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DL/MDA workshop

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Lloyd Currie, Allen Brodsky, Bill Potter, and Jay MacLellan will provide their perspectives and address questions in their presentation on Decision Limits and Minimum Detectable Amount. The presentations will be followed with open discussion by and with the audience

Detection Levels and Minimum Detectable Amounts - Some Background*

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Abstract AbstractAbstractAbstract

A review of the statistical and practical considerations in formulating ways of specifying decision levels and detection capabilities for radioanalytical analyses is very timely. In June, a working group was re-constituted by the Health Physics Society to review the need for revisions of the national standard, ANSI N13.30, "Performance Criteria for Radiobioassay (Health Physics Society 1996). Since I participated from the beginning, about twenty years ago, in the deliberations resulting in the standard, I will review some of the rationale leading to specification of ways of formulating decision levels (DL) and minimum detectable amounts (MDA) in the standard. Then, I will review some of the literature and experience gained since the standard was published. More recent experience suggests a need for some possible revisions to specifications in the standard, or at least some further clarification of the specific purposes and usage of DL and MDA specifications in the context of the standard's objectives.

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* For presentation in the MDA Workshop at the 47th Annual Conference on Bioassay, Analytical and Environmental Radiochemistry, Honolulu, Hawaii, November 3-8, 2001. The views expressed in this paper are those of the authors alone.

INTRODUCTION INTRODUCTIONINTRODUCTIONINTRODUCTION

The principle of open hostility among friends

Before reviewing the history and rationale of DL/MDA formulations, it is urgent that I first review the early-established traditions of open and honest informal discussion at these BAER conferences, which I will call, "**The principle of open hostility among friends**." The first BAER meeting in 1954, which was then called the "bioassay meeting," was formed from a relatively small group of about fifteen analytical chemists, most of whom worked in or headed the analytical chemistry groups of the major Atomic Energy Commission plants or laboratories. I was first sent by Dr. G. Victor Beard to one of these meetings in 1957 as a young AEC health physicist, to encourage the group to provide a report, not as a standard, but as a guide to procedures that were deemed adequate as state-of-the art for determining urinary excretion of tritium, strontium-90, uranium, radium, and plutonium. A compendium, AECU-3066 was published in 1958 (John B. Hursh, Professor, University of Rochester, editor).

To me, these chemists were scientific and professional giants, and they treated me kindly as if I were a junior member of their family. However, I was impressed with how they could argue vociferously with each other over chemical procedures and still enjoy such camaraderie and close friendships at the same time. Some of the leading discussants of this early era were Claude Sill of the Idaho AEC laboratory, Jacob (Jake) Sedlet of Argonne, John Harley of the AEC Health and Safety Laboratory, and Moe Milligan of Los Alamos. These chemists were also interested in the dosimetric applications of their work.

Over time, as I began to make presentations, I sometimes came under attack also. The most vocal attack was when Claude Sill chastised me at a bioassay meeting for leaving out the constant 2.71 in the Currie (1968) formulation of detection limit, pointing out it was much needed in low background counting to avoid stating inappropriately an MDA close to zero. (Claude was very astute regarding the statistical aspects of his analyses, as well as one of the most interesting presenters of his analytical procedures I have ever heard.) This comment forced me back to the "drawing board", and one night, with Roscoe Hall checking me, I re-derived Currie's equation to understand the origin of 2.71. We decided it was an artifact of mixed assumptions, and might as well be set to 3, which at least would in the limit of zero background give the selected exp(-3) = 0.05 probability of a Type II error (Brodsky 1986). Currie agreed to this. Claude's attack (and he is still my good friend), was a good example of the value of the informal atmosphere of the bioassay conferences, and the principle of "open hostility among friends." It forced me back to further study, and I learned much more about my problem. I could give many other examples in the context of our development of the ANSI N13.30 standard, both in committee and in front of the bioassay meeting attendees.

We need to continue this free-speaking tradition. Unfortunately, Dan Strom, Jay MacLellan, Guthrie Miller and I arrived at a temporary cease-fire just before this meeting, and we laid down our arms. However, I am confident that before this meeting is ended, we will find darts to throw again. Please, all you newcomers, feel free to join in the fray and stimulate deep thought and self examination. Skepticism is an important ingredient in our scientific method. Ken Inn and Dave McCurdy, as young as they are, have been here long enough to know what I mean.

