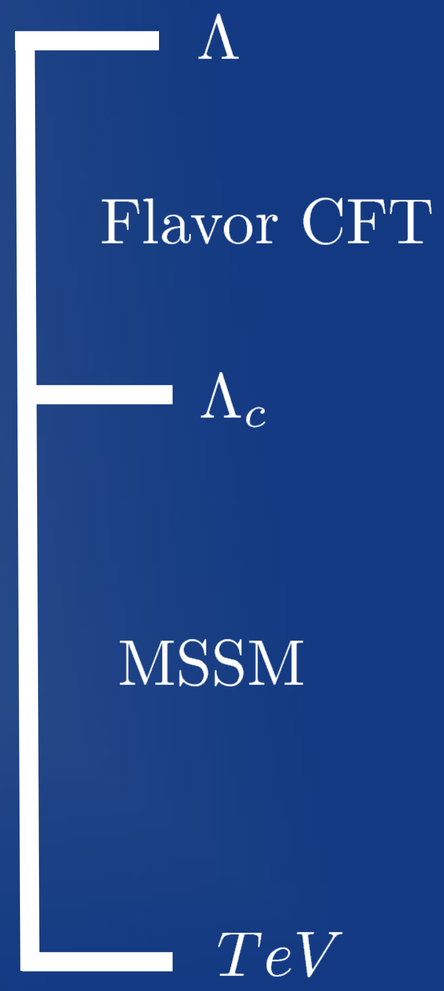
$$W_{int} = T_1 \mathcal{O}_1 + T_2 \mathcal{O}_2 + W_{CFT}$$

- These interactions generate:

$$\epsilon_{T_i}(\mu) = Z_{T_i}^{-1/2}(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\dim(T_i)-1}$$

Nelson-Strassler Models

- Relevant deformations cause exit from CFT regime
- At what scale Λ_c ?



Nelson-Strassler Models

- Relevant deformations cause exit from CFT regime
- At what scale Λ_c?
  - Often W_{int} violates Baryon & Lepton #
  - Landau pole for MSSM gauge couplings
  - Suggests $\Lambda_c \sim M_{GUT}$

(but could be lower in some models)

Nelson-Strassler Models

- In order to evaluate a model, we'd like to calculate the anomalous dimensions
- This is equivalent to finding the 'correct' superconformal $U(1)_R$ symmetry

(since $\dim(\mathcal{O}) = (3/2)R_{\mathcal{O}}$)

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(since $\dim(\mathcal{O}) = (3/2)R_{\mathcal{O}}$)

- In 2000, this could only be uniquely determined if there were a sufficient number of interactions...
- Original models also chiral, so making sure exotic states decouple required even more interactions

Nelson-Strassler Models

	$SU(5)_{\mathrm{GUT}}$	$Sp(12)$	$\mathbf{Z}_2$	dimension
$T_{1,2,3}$	10	1	1	$2, \frac{4}{3}, 1$
$\overline{F}_{1,2,3}, \overline{H}$	$\overline{5}$	1	1	$\frac{5}{3}, 1, 1, 1$
H	5	1	1	1
$\overline{T}$	$\overline{10}$	12	1	$\frac{2}{3}$
A	1	65	1	$\frac{2}{3}$
F	5	12	1	1
Z, U, V	1	12	$1, -1, -1$	$\frac{1}{3}, \frac{7}{6}, \frac{7}{6}$

$$\begin{aligned} W \quad &= \quad T_1\overline{T}Z + T_2\overline{T}ZA + \overline{F}_1 FZ + \overline{T}^3 F + \overline{T}FFZ + AUV \\ &\quad + Z^2UV + Z^2U^2 + Z^2V^2 + W_{exit} \end{aligned}$$

Nelson-Strassler Models

	$SU(5)_{\text{GUT}}$	$Sp(8)$	$Sp(8)'$	dimension
$T_{1,2,3}$	10	1	1	$?,?,1$
$\overline{F}_{1,2,3}, \overline{H}$	$\overline{5}$	1	1	1
H	5	1	1	1
Q	$\overline{10}$	8	1	$?$
L, M	1	8	1	$?,?$
$J_1, J_2, J_3, J_4, J_5, J_6$	1	8	1	$?,?,?,?,\frac{3}{4},\frac{3}{4}$
$\overline{Q}'$	10	1	8	(confined)
$\overline{J}'_1, J'_2$	1	1	8	(confined)

$$\begin{aligned} W &= T_1 Q L + T_2 Q M + (J_1 J_2)^2 + (J_3 J_4)^2 + (J_5 J_6)^2 \\ &\quad + (L J_1)(J_1 J_3) + W_{exit} \end{aligned}$$

a-maximization

- Thankfully, this problem was solved in 2003 by Intriligator and Wecht!

