Exact Likelihood Functions for Measurements Involving Counting

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Many health physics measurements involve counting, usually in conjunction with a background count, which is subtracted in some way. The likelihood function in these cases then involves Poisson distributions (for long decay times). In most analyses this fact is not used in a detailed way. For example the formulas given in Ref.[1] are essentially those obtained using the Gaussian approximation to the Poisson distribution (Ref. [2]} argues that this is valid). Recently, the Gaussian approximation has been challenged in low-level counting situations, and various non-Gaussian formulas for the decision level have been proposed.[3] Other treatments of the problem of low-level paired counting have been given by Little[4] and Potter.[5]

The calibration or normalization factor has important uncertainties in addition to the counting statistics uncertainties. In some cases these uncertainties are known to follow a log-normal distribution (see, for example, Ref. [6] and this would always seem a reasonable assumption. In this paper we assume the calibration factor has an arbitrarily large uncertainty that follows a log-normal distribution.

Use of exact likelihood functions as options in our Bayesian internal dosimetry codes has been implemented using an interpolation-table approach. This means that the exact likelihood functions can be used with no computation time penalty except for the initial setup of the interpolation tables. In cases with only a few measurements involving low-level counts, we find that it is sometimes important to make use of exact likelihood functions rather than the Gaussian approximation in calculating the posterior distribution.

This paper extends the work of Little[7] by incorporating a log-normal distribution of calibration factor, studying the differences between the exact calculation and the Gaussian approximation for priors justified by an Empirical Bayes analysis,[8] and developing the interpolation-table computational technique. We also describe an empirical

Bayesian method for determining the prior probability distribution of background count rates that seems to have important practical advantages.

This BAER poster abstract consists of the introduction to a complete paper that has been submitted for publication (and is available on our website, www.lanl.gov/bayesian)

References

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