Past rationale for formulations of DL and MDA in ANSI N13.30

Of course, I knew that I would not remember today all of the statistical considerations of the working group that developed ANSI N13.30, so I wrote them down for some of the appendices of the standard to help later groups review considerations that might still be applicable. The working group insisted on two revisions to reduce the number of appendix pages that ultimately were included in the standard. However, my boss Bob Alexander suggested that I publish the entire review, which included literature reviews and rationale on accuracy requirements of bioassay analyses for different health physics, legal, and epidemiologic purposes as well as the statistical rationale for the DL, MDA, bias and precision provisions. I obeyed my boss, and after considerable review by peers, published the discussion as a NUREG (Brodsky 1986), with a Preface giving credit to the many individuals who helped develop the statistical concepts and rationale. I really had not intended to put out this report under my own name, since I knew that this would make me the target of many dart throwers.
I refer any newcomers to this report, since it is impossible in this brief abstract to do justice to the 32 pages of the report describing the literature review and rationale for some of the DL and MDA provisions of the standard. Six opening paragraphs presenting the advantages of using standard DL and MDA definitions may be summarized as dealing with: 1) preventing false claims of low detection capability not achieved with a high degree of assurance; 2) avoiding understatements with respect to other laboratories that might result in loss of information or business; 3) requirements for *a priori* adequate determination of precisions and biases to avoid misstatements of confidence and improper recording of results; 4) assurance of prior determination of all calibration and other experimental factors necessary to define DL and MDA so that records will stand up to professional or court scrutiny; 5) allowing selection of appropriate "acceptable minimum detectable amounts" (AMDA) (which were removed from the draft standard before final promulgation); 6) the formation of an industry-wide consensus on these issues to avoid chaos in planning and advertising capabilities, and in the selection of appropriate service laboratories or procedures for specific analyses.

The NUREG report, and much of the appendix material in the standard, also indicated the assumptions and limitations of the DL and MDA formulations under low-background radiometric conditions, and included formulations for different

sample and background counting times. Also, recommendations of Currie (1984) were also summarized and adapted for use in including non-Poisson errors and biases in formulations of DL and MDA. Yet, while the use of other formulations was provided for under special circumstances, the use of the simple formulations for paired sample and blank adapted from Currie (1968), with his limit of detection converted to activity units, was retained in the standard for general use:

$$DL = 2.33\, s_b\ ; \qquad MDA = (4.65\, s_b + 3)/KT,$$

where s_b is the standard deviation (or standard error if multiple blanks measurements are used in a given interpretation) of the blank total count (including non-Poisson random errors), K is a calibration factor to convert, e.g., counts/minute to activity units, and T is the counting time (assumed the same for the blank and the sample).

The NUREG report covers most of the rationale for statistical aspects of ANSI N13.30 only up to the publication of the draft version of the standard (Health Physics Society 1987). Rationales for revisions of the standard are included in appendices to the final standard (Health Physics Society 1996). From the first draft of the standard, an experimental testing program for the efficacy of the standard was carried out under contracts from the NRC and DOE (Robinson et al. 1984; Robinson et al. 1986; Reece et al. 1986). These round robins determined whether the major laboratories could meet the MDA provisions and the accuracy provisions of the draft standard. As a result of these tests and other considerations, the requirement for meeting an AMDA for specific nuclides was removed from the standard. My main original assignment from Bob Alexander was to try to ensure that at least a part of the standard provided the specifications needed to establish a laboratory accreditation program. Ken Heid, the first Chairman, initially objected to this requirement but later agreed to the consensus. Although the NRC later dropped its interest in an accreditation program, the DOE program now under Stan Morton is providing experience with the final standard that can provide guidance on necessary revisions. Ken Heid resigned the chair after the draft standard was published, Roscoe Hall was chair during early revisions, and Matt Lardy steered the standard through many revisions to publication in 1996. All of these chairpersons were indefatigable and excellent chairpersons. Matt has now taken the helm again. He's not one who practices open hostility, but watch out; he's tough as nails.

SUGGESTIONS FOR REVISITING THE DL AND MDA FORMULATIONS

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The many suggestions for revising the DL and MDA formulations in ANSI N13.30

cannot be presented in any detail in this abstract. Reference to papers presented on this subject (MacLellan 2000; Miller et al. 2000; Potter 1999, 2001; Strom and MacLellan 1999, 2001; Tries 1997) can provide a platform for further discussion and understanding of the issue. I am optimistic that a new consensus on the statistical aspects of the standard can be reached within the ANSI working group, with the assistance of presentations and discussions at these BAER meetings.