- The correct R-symmetry maximizes:

$$a(R_t) = 3\mathrm{Tr}(R_t^3) - \mathrm{Tr}(R_t)$$

over all possible “trial” R-charges:

$$R_t = R_0 + \sum_I s_I F_I$$

a-maximization

- Why is it true?
  - Maximizing a is equivalent to:

(1) $$\frac{\partial a}{\partial s_I} = 9\mathrm{Tr}(RRF_I) - \mathrm{Tr}(F_I) = 0$$

(2) $$\frac{\partial^2 a}{\partial s_I \partial s_J} = 18\mathrm{Tr}(RF_I F_J) \quad \text{is negative-definite}$$

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$$\langle \partial J_I \, J_R \, J_R \rangle \sim \langle \partial J_I \, T \, T \rangle$$ by SUSY

(2) $$\frac{\partial^2 a}{\partial s_I \partial s_J} = 18\mathrm{Tr}(RF_I F_J) \quad \text{is negative-definite}$$

unitarity

$$\langle \partial J_R J_I J_J \rangle \sim \langle T J_I J_J \rangle \sim \langle J_I J_J \rangle$$

a-maximization

- This is *extremely* easy to implement:
  - Just maximizing polynomials!
- One important caveat, though:
  - Need to know *all* of the IR flavor symmetries...
  - Accidental symmetries may arise!
    - E.g., gauge invariant operator appears to violate unitarity bound, $R \geq 2/3$

a-maximization

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What is the *simplest* viable model?

Models

- We will focus on:
  - '10-centric' SU(5) models
  - *Vector-like* models
    - Greatly simplifies CFT exit!

Models

- We will focus on:
  - '10-centric' SU(5) models
  - *Vector-like* models
    - Greatly simplifies CFT exit!
- Primary constraints:
  - Proton decay (take $\Lambda_c \sim M_{GUT}$)
  - SU(5) Landau pole should not occur in conformal window!

SU(5) Landau Pole?

- Once we know the correct R-symmetry, can integrate β_{g_5}:

$$\beta_{g_5} = \frac{-3\,\mathrm{Tr}\left[\mathrm{U}(1)_R\,\mathrm{SU}(5)^2_{\mathrm{GUT}}\right]}{16\pi^2\left(1-\frac{5g_5^2}{8\pi^2}\right)} g_5^3$$

- We'll (conservatively) assume the matter content of a minimal SU(5) GUT
- Absence of Landau pole in CFT window is a *very* strong constraint on models

Toy Model

- Let's start with a simple toy model:

	$\mathrm{SU}(5)_{\mathrm{GUT}}$	$\mathrm{SU}(N)$
$X + S$	$\mathbf{10} + \mathbf{1}$	$\square$
$\overline{X} + \overline{S}$	$\overline{\mathbf{10}} + \mathbf{1}$	$\overline{\square}$

$$4 \leq N \leq 7$$

$$W_{int} = T_1 \overline{X} S$$

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$$4 \le N \le 7$$

$$W_{int} = T_1 \overline{X} S$$

- 2 constraints on 5 unknowns:

$$0 = T(G) + \sum_i (R_i - 1) T(r_i)$$

$$2 = R_{T_1} + R_{\overline{X}} + R_S$$

Toy Model

- All we have to do is maximize

$$a(R_X, R_{\overline{X}}, R_S, R_{\overline{S}}, R_{T_1}) =$$
$$2(N^2-1) + \sum_i \dim(r_i)\left(3(R_i-1)^3 - (R_i-1)\right)$$

subject to these 2 constraints.

- Easy to do, e.g., with Mathematica

Toy Model

- This gives:

N	R_{T_1}	R_X	$R_{\overline{X}}$	R_S	$R_{\overline{S}}$
4	.686	.632	.637	.677	.632
5	.771	.683	.546	.533	.533
6	.920	.625	.455	.439	.439
7	1.191	.445	.364	.356	.356

- Larger N leads to a more strongly coupled theory, with larger R_{T_1}
  - Requires a smaller conformal window:

$$\epsilon_{T_1} = \left(\frac{\Lambda_c}{\Lambda}\right)^{\frac{3}{2}R_{T_1}-1}$$

$\mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$ Model

- Simple extension to 2nd generation:

	$\mathrm{SU}(5)_{\mathrm{GUT}}$	$\mathrm{SU}(N)$
$X + \overline{Q} + S$	$\mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$	$\square$
$\overline{X} + Q + \overline{S}$	$\overline{\mathbf{10}} + \mathbf{5} + \mathbf{1}$	$\overline{\square}$

$6 \leq N \leq 10$

$$W_{int} = T_1 \overline{X} S + T_2 X Q$$

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$6 \leq N \leq 10$

$$W_{int} = T_1 \overline{X} S + T_2 X Q$$

- Note that we simply *define* whatever linear combinations appear above to be T_1 and T_2
- Straightforward to check that these interactions violate B&L, so need $\Lambda_c \sim M_{GUT}$

$\mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$ Model

- Maximizing a(R) gives:

N	R_{T_1}	R_{T_2}	$\Lambda_{SU(5)}/\Lambda_c$	Λ/Λ_c
6	.740	.706	$10^{2.48}$	$10^{22.91\pm4.33}$
7	.862	.782	$10^{1.80}$	$10^{8.60\pm1.63}$
8	.992	.885	$10^{1.37}$	$10^{4.96\pm0.77}$
9	1.123	1.021	$10^{1.08}$	$10^{3.26\pm0.27}$
10	1.251	1.196	$10^{0.87}$	$10^{2.35\pm0.01}$

- This has trouble with Landau pole constraints for all N!