However, some cautionary notes should be inserted at this point. We need to be clear that we are all talking at the same time about the same parameters and assumptions, and need to fairly interpret each other's work. Decisions must be made with good science. Strom and MacLellan (1999, 2001) have provided an excellent review of many of the methods proposed in the literature for defining decision levels, in addition to those reviewed elsewhere (Currie 1968, 1984; Brodsky 1986, 1992). They have also proposed some modifications in defining decision level that are worthy of consideration. Yet, their use of the word "wrong" is strong; they have not credited the ANSI working group with their published recognition of the approximations involved in recommending the simple DL and MDA formulas given above. Readers of their paper could get the impression that the ANSI working group was totally incompetent in arriving at the entire standard. Thus, they may be the targets for a few further darts (Brodsky 1993). Further, their results showing that the Type I error they calculate, alpha-prime, is so deviated from the 0.05 level when they calculate DL for 0.05, is based on the assumption that a single measurement of the blank is used as the best estimate of the true mean in calculating DL. They have not referenced my paper (Brodsky 1992), where I calculated exact distributions of the differences in Poisson variates to show, assuming that preliminary laboratory quality assurance procedures required knowledge of blank counts and their stability, that for practical purposes the simple DL and MDA formulations are not so bad over most of the low background range. Potter (1999, 2001) has shown my calculation to be correct, using a theorem involving modified Bessel functions to check my algorithms, and also to propose other ways of determining DL and reducing the chance of false positives. For purely Poisson fluctuations, there can not be formulations that give exact alpha or beta probabilities at very low counts; extra-Poisson variance introduced by fluctuations in calibration factors, yields, or other continuous variables, is necessary to remove the discrete nature of net Poisson counts (Currie 1984; Brodsky 1992; Tries 1997).

Miller et al. (2000) have used the work "weak" to describe the ANSI DL, suggesting instead definitions based on Bayesian formalisms. However, for a well-defined prior blank distribution, which is desirable (when attainable) before a laboratory procedure is placed in use for analyzing human samples, it is not appropriate to assume a prior, as it might be in evaluating actual human exposures *a posteriori* (Brodsky 2001a). Also, as pointed out by Martz (2000, page 66), one of the disadvantages of the Bayesian approach for our purpose is that a practitioner may choose a prior distribution (rather than use a confirmed

one) that is "…inappropriate (at best) or self-serving (at worst)." This kind of unfair choice to make exaggerated claims of detection capability is what we want to avoid in revising the ANSI N13.30 standard.

Other ways of reducing in practice the chance of reporting excessive false positives in routine bioassay analyses would seem more appropriate (MacLellan 2000, Strom and MacLellan 2001). For example, Strom and MacLellan (2001) have suggested adding a constant to some formulations. MacLellan (2000) has suggested using twice the total propagated error. Another way could be to define the DL as 0.5 MDA, since the limit of MDA at zero counts is 3/KT; this would be consistent with the DL/MDA relationship at high counts, as shown by Currie (1968). I would be happy and remain friends with anyone who might suggest formulations that would not increase the chance of false claims of detection capability. We adopted in the previous ANSI working group a detection level of more than 4.65 times the standard deviation of a blank by using Currie's approach, when some laboratories were still using two or three times the standard deviation to define detection limits; it is professionally hazardous to make claims on such bases.

AN EXAMPLE OF DEFINING DL AND MDA IN PRACTICE

Just one example is presented of the importance of deciding what population comparison is appropriate for defining DL and MDA. Consider the distribution in Figure 1 of plutonium in a single day's urine among persons exposed only to global fallout (Brodsky 2000; Barss et al. 2001). The 95 percentile of 85 aCi could be used as a DL for another individual's sample to (tentatively) be considered above the population distribution. Figure 2 shows the uncertainty distribution in determining single sample amounts, as derived for a fission track analysis (FTA) procedure in which it was found that errors were not normally distributed; rather, the ratio of an interpretation divided by the proper regression estimate was found to be lognormal (Klemm et al. 2001; Brodsky et al. 2001b). Examination of Figure 2 shows that an MDA in urine, by the latter FTA procedure, would need to be about 300 aCi median before it could be assured that 95% of the interpretations would be above 85 aCi. Thus, when using here one of the lowest detection methods for plutonium, we completely avoid the problem of erroneous alpha levels; and we can avoid excess false positives (excessive administratively and in regard to risk) by lowering the selected alpha level further as desired.

Food for thought. Let us deliberate among friends. And let us celebrate with our great friends and colleagues 50 years of such deliberation since Jack Schubert (who established the first bioassay program at Argonne and turned it over to the great Jake Sedlet) introduced the subject of radiobioassay with his three seminal review papers (Schubert 1951).

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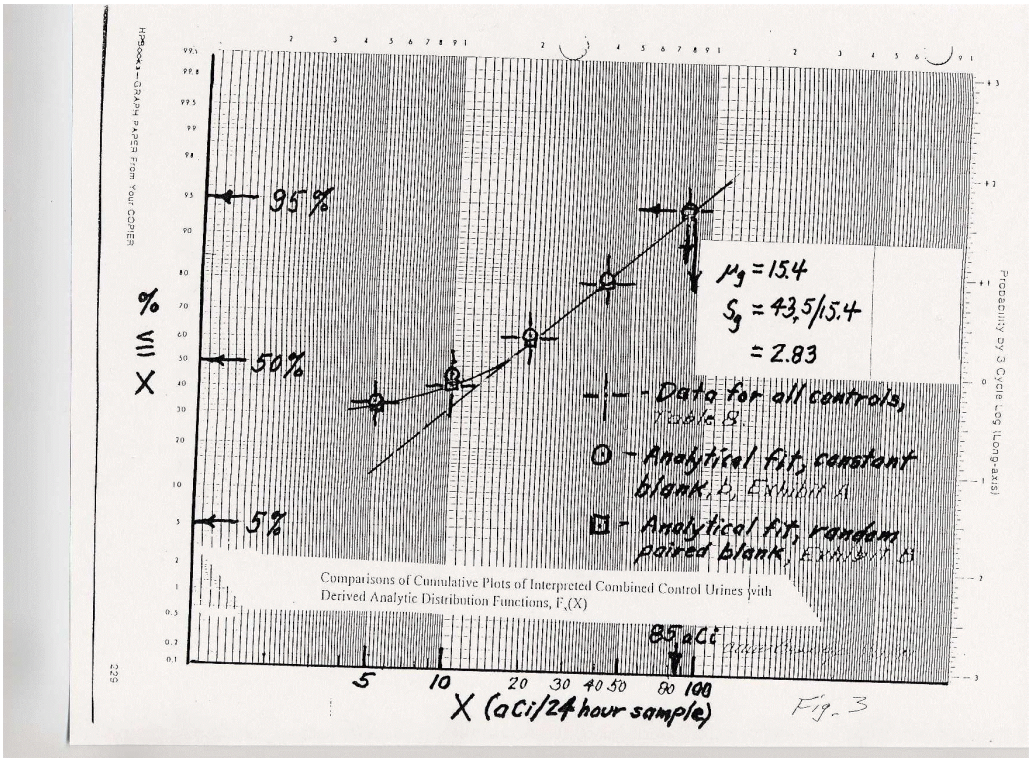

Figure 1 – U. S. Population Urinary Excretion of Pu (Brodsky 2000; Barss et al. 2001).
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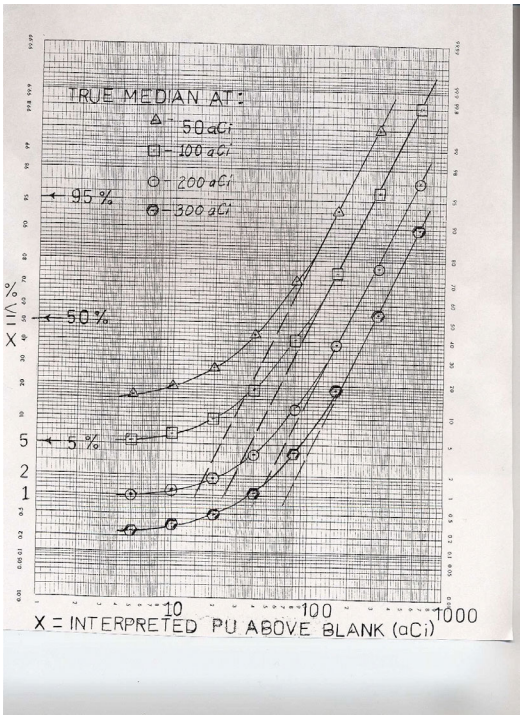

Figure 2 – Interpretation Distributions at Several True Levels (Klemm et al. 2001; Brodsky et al. 2001).