Our Quest

- Can *any* simple models avoid this problem?
- We need a sector that is as efficient as possible!
  - Minimize SU(5) representations while staying strongly coupled
- We find many models with right group theory structure, but very few that can avoid this bound...

$\mathrm{Sp}(2N)$ Models

	$\mathrm{SU}(5)_{\mathrm{GUT}}$	$\mathrm{Sp}(2N)$
$Q+\overline{Q}$	$\mathbf{5}+\overline{\mathbf{5}}$	$\square$
A	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$

$N \geq 4$

$$W_{int} = T_1\overline{QQ} + T_2\overline{Q}A\overline{Q}$$

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$N \geq 4$

$$W_{int} = T_1\overline{QQ} + T_2\overline{Q}A\overline{Q}$$

- Only a single $\overline{\mathbf{5}}$ needed, because both the $\mathrm{SU}(5)$ and $\mathrm{Sp}(2N)$ contractions are anti-symmetric!
- Again can check that interactions violate B&L, so we need $\Lambda_c \sim M_{GUT}$

$\mathrm{Sp}(2N)$ Models

- Maximizing a(R) gives:

N	R_{T_1}	R_{T_2}	$\Lambda_{\mathrm{SU}(5)}/\Lambda_c$	Λ/Λ_c
4	1.045	.778	$10^{7.09}$	—
5	1.103	.872	$10^{5.02}$	$10^{3.85\pm0.73}$
6	1.154	.950	$10^{3.83}$	$10^{3.45\pm0.65}$
7	1.197	1.014	$10^{3.07}$	$10^{3.09\pm0.51}$
8	1.234	1.067	$10^{2.54}$	$10^{2.76\pm0.34}$
9	1.263	1.111	$10^{2.16}$	$10^{2.55\pm0.26}$
10	1.288	1.147	$10^{1.88}$	$10^{2.40\pm0.20}$

- Evades bound for N = 5,6,7,8
- Maybe some tension fitting between M_{GUT} and M_{Pl}

$\mathrm{Sp}(2N)$ Models

- $$W = T_1 \overline{QQ} + T_2 \overline{Q} A \overline{Q} + \mathrm{Tr}[A^3]$$

N	R_{T_1}	R_{T_2}	$\Lambda_{\mathrm{SU}(5)}/\Lambda_c$	Λ/Λ_c
4	1.497	.830	$10^{9.66}$	—
5	1.786	1.119	$10^{8.18}$	$10^{1.57\pm0.22}$

- $$W = T_1 \overline{QQ} + T_2 \overline{Q} A \overline{Q} + \mathrm{Tr}[A^4]$$

N	R_{T_1}	R_{T_2}	$\Lambda_{\mathrm{SU}(5)}/\Lambda_c$	Λ/Λ_c
4	1.331	.831	$10^{6.92}$	—
5	1.531	1.031	$10^{5.88}$	$10^{2.00\pm0.32}$
6	1.787	1.287	$10^{4.72}$	$10^{1.50\pm0.28}$
7	2.000	1.500	$10^{4.64}$	$10^{1.26\pm0.23}$
8	2.200	1.700	$10^{4.24}$	$10^{1.05\pm0.16}$

$\mathrm{Sp}(2N)$ Models

- These models are simple and seem to fit nicely between M_{GUT} and M_{Pl}
- CFT exit occurs when the mass terms $\overline{Q}Q$ and $\mathrm{Tr}[A^2]$ become important
- It is also straightforward to introduce suppressions for $\overline{F}_i, \overline{H}$ by adding additional SM singlets
  - Allows going to smaller $\tan\beta$

Outlook

- We still need a complete picture of GUT physics...
  - Doublet-triplet splitting, proton decay, etc...
  - Use flavor sector for GUT breaking?
  - Study other GUT groups
- SUSY breaking
  - Soft parameters also suppressed...
  - Viable gravity mediation! [NS '01; Kobayashi, Terao '01]
  - Need to know about non-chiral operators...
    - Bound their dimensions? [In progress]

Outlook

- 'Large N' Flavor CFTs have a dual AdS picture

  - "Bulk masses" outputs rather than inputs
  - However, large N is where Landau Pole constraint is strongest...

Summary

- Flavor hierarchies can be generated *dynamically* by CFT dynamics
  - SUSY models are all now 'calculable' with a-maximization!
- But most such models run into Landau poles for visible gauge couplings...
  - For vector-like simple group theories, almost uniquely picks out a model!
- Need a more complete picture, but perhaps flavor can guide us