Exact Critical Levels and Detection Limits for Paired Counting

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Abstract

Utilizing a known relationship between the probability distribution for the net count in paired counting and modified Bessel functions of integral order it is possible to compute exact critical levels and detection limits together with the associated errors of the first and second kinds. An alternative equation for the critical levels for paired counting, when the error of the first kind is 0.05, has been developed. This equation is as follows:

$$Lca = floor(1.414*1.645*sqrt(B) + 0.457).$$

In the above equation the floor function yields the largest integer not greater than it's argument and B is the background count. The results are outstanding as will be demonstrated in this article.

INTRODUCTION

If x and y are Poisson distributed random variables with expectations a and b, respectively, then the probability of observing k net counts can be expressed as follows (Kellam 1946):

$$P(x-y = k) = \exp(-a-b) (\sqrt{a/b})^k I_{|k|}(2\sqrt{ab}).$$

In the above equation I_k is the modified Bessel function of the real variable (2sqrt(ab)) and of integral order k. The function I_k is defined by a power series (Abramowitz and Stegun 1972) and $|k|$ denotes the absolute value of k. A proof of the above equation, from the work of Kellam (1946), is presented in the Appendix. If the background count is not greater than 100 and the errors of the first and second kinds are not less than 0.001, then for the purpose of calculating exact critical levels and detection limits, I_k can be adequately computed using 250 terms in the power series. This fact can be exhibited by looking at the residuals in the well-known recursion relation for the modified Bessel functions of integral order (Abramowitz and Stegun 1972). A function written in C++ to compute I_k is given by Potter (1999).

CRITICAL LEVELS AND DETECTION LIMITS

If the net count is greater than the critical level it is concluded that activity was detected. The probability of concluding there is activity when there is no activity is called the error of the first kind. For discrete random variables, it is usually not possible to determine critical levels yielding errors of the first kind equal to a specified value. Brodsky (1992) and Currie (1984) choose the actual error of the first kind to be equal to or less than a specified error of the first kind. This article follows that approach. (It is possible to use random numbers and make the average error of the first kind, for a specified background count, equal to a specified error of the first kind.) The error of the second kind is the probability of concluding there is no activity on a sample when there is activity present. Because the expected value for the gross count is not restricted to integral values, it is possible to have the error of the second kind equal to a

predetermined value. For the computations of this article, the detection limit is that value, with two decimal places, that yields an error of the second kind closest in absolute value to the specified error of the second kind. In a straightforward manner exact critical levels and detection limits can be determined for paired counting. An article by Potter (2000) further discusses exact results for paired counting.

BRODSKY FORMULATION

The Brodsky formulation (Brodsky 1992) calculates critical levels and the detection limits from the equations below:

$$Lcb = \text{floor}(1.414*1.645\text{sqrt}(B)), \quad Ldb = 3.0 + 4.65\text{sqrt}(B).$$

The quantity B is the background count and the floor function yields the largest integer not greater than it's argument. In this article Lc and Ld are the exact critical levels and detection limits, respectively; Err1 and Err2 are the values for the errors of the first and second kind associated with the exact values, respectively. Furthermore, Err1b and Err2b are the errors of the first and second kind associated with the Brodsky formulation, respectively. Table 1 compares the Brodsky formulation with the exact computation. It is noted that Lcb lags Lc causing errors of the first kind greater than 0.05. It is also noted, for identical values of B, Table 1 of Brodsky (1992) agrees with Table 1 of this article.

Table 1. Critical levels and detection limits for paired counting. The units of B, Lc, Lcb, Ld, and Ldb are counts. Err1, Err1b, Err2, and Err2b are probabilities.

B	Lc	Err1	Lcb	Err1b	Ld	Err2	Ldb	Err2b
0	0	0	0	0	3	0.05	2.996	0.05
0.1	1	0.004248	0	0.08653	4.88	0.04991	4.47	0.01539
0.15	1	0.008832	0	0.1212	4.94	0.05008	4.801	0.01292
0.2	1	0.01454	1	0.01454	5.01	0.04981	5.08	0.04725
0.4	1	0.04343	1	0.04343	5.24	0.05009	5.941	0.0303
0.5	2	0.009271	1	0.05921	6.8	0.05001	6.288	0.026
0.6	2	0.01383	1	0.07485	6.89	0.05005	6.602	0.02288
0.7	2	0.01905	1	0.08996	6.98	0.05002	6.89	0.02052
0.8	2	0.02478	2	0.02478	7.07	0.04992	7.159	0.0473
1	2	0.03724	2	0.03724	7.23	0.05011	7.65	0.03914
1.3	3	0.01608	2	0.0569	8.8	0.04992	8.302	0.03117
1.5	3	0.02211	2	0.06989	8.93	0.05011	8.695	0.02749
1.6	3	0.02532	2	0.07623	9	0.05002	8.882	0.02597
1.7	3	0.02864	3	0.02864	9.07	0.04991	9.063	0.05009
2	3	0.039	3	0.039	9.26	0.04994	9.576	0.04269
2.2	3	0.04607	3	0.04607	9.38	0.05001	9.897	0.03883
2.3	3	0.04962	3	0.04962	9.44	0.05002	10.05	0.03715
2.4	4	0.02031	3	0.05317	10.76	0.05006	10.2	0.03561
2.5	4	0.02228	3	0.05669	10.82	0.04996	10.35	0.03419
2.7	4	0.02634	3	0.06368	10.93	0.04994	10.64	0.03165
2.9	4	0.03053	3	0.07054	11.03	0.0501	10.92	0.02947
3	4	0.03267	4	0.03267	11.09	0.04993	11.05	0.05073
3.5	4	0.04358	4	0.04358	11.34	0.05005	11.7	0.04292
3.7	4	0.04798	4	0.04798	11.44	0.05002	11.94	0.0404
3.8	5	0.02308	4	0.05017	12.71	0.05	12.06	0.03925
4	5	0.02586	4	0.05455	12.8	0.05005	12.3	0.03711
4.2	5	0.0287	4	0.05889	12.89	0.05008	12.53	0.03519
4.4	5	0.03159	4	0.06319	12.98	0.05008	12.75	0.03345

4.7	5	0.03599	5	0.03599	13.12	0.04993	13.08	0.05069
5	5	0.04044	5	0.04044	13.25	0.04992	13.4	0.04718

ALTERNATIVE FORMULA FOR CRITICAL LEVELS

The alternative formulation of this paper calculates critical levels and detection limits from the following two equations:

$$Lca = floor(1.414*1.645sqrt(B) + 0.457), \quad Lda = 3.0 + 4.65sqrt(B).$$

Furthermore, Err1a and Err2a are the errors of first and second kind associated with the alternative formulation, respectively. Table 2 compares the alternative formulation with the exact computation. From the tables values for Err2a are closer to 0.05 than are the values for Err2b. The only instance, In Table 2, for Lca to differ from Lc occurs when B = 3.8. Further investigations support the quality of the alternative approximation.

Table 2. Critical levels and detection limits for paired counting. The units of B, Lc, Lca, Ld, and Lda are counts. Err1, Err1b, Err2, and Err2b are probabilities.

B	Lc	Err1	Lca	Err1a	Ld	Err2	Lda	Err2a
0	0	0	0	0	3	0.05	2.996	0.05
0.1	1	0.004248	1	0.004248	4.88	0.04991	4.47	0.06876
0.15	1	0.008832	1	0.008832	4.94	0.05008	4.801	0.05574
0.2	1	0.01454	1	0.01454	5.01	0.04981	5.08	0.04725
0.4	1	0.04343	1	0.04343	5.24	0.05009	5.941	0.0303
0.5	2	0.009271	2	0.009271	6.8	0.05001	6.288	0.0691
0.6	2	0.01383	2	0.01383	6.89	0.05005	6.602	0.05988
0.7	2	0.01905	2	0.01905	6.98	0.05002	6.89	0.05284
0.8	2	0.02478	2	0.02478	7.07	0.04992	7.159	0.0473
1	2	0.03724	2	0.03724	7.23	0.05011	7.65	0.03914
1.3	3	0.01608	3	0.01608	8.8	0.04992	8.302	0.06489
1.5	3	0.02211	3	0.02211	8.93	0.05011	8.695	0.05658
1.6	3	0.02532	3	0.02532	9	0.05002	8.882	0.05315
1.7	3	0.02864	3	0.02864	9.07	0.04991	9.063	0.05009
2	3	0.039	3	0.039	9.26	0.04994	9.576	0.04269
2.2	3	0.04607	3	0.04607	9.38	0.05001	9.897	0.03883
2.3	3	0.04962	3	0.04962	9.44	0.05002	10.05	0.03715
2.4	4	0.02031	4	0.02031	10.76	0.05006	10.2	0.06443
2.5	4	0.02228	4	0.02228	10.82	0.04996	10.35	0.06169
2.7	4	0.02634	4	0.02634	10.93	0.04994	10.64	0.05682
2.9	4	0.03053	4	0.03053	11.03	0.0501	10.92	0.05262
3	4	0.03267	4	0.03267	11.09	0.04993	11.05	0.05073
3.5	4	0.04358	4	0.04358	11.34	0.05005	11.7	0.04292
3.7	4	0.04798	4	0.04798	11.44	0.05002	11.94	0.0404
3.8	5	0.02308	4	0.05017	12.71	0.05	12.06	0.03925
4	5	0.02586	5	0.02586	12.8	0.05005	12.3	0.06104
4.2	5	0.0287	5	0.0287	12.89	0.05008	12.53	0.05771
4.4	5	0.03159	5	0.03159	12.98	0.05008	12.75	0.0547
4.7	5	0.03599	5	0.03599	13.12	0.04993	13.08	0.05069
5	5	0.04044	5	0.04044	13.25	0.04992	13.4	0.04718

Why does this alternative formulation yield improved results? The alternative formulation is a semi-empirical formulation that attempts to account for the discrete nature of the fundamental probability distribution by utilizing half-integer corrections (Meyer 1975). (Based on half-integer corrections, Lca equals floor(1.414*1.645sqrt(B) + 0.50).) Then a slight perturbation to that approximation, using numerical experimentation, improves the closeness of the match of Lca to Lc. The equation for Lca can be shown to give fine results when B is incremented in increments of 0.01.

Using a similar approach, with a factor different than 0.457, good results are obtained when the errors of the first and second kind are 0.01.

CONCLUSION

An equation has been derived that expresses the probability distribution for the net count, for paired counting, in terms of modified Bessel functions of integral order. From this equation it is obvious that the probability distribution for the net count is symmetric. Results from a code that has the capability to compute exact critical levels and detection limits have been presented. Furthermore the exact results have been compared to the Brodsky formulation. If one looks at average values for the errors of the first and second kind over a range of background counts, then the Brodsky formulation does a good job.

The alternative formula has been shown to do a fine job of matching the exact results for increments of size 0.1 for B. It is asserted that the results for increments of size 0.01 for B are equally fine.

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APPENDIX

Below is a brief proof of the relationship between the probability distribution for the net count in paired counting and the modified Bessel functions of integral order. Take x and y to be Poisson distributed random variables with expectations a and b, respectively. The coefficient of t^r in the expansion of G1(t) = exp(a(t-1)) is the probability of r counts for a Poisson distributed random variable with expected value a. Similarly, the coefficient of t^{-r} in the expansion of G2(t) = exp(b(1/t –1)) is the probability of r counts for a Poisson distributed random variable with expected value b. It then follows that the coefficient of t^r in G(t) is the probability that the net count is exactly r counts where G(t) = G1(t)G2(t). Making the substitutions a = zφ/2 and b =z/2φ, G(t) can then then can be written as

$$G(t) = \exp\{-z/2(\varphi + 1/\varphi) + z/2(\varphi t + 1/\varphi t)\}.$$

The following equation is known to be true (Abramowitz and Stegun 1972):

$$\exp(z/2(t + 1/t)) = \sum_{-\infty}^{\infty} t^n I_n(z).$$

In the above equation $I_n(z)$ is the modified Bessel function of integral order n with argument z Consequently G(t) becomes

$$\exp\{-z/2(\varphi + 1/\varphi)\} \sum_{-\infty}^{\infty} \varphi^r t^r I_r(z).$$

It is concluded that the coefficient of t^r is

$$\exp\{-z/2(\varphi + 1/\varphi)\}\varphi^r I_r(z) = \exp(-a-b)(\mathrm{sqrt}(a/b))^r I_{|r|}(2\mathrm{sqrt}(ab)).$$

In the last equation the identity $I_r = I_{-r}$ was utilized (Abramowitz and Stegun 1972).